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EXPONENTIAL TEMPERATURE EFFECT ON FREQUENCIES OF A RECTANGULAR PLATE OF NON-LINEAR VARYING THICKNESS: A QUINTIC SPLINE TECHNIQUE

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The differential equation governing the transverse motion of an elastic rectangular plate of non-linear thickness variation with thermal gradient has been analyzed on the basis of classical plate theory. Following Levy's approach, i.e. the two parallel edges are simply supported, the fourth-order differential equation governing the motion of such plates of non-linear varying thickness in one direction with exponentially temperature distribution has been solved by using the quintic splines interpolation technique for two different combinations of clamped and simply supported boundary conditions at the other two edges. An algorithm for computing the solution of this differential equation is presented for the case of equal intervals. The effect of thermal gradient together with taper constants on the natural frequencies of vibration is illustrated for the first three modes of vibration.

Key words: exponentially temperature, non-linear, thickness variation, vibration, rectangular plate

1. Introduction

In this era of science and technology, plates of various shapes and variable thickness may be regarded as a first approximation to wings and blades and occur as panels in many forms of engineering structures. Thus knowledge of their natural frequencies is of considerable importance at the design stage in order to avoid resonances excited by internal or external forces. Therefore, their design requires an accurate determination of their natural frequencies and mode shapes.

With the advancement of technology, plates of variable thickness are being extensively used in civil, electronic, mechanical, aerospace and marine engineering applications. Nowadays, it becomes very necessary to study the vibration behavior of plates to avoid resonance excited by internal or external forces. Modern engineering structures are based on different types of design, which involve various types of anisotropic and non-homogeneous materials in the form of their structure components. Depending upon the requirement, durability and reliability, materials are being developed so that they can be used to give better strength and efficiency. In the recent past, there has been a phenomenal increase in the development of elastic materials due to high demand for lightweight, high strength, corrosion resistance and high-temperature performance requirements in modern technology. Plates of composite materials are widely used in many engineering structures and machines.

A number of researchers have worked on free vibration analysis of plates of different shapes and variable thickness. Rectangular plates of non-linear varying thickness are widely used in various structures; however, they have been poorly studied, unlike linearly varying thickness. Rectangular plates of non-linear varying thickness with thermal gradient find various applications in the construction of modern high speed air craft. The vibration characteristics of such plates are of interest to the designer.

An extensive review of the work up to 1985 on linear vibration of isotropic/anisotropic plates of various geometries was given by Leissa (1969). The studies on vibration of rectangular plates with uniform/non-uniform thickness with various edge conditions after 1985 were carried out by a number of researchers and were reported by Leissa (1977, 1978, 1981, 1987).

Here, a quintic splines procedure is developed for obtaining the natural frequencies of a rectangular plate of nonlinear varying thickness with the thermal gradient effect. The consideration of the present type of thickness variation was taken earlier by Gupta *et al.* (2006) for a circular plate. The plate type structural components in aircraft and rockets have to operate under elevated temperatures which causes non-homogeneity in the plate material, i.e. elastic constants of the material become functions of the space variables. In an up-to-date survey of literature, authors have come across various models to account for non-homogeneity of plate materials proposed by researchers dealing with vibration.

Gupta *et al.* (2010a) studied the thermal gradient effect on vibration of a non-homogeneous orthotropic rectangular plate having bi-direction linearly thickness variation. Gupta *et al.* (2011a) did the vibration analysis of a visco-elastic orthotropic parallelogram plate with linearly thickness variation in both directions. Lal *et al.* (1997) studied the transverse vibrations of non-uniform orthotropic rectangular plates by Quintic splines method. Gupta and Kaur (2008) studied the effect of thermal gradient on free vibration of clamped visco-elastic rectangular plates with linearly thickness variation in both directions. Gupta and Khanna (2007) studied the vibration of a visco-elastic rectangular plate with linearly thickness variations in both directions. Gupta *et al.* (2007) observed the thermal effect on vibration of a non-homogeneous orthotropic rectangular plate having bi-directional parabolically varying thickness. Tomar and Gupta (1983, 1985) studied the effect of thermal gradient on frequencies of an orthotropic rectangular plate of variable thickness in one and two directions. Gupta *et al.* (2010c, 2011b) studied the thermal effect on vibration of a parallelogram plate of linearly varying thickness and bi-directional linearly varying thickness. Gupta *et al.* (2010b) did the vibration study of a visco-elastic parallelogram plate of linearly varying thickness.

