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ON THE EFFECTIVENESS OF VARIATION OF PHYSICAL VARIABLES ON STEADY FLOW BETWEEN PARALLEL PLATES WITH HEAT TRANSFER IN A POROUS MEDIUM

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The effect of variation of physical variables on a steady flow through a porous medium with heat transfer between parallel plates is examined. The viscosity and the thermal conductivity are assumed to be temperature dependent. A constant pressure gradient is applied in the axial direction and the two plates are kept at two constant but different temperatures, while the viscous dissipation is considered in the energy equation. A numerical solution for the governing non-linear coupled equations of motion and the energy equation is determined. The effect of porosity of the medium, the variable viscosity, and the variable thermal conductivity on both the velocity and temperature distributions is reported.

Key words: variable properties, porous medium, heat transfer, parallel plates, steady state

List of symbols

a, b	–	viscosity and thermal conductivity parameter
c_p	–	specific heat at constant pressure
Ec, Pr	–	Eckert and Prandtl number
k	–	thermal conductivity
K	–	Darcy permeability
P	–	pressure gradient
M	–	porosity parameter
T	–	temperature of fluid
T_1, T_2	–	temperature of lower and upper plate
u	–	velocity component in x-direction
v	–	velocity of fluid
x, y	–	axial and distance in vertical direction
μ, ρ	–	viscosity and density of fluid

1. Introduction

The flow between infinite horizontal parallel plates has received great attention due to its applications in magnetohydrodynamic (MHD) power generators and pumps, etc. Hartmann and Lazarus (1937) examined the effect of a transverse uniform magnetic field on the flow of a viscous incompressible electrically conducting fluid between two infinite parallel plates. Analytical solutions for the velocity fields were developed by Tao (1960), Alpher, 1961, Sutton and Sherman (1965), Cramer and Pai (1973) under different physical effects. Some exact and numerical solutions for the heat transfer problem were obtained in Nigam and Singh (1960). Soundalgekar *et al.* (1970), Soundalgekar and Uplekar (1986) studied the effect of Hall currents on the steady MHD Couette flow with heat transfer. The temperatures of the two plates were assumed either to be constant (Soundalgekar *et al.*, 1970) or linearly varying along the plates in the direction of the flow (Soundalgekar and Uplekar, 1986). Attia (1998) studied the Hall current effects on the velocity and temperature fields of an unsteady Hartmann flow with uniform suction and injection.

Most of these studies were based on constant physical properties, however more accurate prediction for the flow and heat transfer can be obtained considering the dependence of the physical properties on temperature (Herwig and Wicken, 1986). Klemp *et al.* (1990) examined the influence of temperature dependent viscosity on the entrance flow in a channel in the hydrodynamic case. Attia and Kotb (1996) handled the steady MHD fully developed flow and heat transfer between two parallel plates with temperature dependent viscosity which was extended to the transient state by Attia (1999). The effect of variation in physical properties in the MHD Couette flow between parallel plates was studied by Joaquín *et al.* (2010), Eguia *et al.* (2011).

In this paper, the steady flow of a viscous incompressible fluid with heat transfer through a porous medium is examined. The flow in the porous medium deals with the analysis in which the governing differential equations are based on Darcy's law which accounts for the drag exerted by the porous medium (Joseph *et al.*, 1982; Ingham and Pop, 2002; Khaled and Vafai, 2003). The viscosity and the thermal conductivity are assumed to change with temperature. The two plates are kept at two constant but different temperatures, and a constant pressure gradient is applied in the axial direction. The coupled nonlinear system of equations of motion and energy equation including the viscous dissipation is solved numerically using finite differences to obtain the velocity and temperature distributions.

2. Formulation of the problem

The fluid is flowing between two infinite parallel plates located at the $y = \pm h$ planes through a porous medium where the Darcy model is assumed (Khaled and Vafai, 2003). The two plates are kept at two constant temperatures T_1 for the lower plate and T_2 for the upper plate with $T_2 > T_1$, and a constant pressure gradient is applied in the x-direction. The viscosity of the fluid is assumed to vary with temperature exponentially, while the thermal conductivity is assumed to depend on temperature linearly.

