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HOMOGENIZED THERMOCONSOLIDATION WITH MICROLOCAL PARAMETERS OF PERIODIC STRATIFIED FLUID-SATURATED POROUS SOLIDS

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The paper deals with modelling problems of periodic stratified fluid-saturated porous thermoelastic bodies. By using the homogenization method with microlocal parameters and the linear theory of thermoconsolidation homogenized models accounting certain local effects of stresses, heat fluxes are derived. An example of application of the obtained model is presented.

1. Introduction

The thermomechanical behaviour of a porous elastic medium containig fluid has attracted the attention of researchers in such fields as soil mechanics, ground water hydrology, geophysics, seismology, the theory of filtration and purification, the study of sound-absorbing materials, biomechanics, and so on. The list of papers on fluid-saturated porous body is rather extensive and will not be wholly discussed here. The problem of thermomechanical coupling in the theory of consolidation was presented by Schiffman (1970), Deresiewicz, Pecker (1973), Kończak (1975), Derski, Kowalski (1978), Derski (1979), where the basis constitutes the theory of consolidation given by Biot (1956), (1962) and Biot, Willis (1957). The equations of thermoconsolidation were derived under traditional assumptions made in the consolidation theory and an ideal heat exchange occuring between a skeleton and liquid being the components of porous solids.

The purpose of this paper is to derive from equations of the linear theory of thermoconsolidation a certain class of homogenized models of periodically stratified fluid-saturated porous solids. The basic unit (fundamental layer) is assumed to be composed of $(n+1)$-different porous layers consisting of a homogeneous, isotropic, elastic matrix whose interstices are filled with a compressible viscous liquid. Perfect bonding and perfect thermal contact between the layers are assumed. The considerations are based on the linear theory of thermoconsolidation presented by Derski and Kowalski (1978), Derski (1979). To obtain the homogenized model of stratified bodies the homogenization procedure established by Woźniak (1986), (1987), Matysiak and Woźniak (1987) is employed. The approach is based on some concepts of the nonstandard analysis combined with some a priori postulated physical assumptions. Application of the homogenization procedure leads to equations given in terms of unknown macrodisplacements of skeleton and fluid, macrotemperature as well as some extra unknowns called kinematical microlocal parameters of skeleton and fluid and thermal microlocal parameters of the aggregate. The main feature of the homogenized model is a possibility of modelling not only mean but also local values of deformation gradients, strains, stresses in every material components and heat fluxes in the periodically stratified fluid-saturated porous solids. The homogenized models of periodic stratified porous solids (without taking into account thermal effects) was derived by Matysiak (1992).

The presented models can be applied to some problems of rock and soil thermomechanics (to description of sandstone-slote, sandstone-shale, shale, thin-layered limestone, varved clays, flotation wastes accumulated in storage ponds). The possibility of application of the homogenized models to a geologic strata is discussed by Kaczyński and Matysiak (1993).

2. Preliminaries

Consider a fluid-saturated porous elastic body which occupies a regular region B in the Euclidean 3-space reffered to a fixed Cartesian coordinate system $\boldsymbol{x} = (x_1, x_2, x_3)$. The nonhomogeneous body in a natural (undeformed) configuration is composed of periodically repeated $(n+1)$-different fluid-saturated porous elastic layers (see Fig.1). Let $h_1, ..., h_{n+1}$ be the layer thicknesses, and δ be the thickness of each basic unit of the body, so $\delta = h_1 + ... + h_{n+1}$. The axis x_2 is assumed to be normal to the layering. Perfect bonding and perfect contact between the layers are assumed. Let $\bar{\rho}$ denote the density of free fluid and $\rho^{(r)}$, $r = 1, ..., n+1$, be the densities of skeletons and non-free fluid of the subsequent layers, respectively. By $K^{(r)}$, $L^{(r)}$, $M^{(r)}$, $N^{(r)}$, $r = 1, ..., n+1$, we denote material constants of the consolidating layers (modulae of skeleton volumetric deformations, coupling