As the thickness variation is not perfectly linear and the same for quadratic, therefore nonlinear variation in thickness is very useful for scientists and engineers to study vibration of the plate and find modes of vibrations.

Since there is no work available on the non-linear thickness variation on thermally induced vibration of rectangular plates, in this paper, the thermal effect on vibration of a rectangular plate with non-linear varying thickness is studied. Here, vibration of a rectangular plate with non-linear varying thickness under a steady exponential temperature distribution is examined. The effect of temperature on the modulus of elasticity is assumed to vary exponentially along the x-axis. The non-linear thickness variation is taken as a combination of linear and parabolical variation factor. The differential equation of motion has been solved by the quintic spline interpolation technique. The two edges parallel to the x-axis ($y = 0$ and $y = b$) are assumed to be simply supported. Different sets of boundary conditions have been imposed at the other two edges. The frequency parameters for the first three modes of vibrations for C-S-C-S- and S-S-S-S- boundary conditions and for various values of taper constants, thermal constant and a fixed value of length-to-breadth ratio, are obtained. The results are presented in tabular form.

2. Analysis and equation of motion

Let us consider a rectangular plate which is subjected to an exponential temperature distribution along the length, i.e. in the x-direction

$$T = T_0 \frac{\mathrm{e} - \mathrm{e}^X}{\mathrm{e} - 1} \tag{2.1}$$

where T denotes the temperature excess above the reference temperature at any point at the distance $X = x/a$ and T_0 denotes the temperature excess above the reference temperature at the end, i.e. $x = a$ or $X = 1$.

The temperature dependence of the modulus of elasticity for most of engineering materials is given by Nowacki (1962)

$$E(T) = E_0(1 - \gamma T) \tag{2.2}$$

where E_0 is the value of Young's modulus at the reference temperature, i.e. $T = 0$, and γ is the slope of the variation of E with T.

Taking as the reference temperature, the temperature at the end of the plate, i.e. at $X = 1$, the modulus variation in view of (2.1) and (2.2) becomes

$$E(X) = E_0\Big(1 - \alpha\frac{\mathrm{e} - \mathrm{e}^X}{\mathrm{e} - 1}\Big) \tag{2.3}$$

where $\alpha = \gamma T_0$ $(0 \leqslant \alpha < 1)$ is a constant known as the temperature constant.

The differential equation governing the free transverse motion of an elastic rectangular plate of the length a, breadth b, thickness h and density ρ is

$$\begin{aligned} &D\nabla^4 w + 2\frac{\partial D}{\partial x}\frac{\partial}{\partial x}\nabla^2 w + 2\frac{\partial D}{\partial y}\frac{\partial}{\partial y}\nabla^2 w + \nabla^2 D\nabla^2 w \\ &\quad + (\nu - 1)\Big(\frac{\partial^2 D}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\frac{\partial^2 D}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y} + \frac{\partial^2 D}{\partial y^2}\frac{\partial^2 w}{\partial x^2}\Big) + \rho h\frac{\partial^2 w}{\partial t^2} = 0 \end{aligned} \tag{2.4}$$

where w is the transverse displacement.

