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NEW ANALYSIS OF NATURAL CONVECTION BOUNDARY LAYER FLOW ON A HORIZONTAL PLATE WITH VARIABLE WALL TEMPERATURE

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In this study, steady laminar free convection boundary layer flow on a horizontal plate is investigated through analytical solutions. By transforming the governing non-dimensional boundary layer equations into an ordinary differential equation, the application of the Homotopy Analysis Method can be practical. So in this case, the analytical results for different Prandtl numbers and constant M values which portray the power index are achieved. The trend goes on large amounts of $(M \gg 1)$ and the results are compared with other efforts. Furthermore, the effects of different values of Prandtl number and M values on temperature and velocity profiles are verified.

Key words: free convection, HAM (Homotopy Analysis Method), analytical solution

1. Introduction

Steady free convection boundary layer flow past a horizontal plate is a very practical and basical issue that is worthy enough to be discussed perfectly. So researchers in the heat transfer field paid tangible attention toward this problem. Here, because of the significant effect of buoyancy on the flow field, going through depth of the layer pressure gradient changes. Despite being predetermined at the edge of the boundary layer, it should be founded as a part of solving process in accordance with velocity and temperature. Former studies verified the effect of buoyancy for this problem, and in their effort it is gotten that they focused on flows over plates which are smoothly cooled or heated. Stewartson (1958) concentrated on an isothermal horizontal semi-infinite plate and his results, which were later discussed by Gill *et al.* (1965), announced the existence of similarity solutions just for below and above a cooled and heated surface, respectively. Rotem and Claassen (1969) and Raju *et al.* (1984) discussed on the heated downward-facing or cooled upward-facing plates numerically. Identically, Clifton and Chapman (1969) utilized an integral method for this problem. Jones (1973), and Pera and Gebhart (1973) performed their study on an inclined plate with a isothermal condition. Also Yu and Lin (1988), and Lin *et al.* (1989) verified an arbitrary inclined plate. Afzal *et al.* (1986), Lin and Yu (1988), Brouwers (1993) and Chen *et al.* (1993) studied the influence of suction or blowing on the free convection of a horizontal flat plate. Ackroyd (1976) surveyed the non-Boussinesq effects.

Chen *et al.* (1986) verified different occasions for a plate (horizontal, inclined, and vertical) with changing wall temperature or surface heat flux. A finite-difference method was applied to have a numerical solution for the non-dimensional form of the governing equation. Mahajan and Gebhart (1980) and Afzal (1985) studied higher order effects in free convection flow over horizontal surfaces. Daniels (1992) studied an insulated horizontal wall over which there was a thermal boundary layer flowing. In this flow, the velocity and temperature fields are coupled by buoyancy. Many researchers worked on free convection and made great effort on this issue such as Rotem and Classen (1969), Goldstein *et al.* (1973) and Kitamura and Kimura (1995) who had technical papers.

However, little by little more theoretical studies were performed on scrutinizing the effect of buoyancy on thermal boundary layers over horizontal flat surfaces in which wall temperature and surface heat flux vary. The Homotopy Analysis Method was another approach which was handled by other researchers in many previous papers (Liao, 1992, 2003; Hayat and Sajid, 2007; Domairry and Nadim 2008, Domairry and Fazeli 2009; Abouei Mehrizi *et al.*, 2011). Here, the main aim is to consider free convection boundary layer equations governing the flow on a heated horizontal flat plate facing upward when the non-dimensional surface temperature is given by $T_w(x) = x^M$. In this form, M is a positive constant and x is the coordinate measured along the plate.

2. Description of the problem and governing equations

Consider the problem of free convection boundary layer flow on a heated horizontal flat plate facing upward when the non-dimensional surface temperature is given by $T_w(x) = x^M$, where x is the coordinate measured along the plate from the leading edge and M is a constant. The governing boundary layer equations in dimensional form are

$$\begin{array}{ll} \dfrac{\partial \overline{u}}{\partial \overline{x}} + \dfrac{\partial \overline{v}}{\partial \overline{y}} = 0 & \overline{u}\dfrac{\partial \overline{u}}{\partial \overline{x}} + \overline{v}\dfrac{\partial \overline{u}}{\partial \overline{y}} = -\dfrac{1}{\rho}\dfrac{\partial \overline{p}}{\partial \overline{x}} + \upsilon\dfrac{\partial^2 \overline{u}}{\partial \overline{y}^2} \\ -\dfrac{1}{\rho}\dfrac{\partial \overline{p}}{\partial \overline{y}} + g_r\beta(\overline{T} - T_\infty) = 0 & \overline{u}\dfrac{\partial \overline{T}}{\partial \overline{x}} + \overline{v}\dfrac{\partial \overline{T}}{\partial \overline{y}} = \alpha\dfrac{\partial^2 \overline{T}}{\partial \overline{y}^2} \end{array} \tag{2.1}$$