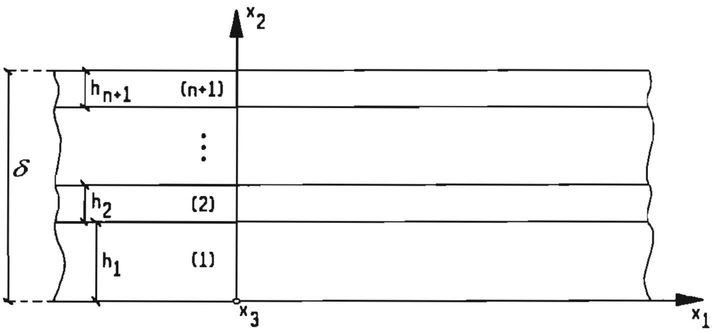

Fig. 1. In the rth layer: $\bar{\rho}$, ρ^r, N^r, M^r, K^r, L^r, b^r, γ^r, $\bar{\gamma}^r$, λ^r, c^r; $r = 1,...,n+1$

coefficients of skeletons and fluid volumetric deformations, modulae of fluid volumetric deformations, shear modulae of the skeletons, respectively). Let $b^{(r)}, r = 1,...,n+1$, be the dissipation coefficients and $\lambda^{(r)}, r = 1,...,n+1$, be the heat conductivity coefficients of the subsequent layers. By $\gamma^{(r)}$ and $\bar{\gamma}^{(r)}$ we denote

$$\begin{aligned}\gamma^{(r)} &= (2N^{(r)} + 3K^{(r)})\alpha^{(r)} + 3L^{(r)}\bar{\alpha}\\ \bar{\gamma}^{(r)} &= 3(L^{(r)}\alpha^{(r)} + M^{(r)}\bar{\alpha})\end{aligned}$$

where $\alpha^{(r)}$, $r = 1,...,n+1$, and $\bar{\alpha}$ are the coefficients of the linear expansion of the skeletons and fluid, respectively. Let t, $t \in [t_0, t_1)$ denote time, $\boldsymbol{u}(\boldsymbol{x},t) \equiv (u_1,u_2,u_3)(\boldsymbol{x},t)$ be the displacement vector of the skeleton, $\boldsymbol{U}(\boldsymbol{x},t) \equiv (U_1,U_2,U_3)(\boldsymbol{x},t)$ be the displacement vector of the fluid and $\vartheta \equiv \vartheta(\boldsymbol{x},t)$ be the relative temperature.

Following Derski and Kowalski (1978), Derski (1979) the constitutive relations of the linear theory of thermoconsolidation take the form[1]

$$\begin{aligned}\sigma_{ij}^{(r)} &= 2N^{(r)}e_{ij} + (K^{(r)}e_{kk} + L^{(r)}U_{k,k} - \gamma^{(r)}\vartheta)\delta_{ij}\\ \sigma^{(r)} &= L^{(r)}e_{kk} + M^{(r)}U_{k,k} - \bar{\gamma}^{(r)}\vartheta\end{aligned} \tag{2.1}$$

where

[1]Throughout the paper indices i,j,k run over $1,2,3$ and are related to the spatial coordinates. Summation convection holds with respect to all repeated indices and $f_{,i} \equiv \partial f/\partial x_i$

$$
\begin{aligned}
e_{ij} &= \frac{1}{2}(u_{i,j} + u_{j,i}) \\
\delta_{ij} &= \begin{cases} 1 & \text{for} \quad i = j \\ 0 & \text{for} \quad i \neq j \end{cases}
\end{aligned}
\tag{2.2}
$$

and $\sigma_{ij}^{(r)}$ are components of stress tensor of the skeleton in a layer of the rth kind referring to the total unit surface, $\sigma^{(r)}$ are stresses in fluid in a layer of the rth kind (fluid pressures inside the pores).

According to the results of Derski and Kowalski (1978), Derski (1979) equations of the linear theory of thermoconsolidation accounting for dissipation have the following form

$$
\begin{aligned}
&\sigma_{ij}^{(r)},_j + \sigma^{(r)},_i + \rho^{(r)} X_i = \rho^{(r)} u_{i,tt} + b^{(r)}(u_{i,t} - U_{i,t}) \\
&\sigma^{(r)},_i + \bar{\rho} X_i = \bar{\rho} U_{i,tt} - b^{(r)}(u_{i,t} - U_{i,t}) \qquad r = 1, ..., n+1
\end{aligned}
\tag{2.3}
$$

The heat conduction equation in the case of theory of thermal stresses can be written

$$
\lambda^{(r)} \vartheta_{,ii} - c^{(r)} \vartheta_{,t} = -W_0 \tag{2.4}
$$

where $c^{(r)}$, $r = 1, ..., n+1$, are specific heats of the aggregates of the subsequent layers (at the constant deformation referring to the unit volume) and W_0 is the heat source function.

The assumption of the perfect bonding and perfect thermal contact between the layers implies the continuity of the displacement vectors of skeletons and fluids, temparature as well as the stress vectors of skeletons and fluid pressure, and heat flux vectors on the interfaces (planes between the layers).