Assume now that the two opposite edges of the plate y=0 and y=b are simply supported. Further, let thickness vary non-linearly in the x-direction only. Thus, the thickness h and flexural rigidity D of the plate become a function of x only. For harmonic vibration, w can be expressed as

$$w(x, y, t) = W_1(x)\sin\Big(\frac{m\pi y}{b}\Big)\mathrm{e}^{\mathrm{i}pt} \tag{2.5}$$

where p is the circular frequency and m is a positive integer.

Substitution of equation (2.5) into (2.4) gives

$$\begin{aligned} &DW_{1,xxxx} + 2D_{,x}W_{1,xxx} + \Big(-\frac{2m^2\pi^2}{b^2}D + D_{,xx}\Big)W_{1,xx} + \Big(-\frac{2m^2\pi^2}{b^2}D_{,x}\Big)W_{1,x} \\ &\quad + \Big(\frac{m^4\pi^4}{b^4}D - \frac{\nu m^2\pi^2}{b^2}D_{,xx}\Big)W_1 = \rho h p^2 W_1 \end{aligned} \tag{2.6}$$

A comma followed by a suffix denotes partial differentiation with respect to that variable.

Thus equation (2.6) reduces to a form independent of y and on introducing the non-dimensional variables

$$H = \frac{h}{a} \qquad W = \frac{W_1}{a} \qquad X = \frac{x}{a} \qquad D_1 = \frac{D}{a^3} \tag{2.7}$$

differential equation (2.6) reduces to

$$\begin{aligned} &D_1 W_{,XXXX} + 2D_{1,X}W_{,XXX} + (D_{1,XX} - 2r^2 D_1)W_{,XX} - 2r^2 D_{1,X}W_{,X} \\ &\quad + r^2(r^2 D_1 - \nu D_{1,XX})W = \rho H a^2 p^2 W \end{aligned} \tag{2.8}$$

where $r^2 = (m\pi a/b)^2$.

Since the thickness varies non-linearly in the x-direction only, therefore, one can assume

$$H = H_0(1 + \beta_1 X + \beta_2 X^2) \tag{2.9}$$

where β_1 and β_2 are taper constants such that $|\beta_1| \leqslant 1$, $|\beta_2| \leqslant 1$ and $\beta_1 + \beta_2 > -1$, H_0 is the thickness at $X = 0$.

Considering equation (2.3) and (2.9) with the help of (2.7), the expression for rigidity D_1 comes out as

$$D_1 = D_0\Big(1 - \alpha\frac{\mathrm{e} - \mathrm{e}^X}{\mathrm{e} - 1}\Big)(1 + \beta_1 X + \beta_2 X^2)^3 \tag{2.10}$$

where $D_0 = E_0 H_0^3/[12(1 - \nu^2)]$.