The flow of the fluid is governed by the Navier-Stokes equation

$$\rho(\mathbf{v}\cdot\nabla v) = -\nabla P + \nabla\cdot(\mu\nabla v) + \mathbf{f}_b \tag{2.1}$$

where $\mathbf{v}$ is the velocity vector, P is the pressure, μ is the viscosity of the fluid, and $\mathbf{f}_b$ is the body force per unit volume. Using Eq. (2.1) the component of the Navier-Stokes equation in the x-direction is given by

$$-\frac{dP}{dx} + \frac{d}{dy}\Big(\mu\frac{du}{dy}\Big) - \frac{\mu}{K}u = 0 \tag{2.2}$$

where K is the Darcy permeability (Khaled and Vafai, 2003). The no-slip condition at the plates implies that

$$y = -h : u = 0 \qquad\qquad y = h : u = 0 \tag{2.3}$$

The energy equation for the fluid considering the viscous dissipation is given by Ingham and Pop (2002)

$$\frac{1}{\rho c_p}\Big[\frac{d}{dy}\Big(k\frac{dT}{dy}\Big) + \mu\Big(\frac{du}{dy}\Big)^2\Big] = 0 \tag{2.4}$$

where T is the temperature of the fluid, cp is the specific heat at constant pressure of the fluid, and k is the thermal conductivity of the fluid. The last term in the left side of Eq. (2.4) represents the viscous dissipation respectively.

The temperature of the fluid must satisfy the boundary conditions

$$T = T_1 \qquad y = -h \qquad \text{and} \qquad T = T_2 \qquad y = h \tag{2.5}$$

The viscosity of the fluid is assumed to vary with temperature in the form $\mu = \mu_o f_1(T)$, and μ_o is the viscosity of the fluid at $T = T_1$. By assuming the viscosity to vary exponentially with temperature, the function $f_1(T)$ takes the form $f_1(T) = \exp(-a_1(T - T_1))$, see Klemp *et al.* (1990), White (1991). The thermal conductivity of the fluid varies with temperature as $k = k_o f_2(T)$, where k_o is the thermal conductivity of the fluid at $T = T_1$. We assume a linear dependence for the thermal conductivity with temperature in the form $k = k_o(1 + b_1(T - T_1))$ (Klemp *et al.*, 1990; White, 1991), where the parameter b1 may be positive or negative (Klemp *et al.*, 1990; White, 1991).

Introducing the following non-dimensional quantities

$$\widehat{y} = \frac{y}{h} \qquad \widehat{P} = \frac{P\rho h^2}{\mu_o^2} \qquad \widehat{u} = \frac{u\rho h}{\mu_o} \qquad \widehat{T} = \frac{T - T_1}{T_2 - T_1} \qquad \alpha = -\frac{d\widehat{P}}{d\widehat{x}}$$

and
$\widehat{f}_1(T) = \exp(-a_1(T_2 - T_1)T) = \exp(-aT)$, a is the viscosity parameter,
$\widehat{f}_2(T) = 1 + b_1(T_2 - T_1)T = 1 + bT$, b is the thermal conductivity parameter,
$M = \mu_o h^2/K$ is the porosity parameter,
$\mathrm{Pr} = \mu_o c_p/k_o$ is the Prandtl number,
$\mathrm{Ec} = \mu_o^2/[h^2 c_p \rho^2 (T_2 - T_1)]$ is the Eckert number,
$\tau_L = (\partial\widehat{u}/\partial\widehat{y})\widehat{y} = -1$ is the axial skin friction coefficient at the lower plate,
$\tau_U = (\partial\widehat{u}/\partial\widehat{y})\widehat{y} = 1$ is the axial skin friction coefficient at the upper plate,
$\mathrm{Nu}_L = (\partial T/\partial\widehat{y})\widehat{y} = -1$ is the Nusselt number at the lower plate,
$\mathrm{Nu}_U = (\partial T/\partial\widehat{y})\widehat{y} = 1$ is the Nusselt number at the upper plate.