Subject to the boundary conditions

$$\begin{array}{llll} \overline{u} = \overline{v} = 0 & \overline{T} = T_w(\overline{x}) & & \text{at} \quad \overline{y} = 0 \\ \overline{u} = 0 & \overline{T} = T_\infty & \overline{p} = p_\infty & \text{as} \quad y \to \infty \end{array} \tag{2.2}$$

where $\overline{x}$ and $\overline{y}$ are the coordinates measured along the plate and normal to it, respectively, $(\overline{u}, \overline{v})$ are the velocity components along the $(\overline{x}, \overline{y})$ axes, $\overline{p}$ is the pressure, $\overline{T}$ is the local fluid temperature, g_r is the acceleration due to gravity, ρ is the fluid density, β is the coefficient of thermal expansion, υ is the kinematic viscosity and α is the constant thermal diffusivity of the fluid.

We introduce now the following non-dimensional variables

$$\begin{array}{llll} x = \dfrac{\overline{x}}{L} & y = Gr^{1/5}\Big(\dfrac{\overline{y}}{L}\Big) & u = Gr^{-2/5}\Big(\dfrac{L}{\upsilon}\Big)\overline{u} & v = Gr^{-1/5}\Big(\dfrac{L}{\upsilon}\Big)\overline{v} \\ \overline{T} = T_\infty + \varDelta\overline{T}T & & p = G^{-4/5}\dfrac{\overline{p} - p_\infty}{\rho\upsilon^2/L^2} & \end{array} \tag{2.3}$$

Substituting variables (2.3) into equations (2.1) leads to the following non-dimensional equations

$$\begin{array}{ll} \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 & u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} = -\dfrac{\partial p}{\partial x} + \dfrac{\partial^2 u}{\partial y^2} \\ -\dfrac{\partial p}{\partial y} + T = 0 & u\dfrac{\partial T}{\partial x} + v\dfrac{\partial T}{\partial y} = \dfrac{1}{\Pr}\dfrac{\partial^2 T}{\partial y^2} \end{array} \tag{2.4}$$

and boundary conditions (2.2) become

$$\begin{array}{llll} u = v = 0 & T = T_w(x) = x^M & & \text{at} \quad y = 0 \\ u \to 0 & T \to 0 & p \to 0 & \text{as} \quad y \to \infty \end{array} \tag{2.5}$$

Further, we look for a similarity solution of these equations of the form (Chen *et al.* 1986)

$$\begin{aligned} &\psi = x^{(M+3)/5} f(\eta) \qquad && T = x^M \theta(\eta) \\ &p = x^{(4M+3)/5} g(\eta) \qquad && \eta = y x^{(M-2)/5} \end{aligned} \tag{2.6}$$

where ψ is the stream function, which is defined as

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \tag{2.7}$$

and automatically satisfy continuity equation $(2.4)_1$.

Substitution of (2.6) into equations $(2.4)_2$ to $(2.4)_4$ gives the following ordinary differential equations

$$\begin{aligned} &5f''' + (M+3)ff'' - (2M+1)f'^2 - (4M+2)g - (M-2)\eta g' = 0 \\ &g' = \theta \qquad \frac{5}{\mathrm{Pr}}\theta'' + (M+3)f\theta' - 5Mf'\theta = 0 \end{aligned} \tag{2.8}$$

and boundary conditions (2.5) become

$$\begin{aligned} &f(0) = f'(0) \qquad && \theta(0) = 1 \\ &f'(\infty) = 0 \qquad && \theta(\infty) = 0 \qquad g(\infty) = 0 \end{aligned} \tag{2.9}$$

where primes denote differentiation with respect to η.