Using equations (2.8) to (2.10), one obtains the equation of motion as

$$\begin{aligned}
&\Big(1 - \alpha\frac{\mathrm{e} - \mathrm{e}^X}{\mathrm{e} - 1}\Big)(1 + \beta_1 X + \beta_2 X^2)^2 W_{,XXXX} + 2\Big[\alpha\frac{\mathrm{e}^X}{\mathrm{e} - 1}(1 + \beta_1 X + \beta_2 X^2)^2 \\
&\quad + 3\Big(1 - \alpha\frac{\mathrm{e} - \mathrm{e}^X}{\mathrm{e} - 1}\Big)(1 + \beta_1 X + \beta_2 X^2)(\beta_1 + 2\beta_2 X)\Big]W_{,XXX} \\
&\quad + \Big[\alpha\frac{\mathrm{e}^X}{\mathrm{e} - 1}(1 + \beta_1 X + \beta_2 X^2)^2 + 6\alpha\frac{\mathrm{e}^X}{\mathrm{e} - 1}(1 + \beta_1 X + \beta_2 X^2)(\beta_1 + 2\beta_2 X) \\
&\quad + 6\Big(1 - \alpha\frac{\mathrm{e} - \mathrm{e}^X}{\mathrm{e} - 1}\Big)(\beta_1 + 2\beta_2 X)^2 + 6\Big(1 - \alpha\frac{\mathrm{e} - \mathrm{e}^X}{\mathrm{e} - 1}\Big)(1 + \beta_1 X + \beta_2 X^2)\beta_2 \\
&\quad - 2r^2\Big(1 - \alpha\frac{\mathrm{e} - \mathrm{e}^X}{\mathrm{e} - 1}\Big)(1 + \beta_1 X + \beta_2 X^2)^2\Big]W_{,XX} \\
&\quad - 2r^2\Big[\alpha\frac{\mathrm{e}^X}{\mathrm{e} - 1}(1 + \beta_1 X + \beta_2 X^2)^2 + 3\Big(1 - \alpha\frac{\mathrm{e} - \mathrm{e}^X}{\mathrm{e} - 1}\Big)(1 + \beta_1 X + \beta_2 X^2)(\beta_1 + 2\beta_2 X)\Big]W_{,X} \\
&\quad + r^2\Big[r^2\Big(1 - \alpha\frac{\mathrm{e} - \mathrm{e}^X}{\mathrm{e} - 1}\Big)(1 + \beta_1 X + \beta_2 X^2)^2 - \nu\alpha\frac{\mathrm{e}^X}{\mathrm{e} - 1}(1 + \beta_1 X + \beta_2 X^2)^2 \\
&\quad + 6\alpha\frac{\mathrm{e}^X}{\mathrm{e} - 1}(1 + \beta_1 X + \beta_2 X^2)(\beta_1 + 2\beta_2 X) + 6\Big(1 - \alpha\frac{\mathrm{e} - \mathrm{e}^X}{\mathrm{e} - 1}\Big)(\beta_1 + 2\beta_2 X)^2 \\
&\quad + 6\Big(1 - \alpha\frac{\mathrm{e} - \mathrm{e}^X}{\mathrm{e} - 1}\Big)(1 + \beta_1 X + \beta_2 X^2)\beta_2\Big]W = \lambda^2 W
\end{aligned} \tag{2.11}$$

where

$$\lambda 2 = \frac{p^2 a^2}{E_0/\rho}\frac{12(1 - \nu^2)}{H_0^2} \tag{2.12}$$

is a frequency parameter.

3. Method of solution

Let $f(X)$ be a function with continuous derivatives in the range $(0, l)$. Choose $(n + 1)$ points $X_0, X_1, X_2, \ldots, X_n$, in the range $0 \leqslant X \leqslant l$ such that $0 = X_0 < X_1 < X_2 < X_3 < \ldots < X_n = 1$.

Let the approximating function $W(X)$ for $f(X)$ be a quintic spline with the following properties:

(a) $W(X)$ is a quintic polynomial in each interval (X_k, X_{k+1}),

(b) $W(X_k) = f(X_k)$, $k = 0(1)n$,

(c) $W'(X)$, $W''(X)$, $W'''(X)$ and $W^{\mathrm{iv}}(X)$ are continuous.

By definition, the quintic spline takes the form

$$W(X) = a_0 + \sum_{i=1}^{4} a_i(X - X_0)^i + \sum_{j=0}^{n-1} b_j(X - X_j)_+^5 \tag{3.1}$$

where

$$(X - X_J)_+ = \begin{cases} 0 & \text{if} \quad X < X_J \\ X - X_J & \text{if} \quad X \geqslant X_J \end{cases} \tag{3.2}$$

It is also assumed, for simplicity, that the knots X_i are equally spaced in $(0, l)$ with the spacing interval ΔX, so that

$$\Delta X = \frac{l}{n} \qquad X_i = i\Delta X \qquad i = 0, 1, 2, \ldots, n \tag{3.3}$$

The number of unknown constants in equation (3.1) is $(n+5)$. Satisfaction of differential equation (2.11) by collocation at the $(n + 1)$ knots in the interval $(0, l)$ together with the boundary conditions (to be explained in the next section) gives precisely the requisite number of equations for the determination of unknown constants.