Eqs (2.3) and (2.4) can be expressed in the following integral forms

$$
\begin{aligned}
&\sum_{r=1}^{n+1} \int_{B_r} \Big[\sigma_{ij}^{(r)} v_{i,j} + \sigma^{(r)} v_{i,i} - \rho^{(r)}(X_i - u_{i,tt}) v_i + b^{(r)}(u_{i,t} - U_{i,t}) v_i\Big] \, dB = 0 \\
&\sum_{r=1}^{n+1} \int_{B_r} \Big[\sigma^{(r)} v_{j,j} - \bar{\rho}(X_j - U_{j,tt}) v_j - b^{(r)}(u_{j,t} - U_{j,t}) v_j\Big] \, dB = 0 \\
&\sum_{r=1}^{n+1} \int_{B_r} \Big[\lambda^{(r)} \vartheta_{,i} v_{,i} + c^{(r)} \vartheta_{,t} v - W_0 v\Big] \, dB = 0
\end{aligned}
\tag{2.5}
$$

for each test functions $v_i(\cdot)$, $v(\cdot)$ such that $v_i(x) \in C^1(\bar{B})$, $v(x) \in C^1(\bar{B})$ and $v_i(\cdot)\Big|_{\partial B} = 0$, $v(\cdot)\Big|_{\partial B} = 0$ and where B_r, $r = 1,...,n+1$, denote the part of the region B occupied by the material of the rth kind (see Fig.1).

Since the body is assumed to be periodic the material coefficients are δ-periodic functions taking constant values in the subsequent layers of the body.

3. Homogenized model with microlocal parameters

To obtain a homogenized model of the periodic stratified fluid-saturated porous thermoelastic solids described in Section 2 the approach called the microlocal modelling is applied. This method is based on concepts of the non-standard analysis combined with some postulated a priori heuristic physical assumptions and has been presented by Woźniak (1986), (1987) for periodic thermoelastic composites. Making appeal to the microlocal modelling method, where the exact explanation of the homogenization procedure in terms of the nonstandard analysis is given) we shall derive equations of a homogenized model omitting the presentation of mathematical assumptions and detailed calculations. Similarly to the papers of Woźniak (1987), Matysiak and Woźniak (1987), an approximate solutions of Eqs (2.5) and (2.3) are assumed in the form

$$
\begin{aligned}
u_i(x,t) &= w_i(x,t) + \underline{l_s(x_2)q_{si}(x,t)} \\
U_i(x,t) &= W_i(x,t) + \underline{l_s(x_2)Q_{si}(x,t)} \\
\vartheta(x,t) &= \theta(x,t) + \underline{l_s(x_2)T_s(x,t)} \qquad i = 1,2,3 \qquad s = 1,...,n
\end{aligned}
\tag{3.1}
$$

where $l_s(\cdot) : \mathcal{R} \to \mathcal{R}$, $s = 1,...,n$, are the known *a priori* δ-periodic functions, called the shape functions (cf Woźniak, 1987), given by

$$
l_s(x_2) = \begin{cases} x_2 - \frac{1}{2}\delta_s & \text{for} \quad 0 \le x_2 \le \delta_s \\ \dfrac{\delta_s(x_2-\delta)}{\delta_s - \delta} - \frac{1}{2}\delta_s & \text{for} \quad \delta_s \le x_2 \le \delta \end{cases}
$$

$$
\delta_s \equiv h_1 + ... + h_s \qquad s = 1,...,n \tag{3.2}
$$

$$
\delta = h_1 + ... + h_{n+1} = \delta_{n+1}
$$

The functions $w_i(\cdot)$, $W_i(\cdot)$ and $\theta(\cdot)$ are unknown functions interpreted as the components of macrodisplacement vectors of the skeleton and fluid and macrotemperature, respectively. The functions $q_{si}(\cdot)$, $Q_{si}(\cdot)$ stand for the kinematical microlocal parameters at the skeleton and fluid and $T_s(\cdot)$ stand for the thermal microlocal parameters of the body and they are related with the microperiodic structure of the body.