Equations (2.3) to (2.5) read (the hats are dropped for simplicity)

$$\alpha + f_1(T)\frac{d^2u}{dy^2} + \frac{df_1(T)}{dy}\frac{du}{dy} - f_1(T)Mu = 0 \tag{2.6}$$

$$y = -1 \qquad u = 0 \qquad \text{and} \qquad y = 1 \qquad u = 0 \tag{2.7}$$

$$\frac{1}{\mathrm{Pr}}f_2(T)\frac{d^2T}{dy^2} + \frac{1}{\mathrm{Pr}}\frac{df_2(T)}{dy}\frac{dT}{dy} + \mathrm{Ec}f_1(T)\Big(\frac{du}{dy}\Big)^2 = 0 \tag{2.8}$$

$$T = 0 \qquad y = -1 \qquad \text{and} \qquad T = 1 \qquad y = 1 \tag{2.9}$$

Equations (2.6) and (2.8) represent a coupled system of non-linear ordinary differential equations which are solved numerically under boundary conditions (2.7) and (2.9) using finite differences. A linearization technique is first applied to replace the nonlinear terms at a linear stage, with corrections incorporated in subsequent iterative steps until convergence is reached. Then the Crank-Nicolson implicit method is applied and an iterative scheme is used to solve the linearized system of difference equations. The resulting block tri-diagonal system is solved using the generalized Thomas-algorithm (Ames, 1977). Finite difference equations relating the variables are obtained by writing the equations at the mid point of the computational cell and then replacing the different terms by their second order central difference approximations in the y-direction. The computational domain is divided into meshes each of dimension Δy. We define the variables $v = du/dy$ and $H = dT/dy$ to reduce the second order differential Eqs. (2.6) and

(2.8) to first order differential equations. The finite difference representations for the resulting first order differential take the form

$$\begin{aligned}
&\alpha + \frac{\overline{f}_1(T)_{i+1} + \overline{f}_1(T)_i}{2}\frac{v_{i+1} - v_i}{\Delta y} + \frac{\overline{f}_1(T)_{i+1} - \overline{f}_1(T)_i}{\Delta y}\frac{u_{i+1} - u_i}{2} \\
&\qquad - M\frac{\overline{f}_1(T)_{i+1} + \overline{f}_1(T)_i}{2}\frac{u_{i+1} - u_i}{2} = 0 \\
&\frac{1}{\mathrm{Pr}}\frac{\overline{f}_2(T)_{i+1} + \overline{f}_2(T)_i}{2}\frac{H_{i+1} - H_i}{\Delta y} + \frac{1}{\mathrm{Pr}}\frac{\overline{f}_2(T)_{i+1} - \overline{f}_2(T)_i}{\Delta y}\frac{H_{i+1} + H_i}{2} \\
&\qquad + \mathrm{Ec}\frac{\overline{f}_1(T)_{i+1} + \overline{f}_1(T)_i}{2}\frac{\overline{v}_{i+1} + \overline{v}_i}{2}\frac{v_{i+1} + v_i}{2} = 0
\end{aligned} \tag{2.10}$$

The variables with bars are given initial guesses. The linearized system of difference equations represents a banded matrix that can be solved by different methods. One of the very powerful methods to solve this system is the Thomas algorithm which has the advantage of reducing memory storage and computational time. Equations (2.10) can be written in the following form

$$\begin{aligned}
&a_1u_i + a_2u_{i+1} + a_3v_i + a_4v_{i+1} + a_5T_i + a_6T_{i+1} = a_7 \\
&b_1T_i + b_2T_{i+1} + b_3H_i + b_4H_{i+1} + b_5u_i + b_6u_{i+1} = b_7
\end{aligned} \tag{2.11}$$

where the a's and b's are the coefficients of the difference equations. We set the following difference equations for evaluating the unknowns u, v, T, H

$$\begin{aligned}
&u_i = \overline{u}_iv_i + \widetilde{u}_iH_i + \widehat{u}_i &\qquad& T_i = \overline{T}_iv_i + \widetilde{T}_iH_i + \widehat{T}_i \\
&v_i = \overline{v}_{i+1}v_{i+1} + \widetilde{v}_{i+1}H_{i+1} + \widehat{v}_{i+1} && H_i = \overline{H}_{i+1}v_{i+1} + \widetilde{H}_{i+1}H_{i+1} + \widehat{H}_{i+1}
\end{aligned} \tag{2.12}$$

where $\overline{u}_i$, $\widetilde{u}_i$, $\widehat{u}_i$, $\overline{T}_i$, $\widetilde{T}_i$, $\widehat{T}_i$, $\overline{v}_i$, $\widetilde{v}_i$, $\widehat{v}_i$, $\overline{H}_i$, $\widetilde{H}_i$, $\widehat{H}_i$ are the Thomas coefficients. The equations relating the velocity and temperature to their derivatives using Eq. (2.12) are given as

$$\begin{aligned}
&u_{i+1} = \Big(\overline{u}_i + \frac{\Delta}{2}\Big)v_i + \widetilde{u}_iH_i + \frac{\Delta}{2}v_{i+1} + \widehat{u}_i \\
&T_{i+1} = \overline{T}_iv_i + \Big(\widetilde{T}_i + \frac{\Delta}{2}\Big)H_i + \frac{\Delta}{2}H_{i+1} + \widehat{T}_i
\end{aligned} \tag{2.13}$$