Substitution of $W(X)$ from equation (3.1) into equation (2.11), for satisfaction at the m-th knot, gives

$$\begin{aligned} & B_4a_0 + [B_4(X_q - X_0) + B_3]a_1 + [B_4(X_q - X_0)^2 + 2B_3(X_q - X_0) + 2B_2]a_2 \\ & \quad +[B_4(X_q - X_0)^3 + 3B_3(X_q - X_0)^2 + 6B_2(X_q - X_0) + 6B_1]a_3 \\ & \quad +[B_4(X_q - X_0)^4 + 4B_3(X_q - X_0)^3 + 12B_2(X_q - X_0)^2 + 24B_1(X_q - X_0) + 24B_0]a_4 \\ & \quad + \sum_{i=0}^{n-1}[B_4(X_q - X_i)^5 + 5B_3(X_q - X_i)^4 + 20B_2(X_q - X_i)^3 + 60B_1(X_q - X_i)^2 \\ & \quad +120B_0(X_q - X_i)]b_i = 0 \end{aligned} \tag{3.4}$$

where

$$\begin{aligned} B_0 &= \Big(1 - \alpha\frac{\mathrm{e} - \mathrm{e}^{X_q}}{\mathrm{e} - 1}\Big)(1 + \beta_1X_q + \beta_2X_q^2)^2 \\ B_1 &= 2\Big[\alpha\frac{\mathrm{e}^{X_q}}{\mathrm{e} - 1}(1 + \beta_1X_q + \beta_2X_q^2)^2 + 3\Big(1 - \alpha\frac{\mathrm{e} - \mathrm{e}^{X_q}}{\mathrm{e} - 1}\Big)(1 + \beta_1X_q + \beta_2X_q^2)(\beta_1 + 2\beta_2X_q)\Big] \\ B_2 &= \Big[\alpha\frac{\mathrm{e}^{X_q}}{\mathrm{e} - 1}(1 + \beta_1X_q + \beta_2X_q^2)^2 + 6\alpha\frac{\mathrm{e}^{X_q}}{\mathrm{e} - 1}(1 + \beta_1X_q + \beta_2X_q^2)(\beta_1 + 2\beta_2X_q) \\ & \quad +6\Big(1 - \alpha\frac{\mathrm{e} - \mathrm{e}^{X_q}}{\mathrm{e} - 1}\Big)(\beta_1 + 2\beta_2X_q)^2 + 6\Big(1 - \alpha\frac{\mathrm{e} - \mathrm{e}^{X_q}}{\mathrm{e} - 1}\Big)(1 + \beta_1X_q + \beta_2X_q^2)\beta_2 \\ & \quad -2r^2\Big(1 - \alpha\frac{\mathrm{e} - \mathrm{e}^{X_q}}{\mathrm{e} - 1}\Big)(1 + \beta_1X_q + \beta_2X_q^2)^2\Big] \\ B_3 &= -2r^2\Big[\alpha\frac{\mathrm{e}^{X_q}}{\mathrm{e} - 1}(1+\beta_1X_q + \beta_2X_q^2)^2 + 3\Big(1-\alpha\frac{\mathrm{e} - \mathrm{e}^{X_q}}{\mathrm{e} - 1}\Big)(1+\beta_1X_q + \beta_2X_q2)(\beta_1+ 2\beta_2X_q)\Big] \\ B_4 &= r^2\Big[r^2\Big(1-\alpha\frac{\mathrm{e} - \mathrm{e}^{X_q}}{\mathrm{e} - 1}\Big)(1 + \beta_1X_q + \beta_2X_q^2)^2 - \nu\Big(\alpha\frac{\mathrm{e}^{X_q}}{\mathrm{e} - 1}(1 + \beta_1X_q + \beta_2X_q^2)^2 + 6\alpha\frac{\mathrm{e}^{X_q}}{\mathrm{e} - 1}\Big) \\ & \quad \cdot(1 + \beta_1X_q + \beta_2X_q^2)(\beta_1 + 2\beta_2X_q) + 6\Big(1 - \alpha\frac{\mathrm{e} - \mathrm{e}^{X_q}}{\mathrm{e} - 1}\Big)(\beta_1 + 2\beta_2X_q)^2 \\ & \quad +6\Big(1 - \alpha\frac{\mathrm{e} - \mathrm{e}^{X_q}}{\mathrm{e} - 1}\Big)(1 + \beta_1X_q + \beta_2X_q^2)\beta_2\Big] - \lambda^2 \end{aligned}$$