Quantities of physical importance in this problem are the skin friction τ_w and the heat transfer at the plate q_w, which are given in non-dimensional form by Chen *et al.* (1986, 1993)

$$\tau_w = x^{(3M-1)/5} f''(0) \qquad q_w = x^{2(3M-1)/5}[-\theta'(0)] \tag{2.10}$$

For large values of $M(\gg 1)$, solutions to equations (2.8) subject to boundary conditions (2.9) can be found using the transformation

$$f = M^{(-3/5)} F(z) \qquad \theta = \theta(z) \qquad g = M^{-2/5} G(z) \qquad z = M^{-2/5}\eta \tag{2.11}$$

Substituting (2.11) into equations (2.8) yields

$$\begin{aligned} &5F''' + \Big(1 + \frac{3}{M}\Big)FF'' - \Big(2 + \frac{1}{M}\Big)F'^2 - \Big(4 + \frac{2}{M}\Big)G - \Big(1 - \frac{2}{M}\Big)zG' = 0 \\ &G' = \theta \qquad \frac{5}{\mathrm{Pr}}\theta'' + \Big(1 + \frac{3}{M}\Big)F\theta' - 5F'\theta = 0 \end{aligned} \tag{2.12}$$

Subject to the boundary conditions

$$\begin{aligned} &F(0) = F'(0) \qquad && \theta(0) = 1 \\ &F(\infty) = 0 \qquad && \theta(\infty) = 0 \qquad G(\infty) = 0 \end{aligned} \tag{2.13}$$

where primes now denote differentiation with respect to z.

Substituting equation $(2.12)_2$ into equation $(2.12)_3$, we have

$$\begin{aligned} &5F''' + \Big(1 + \frac{3}{M}\Big)FF'' - \Big(2 + \frac{1}{M}\Big)F'^2 - \Big(4 + \frac{2}{M}\Big)G - \Big(1 - \frac{2}{M}\Big)zG' = 0 \\ &\frac{5}{\mathrm{Pr}}G''' + \Big(1 + \frac{3}{M}\Big)FG'' - 5F'G' = 0 \end{aligned} \tag{2.14}$$

and

$$\begin{aligned} &F(0) = F'(0) \qquad && G'(0) = 1 \\ &F(\infty) = 0 \qquad && G'(\infty) = 0 \qquad G(\infty) = 0 \end{aligned} \tag{2.15}$$

Solving the equations, the functions F and G will be calculated, and to evaluate the values of $f''(0)$ and $-\theta'(0)$ the following relations are used

$$f''(0) = M^{1/5}F''(0) \qquad -\theta'(0) = -g''(0) = -M^{2/5}G''(0) \tag{2.16}$$

3. Homotopy analysis solution

For HAM solution of the governing equations, we choose the auxiliary linear operators $L_1(F)$ and $L_2(G)$ as follows

$$L_1(F) = F''' + F'' \qquad L_2(G) = G''' + G'' \tag{3.1}$$

Solving Eqs. (3.1) with initial conditions of Eqs. (2.15), one could make the initial guesses

$$L_1(C_1 + C_2 + C_3 \mathrm{e}^{-z}) \qquad L_2(C_4 + C_5 + C_6 \mathrm{e}^{-z}) \tag{3.2}$$

and

$$F_0(z) = 0 \qquad G_0(z) = -\mathrm{e}^{-z} \tag{3.3}$$

where c_i $(i = 1, \ldots, 5)$ are considered as constants. $P \in [0, 1]$ denotes the embedding parameter and $\hbar$ indicates non-zero auxiliary parameters. We then construct the following equations.

3.1. Zeroth order deformation equations

$$\begin{aligned}(1-P)L_1[F(z;p) - F_0(z)] &= p\hbar H_1(z) N_1[F(z;p), G(z;p)]\\ (1-P)L_2[G(z;p) - G_0(z)] &= p\hbar H_2(z) N_2[F(z;p), G(z;p)]\end{aligned} \tag{3.4}$$

and

$$\begin{array}{lll} F(0;p) = 0 & F'(0;p) = 0 & F'(\infty;p) = 0 \\ G'(0;p) = 1 & G'(\infty;p) = 0 & G(\infty;p = 0 \end{array} \tag{3.5}$$

and

$$\begin{aligned} N_1[F(z;P)] &= 5\frac{d^3F(z)}{dz^3} + (1 + \frac{3}{M}\Big)F\frac{d^2F(z)}{dz^2} - \Big(2 + \frac{1}{M}\Big)\Big(\frac{dF(z)}{dz}\Big)^2 \\ &\quad - \Big(4 + \frac{2}{M}\Big)G(z) - \Big(1 - \frac{2}{M}\Big)z\frac{dG(z)}{dz} \\ N_2[G(z;p)] &= \frac{5}{\mathrm{Pr}}\frac{d^3G(z)}{dz^3} + \Big(1 + \frac{3}{M}\Big)F(z)\frac{d^2G(z)}{dz^2} - 5\frac{dF(z)}{dz}\frac{dG(z)}{dz} \end{aligned} \tag{3.6}$$

For $p = 0$ and $p = 1$ we have

$$\begin{array}{ll} F(z;0) = F_0(z) & F(z;1) = F(z) \\ G(z;0) = G_0(z) & G(z;1) = G(z) \end{array} \tag{3.7}$$

As the embedding parameter increases from 0 to 1, $F(z;0)$ and $G(z;0)$ vary from the initial guess $F_0(z)$, $G_0(z)$ to the exact solution $F(z)$ and $G(z)$.