Substituting Eqs. (2.12) and (2.13) into Eqs. (2.11) and solving the resulting equations for the unknown coefficients in Eqs. $(2.12)_{3,4}$, we obtain

$$\begin{aligned}
&\overline{v}_{i+1} = \frac{\ell_{31}h_2 - h_{31}l_2}{l_1h_2 - l_2h_1} &\qquad& \widetilde{v}_{i+1} = \frac{\ell_{32}h_2 - h_{32}l_2}{l_1h_2 - l_2h_1} &\qquad& \widehat{v}_{i+1} = \frac{\ell_{33}h_2 - h_{33}l_2}{l_1h_2 - l_2h_1} \\
&\overline{H}_{i+1} = \frac{h_{31}\ell_1 - h_{31}h_1}{l_1h_2 - l_2h_1} && \widetilde{H}_{i+1} = \frac{h_{32}\ell_1 - \ell_{32}h_1}{l_1h_2 - l_2h_1} && \widehat{H}_{i+1} = \frac{h_{33}\ell_1 - \ell_{33}h_1}{l_1h_2 - l_2h_1}
\end{aligned}$$

where

$$\begin{aligned}
&l_1 = a_1\overline{u}_i + a_2\Big(\overline{u}_i + \frac{\Delta}{2}\Big) + a_3 + (a_5 + a_6)\overline{T}_i &\qquad& l_2 = (a_1 + a_2)\widetilde{u}_i + a_5\widetilde{T}_i + a_6\Big(\widetilde{T}_i + \frac{\Delta}{2}\Big) + a_7 \\
&l_{31} = -\Big(a_2\frac{\Delta}{2} + a_4\Big) && l_{32} = -\Big(a_6\frac{\Delta}{2} + a_8\Big) \\
&l_{33} = a_9 - (a_1 + a_2)\widehat{u}_i - (a_5 + a_6)\widehat{T}_i && h_1 = b_5\overline{u}_i + b_6\Big(\overline{u}_i + \frac{\Delta}{2}\Big) + b_7 + (b_1 + b_2)\overline{T}_i \\
&h_2 = (b_5 + b_6)\widetilde{u}_i + b_1\widetilde{T}_i + b_2\Big(\widetilde{T}_i + \frac{\Delta}{2}\Big) + b_3 && h_{31} = -\Big(b_6\frac{\Delta}{2} + b_8\Big) \\
&h_{32} = -\Big(b_2\frac{\Delta}{2} + b_4\Big) && h_{33} = b_9 - (b_5 + b_6)\widehat{u}_i - (b_1 + b_2)\widehat{T}_i
\end{aligned}$$

Then Eqs. (2.13) can be used to determine the rest of the unknown coefficients in Eqs. $(2.12)_{1,2}$. Computation of the coefficients starts from the lower plate $(y = -1)$ with known boundary conditions for both the velocity and temperature problems and proceeds up to the upper plate $(y = 1)$. Consequently, the computation of the unknown functions is done from the upper plate up till the lower one.

In this study, computations have been carried out for $\alpha = 5$, $\mathrm{Pr} = 1$ and $\mathrm{Ec} = 0.2$. Grid-independence studies show that the computational domain $-1 < y < 1$ can be divided into intervals with the step size $\Delta y = 0.005$. Smaller step sizes do not show any significant change in the results. Convergence of the scheme is assumed when all of the unknowns u, v, T and H for the last two approximations differ from unity by less than 10^{-6} for all values of y in $-1 < y < 1$. Less than 7 approximations are required to satisfy these convergence criteria for all ranges of the parameters studied here.