Since $|l_s(x_2)| < \delta$ for every $x_2 \in \mathcal{R}$, then for small δ the underlined terms in equations (3.1) are small and will be neglected (see for an exact explanation in terms of the nonstandard analysis to Woźniak (1986), (1987)). It has been emphasized that $l_{s,2}$, $s = 1,...,n$ are not small and the terms involving l_s, cannot be neglected. So, we have

$$\begin{array}{lll} u_{i,\alpha} \approx w_{i,\alpha} & u_{i,2} \approx w_{i,2} + l_{s,2}\, q_{si} & u_{i,t} \approx w_{i,t} \\ U_{i,\alpha} \approx W_{i,\alpha} & U_{i,2} \approx W_{i,2} + l_{s,2}\, Q_{si} & U_{i,t} \approx W_{i,t} \\ \vartheta_{,\alpha} \approx \theta_{,\alpha} & \vartheta_{,2} \approx \theta_2 + l_{s,2} T_s & \vartheta_{,t} \approx \theta_{,t} \end{array} \tag{3.3}$$

Taking into account the tested functions in the form

$$\begin{aligned} v_i(\boldsymbol{x},t) &= g_i(\boldsymbol{x},t) + \underline{l_p(x_2)G_{pi}(\boldsymbol{x},t)} \\ v(x,t) &= g(\boldsymbol{x},t) + \underline{l_p(x_2)G_p(\boldsymbol{x},t)} \qquad p = 1,...,n \end{aligned} \tag{3.4}$$

and substituting Eqs (3.1) and (3.4) into Eqs (2.1) and (2.5) after some calculations similar to given by Woźniak (1987) we arrive at the following equations

$$\begin{aligned} &<N>w_{i,jj} + <N+K+L>w_{j,ji} + <M+L>W_{j,ji} + <Nl_{s,}>q_{si,j} + \\ &+ <Nl_{s,i}>q_{sj,j} + <(K+L)l_{s,j}>q_{sj,i} + <(M+L)l_{s,j}>Q_{sj,i} + \\ &- <\gamma+\bar{\gamma}>\theta_{,i} + <\rho>X_i = <\rho>w_{i,tt} + <b>(w_{i,t} - W_{i,t}) \\[1em] &<L>w_{k,kj} + <M>W_{k,kj} + <Ll_{s,k}>q_{sk,j} + <Ml_{s,k}>Q_{sk,j} + \\ &- <\bar{\gamma}>\theta_{,j} + \bar{\rho}X_j = \bar{\rho}W_{j,tt} - <b>(w_{j,t} - W_{j,t}) \end{aligned} \tag{3.5}$$

$$<\lambda>\theta_{,ii} + <\lambda l_{s,i}>T_{s,i} - <c>\theta_{,t} = -W_0$$

and

$$\begin{aligned} &<Nl_{s,j}l_{p,j}>q_{si} + <(K+L)l_{s,j}l_{p,i}>q_{sj} + <Nl_{s,i}l_{p,j}>q_{sj} + \\ &+ <(M+L)l_{s,j}l_{p,i}>Q_{sj} = - <Nl_{p,j}>(w_{i,j} + w_{j,i}) + \\ &- <(K+L)l_{p,i}>w_{k,k} - <(M+L)l_{p,i}>W_{k,k} + <(\gamma+\bar{\gamma})l_{p,i}>\theta \end{aligned}$$

$$<Ll_{s,k}l_{p,j}>q_{sk} + <Ml_{s,k}l_{p,j}>Q_{sk} = \tag{3.6}$$

$$= - < Ll_{p,j}> w_{k,k} - < Ml_{p,j}> W_{k,k} + <\bar{\gamma} l_{p,j}> \theta$$

$$< \lambda l_{p,i} l_{s,i}> T_s = - < \lambda l_{p,i}> \theta_{,i}$$
$$s, p = 1, ..., n \qquad i, j, k = 1, 2, 3$$

where the symbol $< f >$ denotes

$$< f >\equiv \frac{1}{\delta} \int\limits_0^\delta f(x_2)\, dx_2 \tag{3.7}$$

for any δ-periodic intergrable function $f(\cdot)$.

Using the formulae (3.7) and (3.2) for an arbitrary δ-periodic function $f(\cdot)$ taking a constant value for f_i in a layer of the ith kind, $i = 1, ..., n+1$, we obtain

$$\begin{aligned} < f > &= \sum_{i=1}^{n+1} f_i \eta_i \\ < f l_{s,2} > &= \sum_{i=1}^{s} f_i \eta_i - \alpha_s \sum_{i=s+1}^{n+1} f_i \eta_i \\ < f l_{s,2}\, l_{p,2} > &= \sum_{i=1}^{p} f_i \eta_i - \alpha_p \sum_{i=p+1}^{s} f_i \eta_i + \alpha_p \alpha_s \sum_{i=s+1}^{n+1} f_i \eta_i \\ p, s &= 1, ..., n \qquad p \leq s \end{aligned} \tag{3.8}$$

where

$$\eta_i \equiv \frac{\delta_i}{\delta} \qquad \alpha_i \equiv \frac{\eta_1 + ... + \eta_i}{1 - (\eta_1 + ... + \eta_i)} \qquad i = 1, ..., n \tag{3.9}$$

Employing Eqs (3.8) we can calculate all material modulae in Eqs (3.5) and (3.6) by substituting for a function $f(\cdot)$ the δ-periodic functions M, N, K, L, ρ, γ, $\bar{\gamma}$, b, λ, c.