Thus, one obtains a homogeneous set of equations in terms of the unknown constants $a_0, a_1, a_2, a_3, a_4, b_0, b_1, \dots, b_{n-1}$, which, when written in matrix notation, takes the form

$$\mathbf{BC} = \mathbf{0} \tag{3.5}$$

where $\mathbf{B}$ is an $(n+1) \times (n+5)$ matrix and $\mathbf{C}$ is an $(n+5) \times 1$ matrix.

4. Boundary conditions and frequency equations

The frequency equations for clamped (C) and simply supported (S) rectangular plates have been obtained by employing the appropriate boundary conditions.

4.1. C-S-C-S-plates

For a rectangular plate clamped at both the edges $X = 0$ and $X = 1$ (and simply supported at the remaining two edges)

$$W\Big|_{X=0,1} = \frac{\partial W}{\partial X}\Big|_{X=0,1} = 0 \tag{4.1}$$

Applying boundary conditions (4.1), to deflection function (3.1), at the two edges $X = 0$ and $X = 1$, one obtains a set of four homogeneous equations in terms of the unknown constants, which can be written as

$$\mathbf{A}_1\mathbf{C} = \mathbf{0} \tag{4.2}$$

where $\mathbf{A}_1$ is an $4 \times (n+5)$ matrix and $\mathbf{C}$ is an $(n+5) \times 1$ matrix.

Equation (4.2) taken together with equation (3.5) gives a complete set of $(n+5)$ equations for a C-S-C-S-plate. These can be written as

$$[\mathbf{B}/\mathbf{A}_1]\mathbf{C} = \mathbf{0} \tag{4.3}$$

For a non-trivial solution of equation (4.3), the characteristic determinant must vanish

$$|\mathbf{B}/\mathbf{A}_1| = 0 \tag{4.4}$$

This is the frequency equation for a C-S-C-S-plate.

4.2. S-S-S-S-plates

For a rectangular plate simply supported at both the edges $X = 0$ and $X = 1$ (and simply supported at the remaining two edges), the following holds

$$W\Big|_{X=0,1} = \frac{\partial^2 W}{\partial X^2}\Big|_{X=0,1} = 0 \tag{4.5}$$

Employing boundary conditions (4.5) to deflection function (3.1) at the two edges $X = 0$ and $X = 1$, one gets the boundary equations for a S-S-S-S-plate as

$$\mathbf{A}_2\mathbf{C} = \mathbf{0} \tag{4.6}$$

where $\mathbf{A}_2$ is an $4 \times (n+5)$ matrix and $\mathbf{C}$ is an $(n+5) \times 1$ matrix.

Hence the frequency equation comes out for S-S-S-S-plate as

$$|\mathbf{B}/\mathbf{A}_2| = 0 \tag{4.7}$$

5. Results and discussion

Frequency equations (4.4) and (4.7) are transcendental equations in λ^2 from which infinitely many roots can be obtained. The frequency parameter λ corresponding to the first three modes of vibration of C-S-C-S- and S-S-S-S-rectangular plates have been computed for $m = 1$ and various values of aspect ratio (a/b), thermal constant (α) and taper constants (β_1, β_2). The value of Poisson's ratio ν has been taken as 0.3.