3. Results and discussion

Figure 1 shows the velocity profile for various values of the parameters a and M and for $b = 0$. It is clear that an increase in the parameter M decreases the velocity u for all values of a as a result of the increasing damping force on u. On the other hand, an increase in the parameter a increases u for all M due to the decrease in the viscosity. The influence of the parameter a on u is more apparent for smaller porosity parameters. The parameter a has a pronounced effect on the symmetry of the velocity profile about the plane $y = 0$.

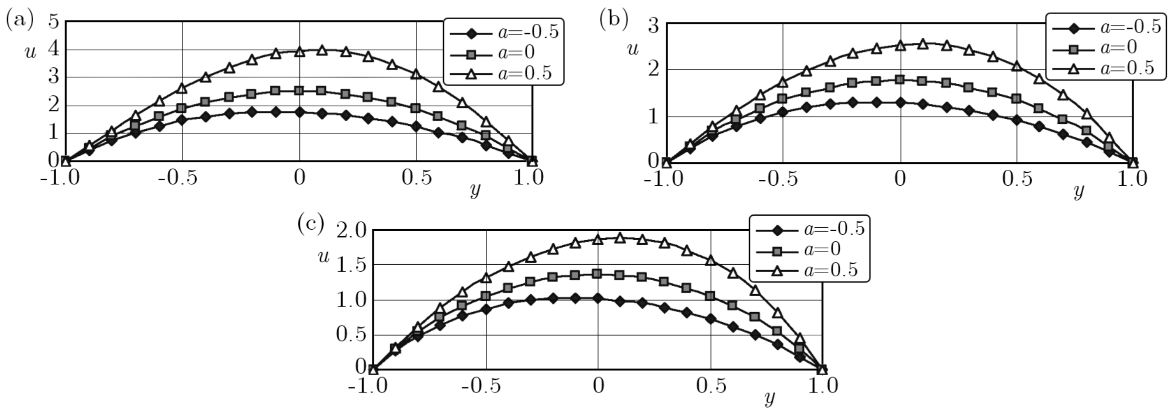

Fig. 1. Variation of u with y for various values of a; (a) $b = 0$, $M = 0$, (b) $b = 0$, $M = 1$, (c) $b = 0$, $M = 2$

Figure 2 shows the temperature profile for various values of the parameters a and M and for $b = 0$. An increase in the parameter M decreases T as a result of decreasing u and, in turn, decreases the viscous dissipation. On the other hand, increasing the parameter a cause an increase in T for all values of M due to its effect in increasing u and consequently the viscous dissipation. It is also clear also that the effect of a on T is more clear for smaller M.

Figure 3 presents the temperature profile for different values of the parameters b and M and for $a = 0$. The figure depicts that the effect of b on T depends on t, and that increasing b increases T for small times, but decreases T with time progression. This occurs because, for small time, the centre of the channel acquires heat by conduction from the upper hot plate, but after large time, when u is large, the viscous dissipation is large at the centre and, consequently centre looses heat by conduction. It is also clear that the effect of b on T is more pronounced for smaller M. The parameter b has no significant effect on u in spite of the coupling between the momentum and energy equations.

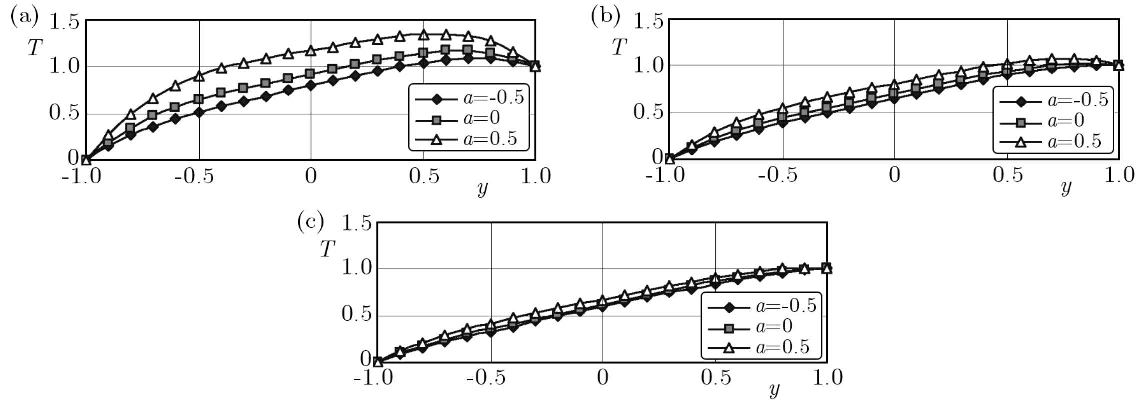

Fig. 2. Variation of T with y for various values of a; (a) $b=0$, $M=0$, (b) $b=0$, $M=1$, (c) $b=0$, $M=2$