To determine the stresses in a layer of the rth kind we substitute Eqs (3.3) into Eq (2.3). Thus, we have

$$\begin{aligned} \sigma_{ij}^{(r)} &= N^{(r)}(w_{i,j} + w_{j,i} + l_{s,j}\, q_{si} + l_{s,i}\, q_{sj}) + [K^{(r)}(w_{k,k} + l_{s,k}\, q_{sk}) + \\ &+ L^{(r)}(W_{k,k} + l_{s,k}\, Q_{sk}) - \gamma^{(r)}\theta]\delta_{ij} \\ \sigma^{(r)} &= L^{(r)}(w_{k,k} + l_{s,k}\, q_{sk}) + M^{(r)}(W_{k,k} + l_{s,k}\, Q_{sk}) - \bar{\gamma}^{(r)}\theta \end{aligned} \tag{3.10}$$

The components of heat flux vector $h^{(r)}$ in the layer of rth kind are given by

$$h^{(r)} = \lambda^{(r)}(\theta_{,i} + l_{s,i} T_s) \tag{3.11}$$

Remark

In Eqs (3.5) to (3.8) and (3.10), (3.11) we have

$$l_{s,1} \equiv 0 \qquad\qquad l_{s,3} \equiv 0 \tag{3.12}$$

Eqs (3.5), (3.6), (3.10) and (3.11) constitute the governing equations of the homogenized model of periodically stratified fluid-saturated porous termoelastic body. It can be easily observed that we have the system of $7n$ linear algebraic equations (see Eqs (3.6)) for the kinematical and thermal microlocal parameters q_{si}, Q_{si}, T_s, $s = 1, ..., n$, $i = 1,2,3$. The microlocal parameters can be eliminated from Eqs (3.5) by using (3.6) which leads to the system of seven linear partial differential equations of the second order with constant coefficients in macrodisplacements w_i, W_i and macrotemperature θ. These equations should be supplemented by appropriate boundary and initial conditions similar to that of the linear theory of thermoconsolidation.

4. Thermoconsolidation of periodic two-layered fluid-saturated porous solids

For the case of periodic two-layered fluid-saturated porous solids we obtain (assuming that $n = 1$ and using Eqs (3.8), (3.9) and (3.2))

$$\begin{aligned} < f >&= \eta_1 f_1 + (1 - \eta_1) f_2 \equiv \widetilde{f} \\ < f l_{1,2} >&= \eta_1 (f_1 - f_2) \equiv [f] \\ < f l_{1,2}^2 >&= \eta_1 f_1 + \frac{\eta_1^2 f_1}{1 - \eta_1} \equiv \widehat{f} \end{aligned} \tag{4.1}$$

where

$$\eta_1 = \frac{h_1}{\delta} \tag{4.2}$$

By substituting Eqs (4.1) into Eq (3.5) we have the following system of equations

$$
\begin{aligned}
&\widetilde{N} w_{i,jj} + (\widetilde{N} + \widetilde{K} + \widetilde{L}) w_{j,ji} + (\widetilde{M} + \widetilde{L}) W_{j,ji} + [N] q_{1i,2} + [N] q_{1j,j} \delta_{i2} + \\
&+ [K + L] q_{12,i} + ([M] + [L]) Q_{12,i} - (\widetilde{\gamma} + \widetilde{\bar{\gamma}}) \theta_{,i} + \widetilde{\rho} X_i = \widetilde{\rho} w_{i,tt} + \widetilde{b}(w_{i,t} - W_{i,t}) \\
&\widetilde{L} w_{k,kj} + \widetilde{M} W_{k,kj} + [L] q_{12,j} + [M] Q_{12,j} - \widetilde{\bar{\gamma}} \theta_{,j} + \bar{\rho} X_j = \bar{\rho} W_{j,tt} - \widetilde{b}(w_{j,t} - W_{j,t})
\end{aligned}
\qquad (4.3)
$$

$$
\widetilde{\lambda} \theta_{,ii} + [\lambda] T_{1,2} - \widetilde{c} \theta_{,t} = -W_0
$$