With expanding $F(z;q)$ and $G(z;q)$ in Taylor series with respect to q, we have

$$F(z;p) = F_0(z) + \sum_{m-1}^{\infty} F_m(z)p^m \qquad G(z;p) = G_0(z) + \sum_{m-1}^{\infty} G_m(z)p^m \tag{3.8}$$

where

$$F_m(z) = \frac{1}{m!}\frac{\partial^m F(z;p)}{\partial p^m} \qquad G_m(z) = \frac{1}{m!}\frac{\partial^m G(z;p)}{\partial p^m} \tag{3.9}$$

3.2. mth order deformation equations

According to definition (3.9) the governing equations can be deduced from the zero order deformation Eqs. (3.4). Define the vector

$$\mathbf{F}_n = [F_0(z), F_1(z), F_2(z), \dots, F_n(z)] \tag{3.10}$$

Differentiating Eqs. (3.4) m times with respect to the embedding parameter p and then setting $p = 0$ and finally dividing them by $m!$, we have the so-called mth order deformation equation

$$\begin{aligned} &L_1[F_m(z) - \chi_m F_{m-1}(z)] = \hbar H(z) R1_m(z) \\ &L_2[G_m(z) - \chi_m G_{m-1}(z)] = \hbar H(z) R2_m(z) \end{aligned} \tag{3.11}$$

and

$$\begin{array}{lll} F(0) = 0 & F'(0) = 0 & F'(\infty) = 0 \\ G'(\infty) = 0 & G'(0) = 1 & G(\infty) = 0 \end{array} \tag{3.12}$$

and

$$\begin{aligned} R1_m(z) &= 5\frac{d^3 F_{m-1}(z)}{dz^3} + \Big(1 + \frac{3}{M}\Big) \sum_{k=0}^{m-1} \Big(F_{m-1-k} \frac{d^2 F_k(z)}{dz^2}\Big) \\ &\quad - \Big(2 + \frac{1}{M}\Big) \sum_{k=0}^{m-1} \Big(\frac{dF_{m-1-k}(z)}{dz} \frac{dF_k(z)}{dz}\Big) - \Big(4 + \frac{2}{M}\Big) G_{m-1}(z) - \Big(1 - \frac{2}{M}\Big) z \frac{dG_{m-1}(z)}{dz} \\ R2_m(z) &= \frac{5}{\mathrm{Pr}} \frac{d^3 G_{m-1-k}(z)}{dz^3} + \Big(1 + \frac{3}{M}\Big) \sum_{k=0}^{m-1} \Big(F_{m-1-k}(z) \frac{d^2 G_k(z)}{dz^2}\Big) \\ &\quad - 5 \sum_{k=0}^{m-1} \Big(\frac{dF_{m-1-k}(z)}{dz} \frac{dG_k(z)}{dz}\Big) \end{aligned} \tag{3.13}$$

where

$$\chi_m = \begin{cases} 0 & m \leqslant 1 \\ 1 & m > 1 \end{cases} \tag{3.14}$$

Finally, according to the third rule of solution expression, the auxiliary function $H(z)$ should be in the form

$$H_1(z) = H_2(z) = \mathrm{e}^{-z} \tag{3.15}$$

Solving Eqs. (3.11) by mathematical software such as Maple, for example with $M = 1$ and $\mathrm{Pr} = 1$, one successively obtains