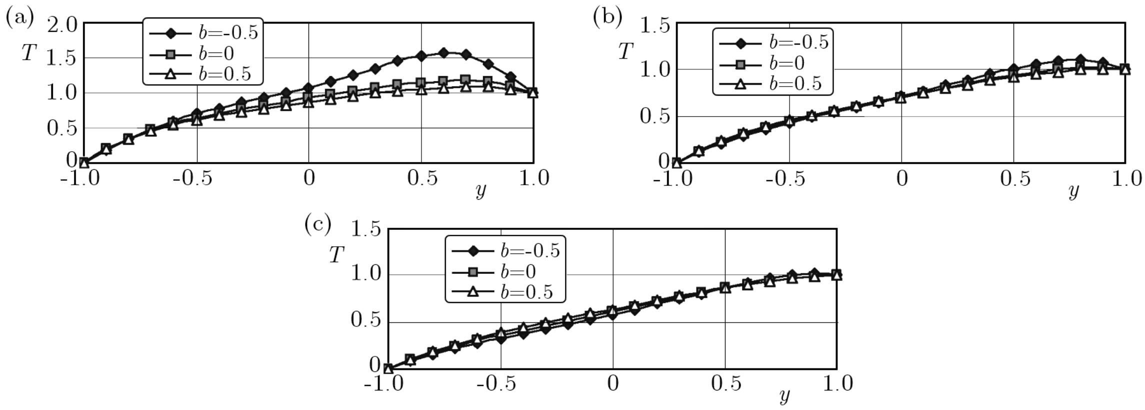

Fig. 3. Variation of T with y for various values of b; (a) $a=0$, $M=0$, (b) $a=0$, $M=1$, (c) $a=0$, $M=2$

Table 1 presents the variation of temperature at the centre of the channel with the parameters a and b for $M=1$. In Table 1, T increases with increasing a for all values of b. On the other hand, for smaller values of a, increasing b increases T, while for higher values of a, increasing b decreases it. This is because a decrease in a decreases the velocity and its gradient which decreases dissipation and makes the centre gain heat by conduction. Higher values of a increases dissipation, and the centre looses heat by conduction which results in a decrease in T with the increment of b.

Table 1. Variation of the steady state temperature T at $y=0$ for various values of a and b ($M=1$)

T	$a=-0.5$	$a=-0.1$	$a=0.0$	$a=0.1$	$a=0.5$
$b=-0.5$	0.6315	0.6885	0.7086	0.7322	0.8822
$b=-0.1$	0.6475	0.6902	0.7046	0.7210	0.8171
$b=0.0$	0.6514	0.6916	0.7049	0.7202	0.8082
$b=0.1$	0.6549	0.6927	0.7052	0.7195	0.8007
$b=0.5$	0.6638	0.6945	0.7045	0.7157	0.7777

Tables 2 and 3 show the influence of the parameters a and b on the skin friction coefficients at both walls τ_L and τ_U, respectively, for $M=1$. Increasing a increases τ_L and the magnitude of τ_U for all b. The effect of b on τ_L and τ_U depends on a. For small a, increasing b increases τ_L but decreases the magnitude of τ_U. On the other hand, for higher values of a, increasing b decreases τ_L but increases the magnitude of τ_U. Tables 4 and 5 show the effect of the parameters a and b on the Nusselt numbers at both walls Nu_L and Nu_U, respectively, for $M = 1$. Increasing a increases Nu_L and the magnitude of Nu_U. However, although an increase in b decreases the magnitude of Nu_U, its effect on Nu_L depends on the value of b. Therefore, increasing b decreases Nu_L, but increasing b increases Nu_L more.