To choose the appropriate interpolation interval ΔX, a computer program has been developed for the evaluation of the frequency parameter λ and run for $n = 10(5)60$. The numerical values show consistent improvement with the increase of the number of knots. In the computation, the authors have fixed $n = 50$, since further increase in n does not improve the results except for the fifth or sixth decimal places. These results are presented in Tables 1 to 3.

Table 1 shows the variation of the frequency parameter (λ) with the thermal constant (α) for different combinations of taper constants (β_1, β_2) and the fixed aspect ratio $(a/b = 1.5)$ corresponding to the first three modes of vibration for C-S-C-S- and S-S-S-S-plates. The value of the frequency parameter decreases with the increase of the thermal constant for both boundary conditions considered here. Furthermore, it can be seen that the frequency parameter, for both boundary conditions, decreases gradually in the third mode of vibrations in comparison to the first two modes of vibration.

Table 1. Values of the frequency parameter (λ) for different thermal constants (α) with different combinations of the taper constant (β_1, β_2) and a fixed aspect ratio $(a/b = 1.5)$ for C-S-C-S- and S-S-S-S-plates for the first three modes of vibrations

β_1, β_2	α	C-S-C-S-plate			S-S-S-S-plate		
		First mode	Second mode	Third mode	First mode	Second mode	Third mode
$\beta_1 = -0.5$ $\beta_2 = -0.5$	0.0	29.3011	63.0984	111.0257	21.4941	53.0078	91.1043
	0.1	28.3908	61.1005	107.6504	20.4906	50.7688	87.9017
	0.2	27.4901	58.8310	102.9045	19.4041	48.4050	84.8101
	0.3	26.4212	56.8112	99.4052	18.3100	46.1205	81.7032
	0.4	25.3101	54.7283	95.3142	17.1794	43.9503	78.6021
	0.5	24.2465	52.4450	91.1528	15.8906	41.9390	75.2167
$\beta_1 = -0.5$ $\beta_2 = 0.5$	0.0	36.0132	72.5490	127.4781	27.3761	63.7524	108.1562
	0.1	35.1001	70.2441	122.8791	26.2001	61.1533	104.7054
	0.2	34.0002	68.1961	118.7376	25.3550	58.9641	101.1645
	0.3	32.9982	66.0348	114.7082	24.2611	56.8300	97.9410
	0.4	31.8908	63.8503	110.5314	23.1530	54.6306	94.7502
	0.5	30.7983	61.7406	106.5164	22.0029	52.3858	91.4213
$\beta_1 = 0.5$ $\beta_2 = 0.5$	0.0	49.4210	106.8851	191.4330	39.7562	97.2203	171.4612
	0.1	48.4301	104.4254	187.3517	38.5401	94.8203	169.7908
	0.2	47.4209	102.1481	184.3200	37.5304	92.8103	164.7800
	0.3	46.3211	99.8605	181.0769	36.5041	90.6234	161.5073
	0.4	45.1899	97.6103	177.0027	35.4801	88.5328	158.4087
	0.5	44.0277	95.6025	172.6536	34.4327	86.3658	155.1998

The results presented in Table 2 show a marked effect of variation of the taper constant (β_1) on the frequency parameter for the taper constant $(\beta_2 = 0.5)$, two values of the thermal constant $(\alpha = 0.0, 0.4)$ and a fixed aspect ratio $(a/b = 1.5)$ corresponding to the first three modes of vibration. It is observed that the frequency parameter increases with the increase of the taper constant for both boundary conditions considered here.