Combining Eqs (3.6) and (4.1) we obtain the formulae for kinematical and thermal microlocal parameters

$$
\begin{aligned}
&\widehat{N} q_{11} = -[N](w_{1,2} + w_{2,1}) \\
&(2\widehat{N} + \widehat{K} + \widehat{L}) q_{12} + (\widehat{M} + \widehat{L}) Q_{12} = -2[N] w_{2,2} - ([K] + [L]) w_{k,k} + \\
&\qquad - ([M] + [L]) W_{k,k} + ([\gamma] + [\bar{\gamma}]) \theta \\
&\widehat{N} q_{13} = -[N](w_{3,2} + w_{2,3}) \\
&\widehat{L} q_{12} + \widehat{M} Q_{12} = -[L] w_{k,k} - [M] W_{k,k} + [\bar{\gamma}] \theta \\
&\widehat{\lambda} T_1 = -[\lambda] \theta_{,2}
\end{aligned}
\qquad (4.4)
$$

To determine stresses $\sigma_{ij}^{(r)}$, fluid pressures $\sigma^{(r)}$ and heat fluxes $h^{(r)}$ in a layer of rth kind, $r = 1, 2$, we employ Eqs (3.10), (3.11), (4.1) and (3.2). Thus, we have

$$
\begin{aligned}
\sigma_{11}^{(r)} &= 2N^{(r)} w_{1,1} + K^{(r)}(w_{k,k} + l_{1,2} q_{12}) + L^{(r)}(W_{k,k} + l_{1,2} Q_{12}) - \gamma^{(r)} \theta \\
\sigma_{12}^{(r)} &= N^{(r)}(w_{1,2} + w_{2,1} + l_{1,2} q_{11}) \\
\sigma_{13}^{(r)} &= N^{(r)}(w_{1,3} + w_{3,1}) \\
\sigma_{22}^{(r)} &= 2N^{(r)}(w_{2,2} + l_{1,2} q_{12}) + K^{(r)}(w_{k,k} + l_{1,2} q_{12}) + \\
&\quad + L^{(r)}(W_{k,k} + l_{1,2} Q_{12}) - \gamma^{(r)} \theta \\
\sigma_{23}^{(r)} &= N^{(r)}(w_{2,3} + w_{3,2} + l_{1,2} q_{13}) \\
\sigma_{33}^{(r)} &= 2N^{(r)} w_{3,3} + K^{(r)}(w_{k,k} + l_{1,2} q_{12}) + L^{(r)}(W_{k,k} + l_{1,2} Q_{12}) - \gamma^{(r)} \theta \\
\sigma^{(r)} &= L^{(r)}(w_{k,k} + l_{1,2} q_{12}) + M^{(r)}(W_{k,k} + l_{1,2} Q_{12}) - \bar{\gamma}^{(r)} \theta \\
h^{(r)} &= \lambda^{(r)}(\theta_{,i} + l_{1,2} T_1)
\end{aligned}
\qquad (4.5)
$$

where

$$
l_{1,2} = \begin{cases} 1 & \text{for} \quad r = 1 \\ -\dfrac{\eta_1}{1 - \eta_1} & \text{for} \quad r = 2 \end{cases}
\qquad (4.6)
$$

and $q_{11}, q_{12}, q_{13}, Q_{12}, T_1$ are given by Eqs (4.4).

5. Example

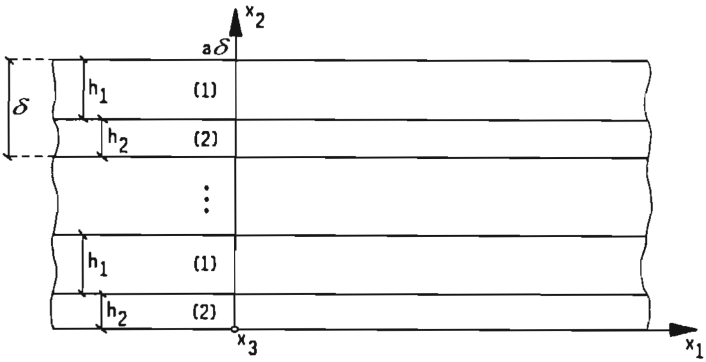

Fig. 2. In the rth layer: $\bar{\rho}$, ρ^r, N^r, M^r, K^r, L^r, b^r, γ^r, $\bar{\gamma}^r$, λ^r, c^r; $r = 1,2$