Table 2. Variation of the steady state skin friction coefficient at the lower plate τ_L for various values of a and b $(M = 1)$

τ_L	$a = -0.5$	$a = -0.1$	$a = 0.0$	$a = 0.1$	$a = 0.5$
$b = -0.5$	3.4975	3.7445	3.8098	3.8764	4.1570
$b = -0.1$	3.5081	3.7483	3.8097	3.8717	4.1226
$b = 0.0$	3.5096	3.7488	3.8098	3.8710	4.1184
$b = 0.1$	3.5111	3.7493	3.8097	3.8705	4.1143
$b = 0.5$	3.5164	3.7509	3.8097	3.8688	4.1031

Table 3. Variation of the steady state skin friction coefficient at the lower plate τ_U for various values of a and b $(M = 1)$

τ_U	$a = -0.5$	$a = -0.1$	$a = 0.0$	$a = 0.1$	$a = 0.5$
$b = -0.5$	−2.4874	−3.4992	−3.8097	−4.1478	−5.8263
$b = -0.1$	−2.4739	−3.4942	−3.8097	−4.1545	−5.8958
$b = 0.0$	−2.4718	−3.4936	−3.8097	−4.1553	−5.9022
$b = 0.1$	−2.4699	−3.4929	−3.8097	−4.1559	−5.9076
$b = 0.5$	−2.4645	−3.4913	−3.8097	−4.1579	−5.9197

Table 4. Variation of the steady state Nusselt number at the lower plate Nu_L for various values of a and b $(M = 1)$

Nu_L	$a = -0.5$	$a = -0.1$	$a = 0.0$	$a = 0.1$	$a = 0.5$
$b = -0.5$	1.1654	1.3366	1.3910	1.4521	1.7902
$b = -0.1$	1.1334	1.3056	1.3593	1.4185	1.7358
$b = 0.0$	1.1347	1.3073	1.3609	1.4199	1.7342
$b = 0.1$	1.1380	1.3110	1.3646	1.4234	1.7350
$b = 0.5$	1.1611	1.3354	1.3888	1.4474	1.7511

Table 5. Variation of the steady state Nusselt number at the upper plate Nu_U for various values of a and b $(M = 1)$

Nu_U	$a = -0.5$	$a = -0.1$	$a = 0.0$	$a = 0.1$	$a = 0.5$
$b = -0.5$	−0.3986	−0.7259	−0.8288	−0.9434	−1.5755
$b = -0.1$	−0.1596	−0.3464	−0.4046	−0.4689	−0.8189
$b = 0.0$	−0.1396	−0.3084	−0.3609	−0.4189	−0.7321
$b = 0.1$	−0.1255	−0.2796	−0.3274	−0.3802	−0.6632
$b = 0.5$	−0.1006	−0.2148	−0.2499	−0.2886	−0.4919

4. Conlusions

The steady flow through a porous medium between two parallel plates is examined. The viscosity and thermal conductivity of the fluid are assumed to vary with temperature. The effects of the porosity parameter M, the viscosity parameter a, and the thermal conductivity parameter b

on the velocity and temperature fields are investigated. It is found that the parameters a and b have a more apparent effect on the velocity and temperature distributions for smaller values of M. On the other hand, the parameter b has no significant effect on the velocity u, however, it has a marked effect on the temperature, and its effect greatly depends on the parameters M and a. It is of interest to find that the variation of the Nusselt number at the lower plate with the thermal conductivity parameter b depends on the values of b.

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O efektywności uzmienniania parametrów fizycznych stacjonarnego przepływu przez ośrodek porowaty z uwzględnieniem przesyłu ciepła pomiędzy równoległymi płytami

Streszczenie

W pracy zbadano wpływ zmiany parametrów fizycznych na ustalony przepływ przez porowaty ośrodek przy jednoczesnym przesyle ciepła pomiędzy równoległymi płytami ograniczającym obszar ośrodka. Założono, że lepkość i współczynnik przewodnictwa cieplnego są zależne od temperatury. Do utrzymania stacjonarnego przepływu wprowadzono stały gradient ciśnienia w kierunku osiowym, a dwie płyty utrzymano w stałych, lecz różniących się temperaturach. Efekt dyssypacji wiskotycznej uwzględniono w równaniu energii. Zaprezentowano numeryczne rozwiązanie nieliniowych, sprzężonych równań ruchu oraz równania energii. Omówiono wpływ porowatości ośrodka, zmiennej lepkości i współczynnika przewodnictwa cieplnego na rozkład prędkości i temperatury w obszarze przepływu.

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