$$\begin{aligned} &f_0(z) = 0 \qquad\qquad f_1(z) = -2h\mathrm{e}^{-2z} - \frac{1}{4} h\mathrm{e}^{-2z} z + \frac{15}{4} h\mathrm{e}^{-z} - \frac{7}{4} h \\ &g_0(z) = -\mathrm{e}^{-z} \qquad\quad g_1(z) = -\frac{1}{40} h\mathrm{e}^{-2z} - \frac{7}{20} h\mathrm{e}^{-2z} z - \frac{3}{10} h\mathrm{e}^{-z} \\ &f_2(z) = -\frac{1}{432} h(\mathrm{e}^{-3z}(2090h + 300, hz) + \mathrm{e}^{-2z}(864 - 4158h - 270hz - 108z) \\ &\qquad + \mathrm{e}^{-z}(-1620 + 2130h) - 35h + 756 \\ &g_2(z) = \frac{1}{288} h\mathrm{e}^{-z}[\mathrm{e}^{-3z}(72h + 9hz) + \mathrm{e}^{-2z}(-860h) + \mathrm{e}^{-z}(396h - 360) + 1509h + 720 \end{aligned} \tag{3.16}$$

4. Convergence of HAM solution

HAM provides us with great freedom in choosing the solution of a nonlinear problem by different base functions and has a great effect on the convergence region. Therefore, we should ensure that the solution converges. On the other hand, the convergence and rate of approximation for the HAM solution strongly depends on the value of the auxiliary parameter h. By means of the so-called h-curves, it is easy to find out the so-called valid regions of auxiliary parameters to gain a convergent solution series.

To demonstrate the influence of h on the convergence of solution, we plotted the so-called h-curve of $f(0)$ and $\theta(0)$ by 11th to 17th order approximation of the solution as shown in Fig. 1a and 1b.

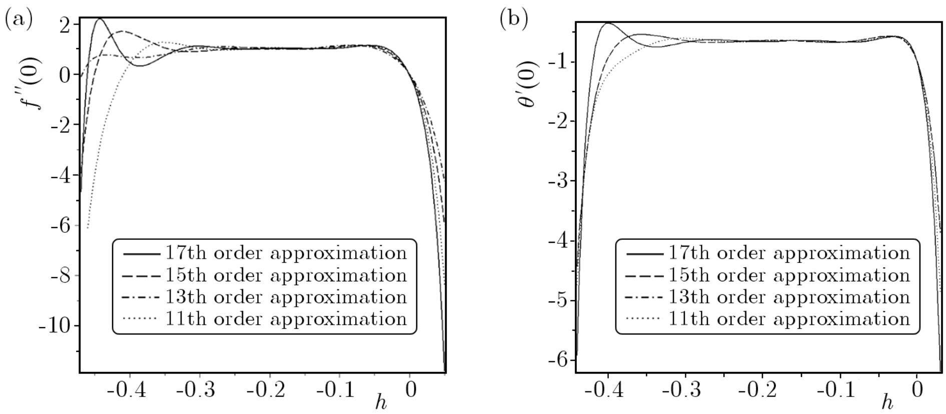

Fig. 1. The h-validity of $f''(0)$ (a) and $\theta'(0)$ (b)

5. Results and discussion

In the literature, the governing equation is solved with an analytical method called HAM and, as it is portrayed, good and acceptable agreement is achieved in comparison with other efforts. In Table 1, the amounts of the heat transfer coefficient, $-\theta'(0)$ are presented for $M = 0$ and three values of the Prandtl number. For $\mathrm{Pr} = 1$, the results by Pop *et al.* (2009), Raju *et al.* (1984) and Lin *et al.* (1989) are also put in this table, and it is seen that the gratifying correspondence exists.

Table 1. Amounts of $-\theta'(0)$ for $M = 0$ and different Pr

Pr	Present	Pop *et al.* (2009)	Raju *et al.* (1984)	Lin *et al.* (1989)
1	0.3866	0.3905	0.3881	0.3905
7	0.6317	0.6300		
10	0.6813	0.6833		

Normally, the Prandtl number has a reciprocal relationship with thermal conductivity. So when Pr increases, because of a higher heat transfer rate at the surface, the heat transfer coefficient increases, and this trend is shown in Table 1. Subsequently, Table 2 represents the values of heat transfer coefficient $-\theta'(0)$ and skin coefficient $f''(0)$ when $\mathrm{Pr} = 1$ but M varies. On the other hand, Table 3 demonstrates the values of $-\theta'(0)$ and $f''(0)$ when $M = 2$ but Pr varies. Finally, Table 4 shows the amounts of heat transfer coefficient and skin fiction coefficient when $\mathrm{Pr} = 7$ but M varies. As it is vivid, acceptable agreement is achieved, and by comparing the results with numerical data, it is gotten that enhancing the value of M makes more inclination between the results. Alternatively, for lower values of M and Prandtl number, more precise results are gained.