Consider now a periodic two-layered fluid-stratified layer resting on the rigid impermeable subsoil (see Fig.2). Let $a\delta$ be the thickness of the stratified layer, where a is a sufficiently large natural number. We assume that no heat sources are present inside the layer, and that the body forces are omitted. Let the upper and lower planes of layer be subjected to constant temperatures θ_1 and θ_0, respectively, as well as the upper one is free of tractions. Thus, the considered problem is a stationary and one-dimensional. The displacement vectors of the skeleton, fluid and temperature can be taken in the form

$$\begin{aligned} \boldsymbol{u}(x_2) &= (0, u_2(x_2), 0) \\ \boldsymbol{U}(x_2) &= (0, U_2(x_2), 0) \\ \theta &= \theta(x_2) \end{aligned} \tag{5.1}$$

Substituting Eqs (5.1) into Eqs (4.4) we obtain

$$\begin{aligned} q_{11} &= 0 & q_{12} &= d_1 w_{2,2} + d_2 W_{2,2} + d_3\theta \\ q_{13} &= 0 & Q_{12} &= d_4 w_{2,2} + d_5 W_{2,2} + d_6\theta \\ T_1 &= -\frac{[\lambda]\theta_{,2}}{\widehat{\lambda}} \end{aligned} \tag{5.2}$$

where

$$d_1 = \frac{-2[N]\widehat{M} - [K]\widehat{M} + [L]\widehat{L}}{A} \qquad d_2 = \frac{-[L]\widehat{M} + [M]\widehat{L}}{A}$$

$$d_3 = \frac{[\gamma]\widehat{M} - [\bar{\gamma}]\widehat{L}}{A} \qquad d_4 = \frac{-2\widehat{N}[L] - \widehat{K}[L] + 2[N]\widehat{L} + [K]\widehat{L}}{A} \tag{5.3}$$

$$d_5 = \frac{-2\widehat{N}[M] - \widehat{K}[M] + \widehat{L}[L]}{A} \qquad d_6 = \frac{2\widehat{N}[\bar{\gamma}] + \widehat{K}[\bar{\gamma}] - \widehat{L}[\gamma]}{A}$$

$$A = 2\widehat{N}\widehat{M} + \widehat{K}\widehat{M} - \widehat{L}^2$$

Eqs (5.1), (5.2) and (4.3) yield

$$\begin{aligned} &A_1 w_{2,22} + A_2 W_{2,22} + A_3 \theta_{,2} = 0 \\ &B_1 w_{2,22} + B_2 W_{2,22} + B_3 \theta_{,2} = 0 \\ &\theta_{,22} = 0 \end{aligned} \tag{5.4}$$

where

$$\begin{aligned} A_1 &= 2\widetilde{N} + \widetilde{K} + \widetilde{L} + (2[N] + [K] + [L])d_1 + ([M] + [L])d_4 \\ A_2 &= \widetilde{M} + \widetilde{L} + (2[N] + [K] + [L])d_2 + ([M] + [L])d_5 \\ A_3 &= -(\widetilde{\gamma} + \widetilde{\bar{\gamma}}) + (2[N] + [K] + [L])d_3 + ([M] + [L])d_6 \\ B_1 &= \widetilde{L} + [L]d_1 + [M]d_4 \\ B_2 &= \widetilde{M} + [L]d_2 + [M]d_5 \\ B_3 &= -\widetilde{\bar{\gamma}} + [L]d_3 + [M]d_6 \end{aligned} \tag{5.5}$$

Since $A_1B_2 - A_2B_1 \neq 0$ then from Eqs (5.4) it follows that

$$w_{2,22} = f_1\theta_{,2} \qquad W_{2,22} = f_2\theta_{,2}$$
$$\theta(x_2) = n_1x_2 + n_2 \tag{5.6}$$

where

$$f_1 = \frac{A_2B_3 - A_3B_2}{A_1B_2 - A_2B_1} \qquad f_2 = \frac{A_3B_1 - A_1B_3}{A_1B_2 - A_2B_1} \tag{5.7}$$

and n_1, n_2 are arbitrary constants which should be determined for appropriate boundary conditions.

Bearing in mind the above given assumptions we consider the following boundary conditions

$$\begin{aligned} &\theta(x_2 = 0) = \theta_0 &\qquad &\theta(x_2 = a\delta) = \theta_1 \\ &w_2(x_2 = 0) = 0 &\qquad &\sigma_{22}^{(1)}(x_2 = a\delta) = 0 \\ &W_2(x_2 = 0) = 0 &\qquad &\sigma^{(1)}(x_2 = a\delta) = 0 \end{aligned} \tag{5.8}$$

where θ_0, θ_1 are given constants.