Table 2. Values of the frequency parameter (λ) for different taper constants (β_1) with different combinations of the thermal constant (α) and a fixed aspect ratio $(a/b = 1.5)$ for C-S-C-S- and S-S-S-S-plates for the first three modes of vibrations; $\beta_2 = 0.5$

α	β_1	C-S-C-S-plate			S-S-S-S-plate		
		First mode	Second mode	Third mode	First mode	Second mode	Third mode
0.0	−0.5	36.0132	72.5490	127.4781	27.3761	63.7524	108.1562
	−0.3	38.3782	78.1352	137.9481	29.4861	69.2189	118.7010
	−0.1	40.5601	83.7103	148.5275	31.5908	74.7902	129.0211
	0.0	42.5324	89.0914	158.7945	33.3920	79.8053	138.8386
	0.1	44.8210	94.5601	169.4642	35.3904	85.2100	149.0842
	0.3	47.0572	100.4805	180.3443	37.3026	91.0409	160.0490
	0.5	49.4210	106.8851	191.4330	39.7562	97.2203	171.4612
0.4	−0.5	31.8908	63.8503	110.5314	23.1530	54.6306	94.7502
	−0.3	34.1304	69.5461	122.0372	25.4328	60.4220	105.6032
	−0.1	36.3308	75.3196	133.0619	27.5038	65.6027	116.1181
	0.0	38.1510	80.1950	143.6991	29.1401	70.7212	125.8526
	0.1	40.4302	86.1491	156.8407	31.1999	76.1082	136.4291
	0.3	42.8410	91.9428	165.8413	33.2632	82.2734	147.4841
	0.5	45.1899	97.6103	177.0027	35.4801	88.5328	158.4087

In Table 3, the effect of the taper constant (β_2) on the frequency parameter for the taper constant $(\beta_1 = 0.5)$, two values of the thermal constant $(\alpha = 0.0, 0.4)$ and a fixed aspect ratio $(a/b = 1.5)$ corresponding to the first three modes of vibration for C-S-C-S- and S-S-S-S-plates is shown. From this table, one can observe that the frequency parameter in the first three modes of vibration increases with the increase of the taper constant for C-S-C-S- and S-S-S-S-plates.

Table 3. Values of the frequency parameter (λ) for different taper constants (β_2) with different combinations of the thermal constant (α) and a fixed aspect ratio $(a/b = 1.5)$ for C-S-C-S- and S-S-S-S-plates for the first three modes of vibrations; $\beta_1 = 0.5$

α	β_2	C-S-C-S-plate			S-S-S-S-plate		
		First mode	Second mode	Third mode	First mode	Second mode	Third mode
0.0	−0.5	37.5320	77.7213	142.9941	27.8301	65.7224	116.9956
	−0.3	39.4712	82.2052	151.0998	29.7861	70.8189	126.0010
	−0.1	41.4011	86.9910	159.2275	31.7408	75.9702	135.1221
	0.0	43.1534	91.1293	167.0042	33.4221	80.7053	143.6186
	0.1	45.1210	96.0601	175.1864	35.3090	86.0030	152.7484
	0.3	47.1572	101.2805	183.2443	37.3026	91.3409	162.0060
	0.5	49.4210	106.8851	191.4330	39.7562	97.2203	171.4612
0.4	−0.5	33.2720	76.0091	126.9418	23.6881	60.1602	105.0480
	−0.3	35.1142	79.7146	135.5856	25.6627	64.9400	114.0071
	−0.1	37.2008	83.5046	143.9999	27.7293	69.7901	122.9085
	0.0	38.8180	86.8696	152.1187	29.4711	74.2420	131.2102
	0.1	40.8802	90.5247	160.9991	31.4209	78.9999	140.3121
	0.3	43.1101	94.3131	169.1411	33.4382	83.6312	149.3722
	0.5	45.1899	97.6103	177.0027	35.4801	88.5328	158.4087

Moreover, it can be seen in Tables 2 and 3 that the frequency parameter, for both boundary conditions, increases gradually in the third mode of vibrations in comparison to the first two modes of vibration.

Also, one can observe from Tables 1 to 3, that the frequency parameter of the C-S-C-S-plate is higher than that of the S-S-S-S-plate.

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