Solving Eqs (5.6) under the boundary conditions (5.8) (by using Eqs (4.5) and (5.1)) we obtain

$$\begin{aligned}\theta(x_2) &= \frac{\theta_1 - \theta_0}{a\delta}x_2 + \theta_0 \\ w(x_2) &= \frac{1}{2}f_1 x_2^2 + e_1 x_2 \\ W(x_2) &= \frac{1}{2}f_2 x_2^2 + e_2 x_2\end{aligned} \tag{5.9}$$

where

$$\begin{aligned}e_1 &= \frac{-(g_1 f_1 a\delta + g_2 f_2 a\delta + g_3\theta_1)g_5 + (g_4 f_1 a\delta + g_5 f_2 a\delta + g_6\theta_1)g_4}{g_1 g_5 - g_2 g_4} \\ e_2 &= \frac{-(g_4 f_1 a\delta + g_5 f_2 a\delta + g_6\theta_1)g_1 + (g_1 f_1 a\delta + g_2 f_2 a\delta + g_3\theta_1)g_4}{g_1 g_5 - g_2 g_4}\end{aligned}$$

$$\begin{aligned}g_1 &= (2N^{(1)} + K^{(1)})(1 + d_1) + L^{(1)}d_4 \\ g_2 &= (2N^{(1)} + K^{(1)})d_2 + L^{(1)}(1 + d_5) \\ g_3 &= (2N^{(1)} + K^{(1)})d_3 + L^{(1)}d_6 - \gamma^{(1)} \\ g_4 &= L^{(1)}(1 + d_1) + M^{(1)}d_4 \\ g_5 &= L^{(1)}d_2 + (1 + d_5)M^{(1)} \\ g_6 &= L^{(1)}d_3 + M^{(1)}d_6 - \bar{\gamma}^{(1)}\end{aligned} \tag{5.10}$$

Knowing the macrotemperature $\theta(\cdot)$ and macrodisplacements $w_2(\cdot)$, $W_2(\cdot)$ (see Eqs (5.9)) one can determine the stresses $\sigma_{ij}^{(r)}$, $\sigma^{(r)}$ and heat fluxes $h^{(r)}$ on the basis of Eqs (4.5), (4.6) and (5.2).

6. Concluding remarks

The homogenized model of periodic stratified fluid-saturated porous thermoelastic solids derived and discussed throughout the paper can be treated as a basis of a theory and a starting point for applications in geophysics, geotechnical engineering, composite materials. The obtained model describes not only mean but also some local stresses and heat fluxes connected with the periodic layered structure of the body.

Assuming that the skeleton is homogeneous, so

$$\begin{gathered}\left\{\rho^{(r)}, K^{(r)}, L^{(r)}, M^{(r)}, N^{(r)}, b^{(r)}, \gamma^{(r)}, \bar{\gamma}^{(r)}, \lambda^{(r)}\right\} = \\ = \left\{\rho^{(s)}, K^{(s)}, L^{(s)}, M^{(s)}, N^{(s)}, b^{(s)}, \gamma^{(s)}, \bar{\gamma}^{(s)}, \lambda^{(s)}\right\} \\ r,s \in \{1,...,n+1\}\end{gathered} \tag{6.1}$$

we obtain form Eqs (3.8) and (3.6) that

$$q_{si} = 0 \qquad Q_{si} = 0 \qquad T_s = 0 \qquad \left\{\begin{array}{l} s = 1,...,n \\ i = 1,2,3 \end{array}\right. \tag{6.2}$$

Substituting Eqs (6.2) into Eqs (3.5) and using Eqs (6.1), (3.8) we arrive at the relations of the linear theory of thermoconsolidation given by Derski and Kowalski (1978), Derski (1979).

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Homogenizowana termokonsolidacja z parametrami mikrolokalnymi periodycznie uwarstwionych wypełnionych cieczą ciał porowatych

Streszczenie

W pracy rozpatrzono zagadnienia modelowania periodycznie uwarstwionych, wypełnionych cieczą, termosprężystych ciał porowatych. Stosując metodę homogenizacji z parametrami mikrolokalnymi oraz liniową teorię termokonsolidacji wyprowadzono homogenizowane modele uwzględniające pewne lokalne efekty dla naprężeń, strumieni ciepła. Przedstawiono również przykład, w którym zastosowano otrzymany model.

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