Table 2. Amounts of $-\theta'(0)$ and $f''(0)$ when $\mathrm{Pr} = 1$ but M varies

	HAM		Numeric		Error [%]	
M	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
1	0.9953	0.6537	0.9910	0.6532	0.433	0.076
2	1.0857	0.8142	1.0811	0.8129	0.425	0.159
3	1.1494	0.9311	1.1499	0.9360	0.043	0.523
4	1.2053	1.0366	1.2059	1.0388	0.049	0.211
5	1.2530	1.1118	1.2532	1.1283	0.015	1.462
6	1.2944	1.1867	1.2945	1.2083	0.007	1.787
7	1.3311	1.2751	1.3311	1.2811	0	0.468
8	1.3640	1.337	1.3641	1.3482	0.007	0.830
9	1.3945	1.396	1.3943	1.4106	0.014	1.035
10	1.4213	1.4580	1.4220	1.4692	0.049	0.762

Table 3. Amounts of $-\theta'(0)$ and $f''(0)$ when $M = 2$ but Pr varies

	HAM		Numeric	
Pr	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
1	1.083	0.8127	1.0811	0.8129
7	0.5149	1.2536	0.4911	1.2480
10	0.4682	1.3709	0.4259	1.3464

Table 4. Amounts of $-\theta'(0)$ and $f''(0)$ when $\mathrm{Pr} = 7$ but M varies

	HAM		Numeric		Error [%]	
M	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
1	0.4523	1.0126	0.4560	1.0144	0.811	0.177
2	0.5149	1.2536	0.4911	1.2480	4.846	0.448
3	0.5507	1.4463	0.5195	1.4291	6.006	1.203
4	0.5785	1.5728	0.5431	1.5809	6.518	0.512
5	0.5991	1.7068	0.5635	1.7135	6.317	0.391
6	0.6100	1.8263	0.5810	1.8325	4.991	0.338
7	0.6341	1.9341	0.5973	1.9410	6.161	0.355
8	0.6496	2.0335	0.6117	2.0405	6.195	0.343
9	0.6625	2.1262	0.6249	2.1336	6.016	0.346
10	0.6751	2.2124	0.6371	2.2209	5.964	0.382

Passing through this information, velocity profiles and temperature profiles are investigated, too. Figure 2a shows the velocity profiles $f'(\eta)$, when $\mathrm{Pr} = 1$ and M has different amounts. Admitting the information of Table 2, here this picture claims that when there is larger amount of M, the velocity gradient at the surface is larger and this leads to having larger skin friction. Furthermore, Fig. 2b admits Table 2, and by demonstrating the temperature profiles $\theta(\eta)$ it is seen that the temperature gradient at the surface and M values have a direct relationship. Additionally, Fig. 3a presents the velocity profile when $M = 2$ but Pr has different amounts. By changing the amounts of the Prandtl number, this section shows that when there are smaller values for Pr, larger amounts of the velocity gradient is found at the surface and again this leads to a large skin friction coefficient. This data reviews the information presented in Table 3.

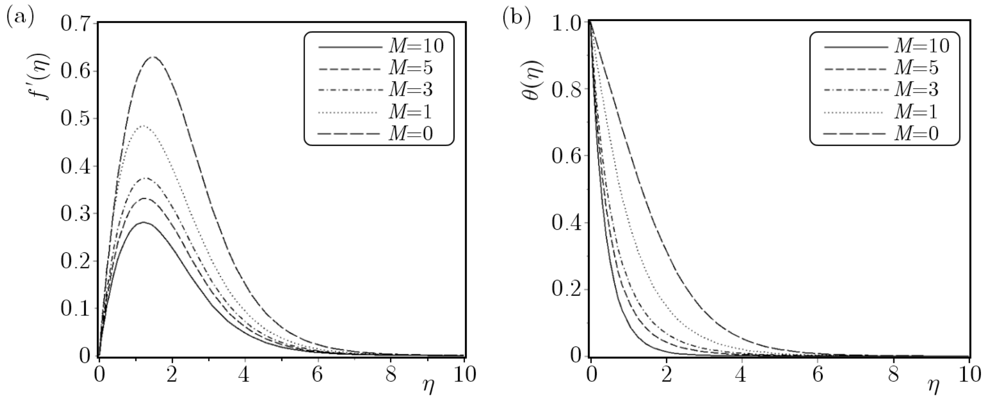

Fig. 2. Velocity profiles $f'(\eta)$ (a) and temperature profiles $\theta(\eta)$ (b) for $\mathrm{Pr} = 1$ and different values of M

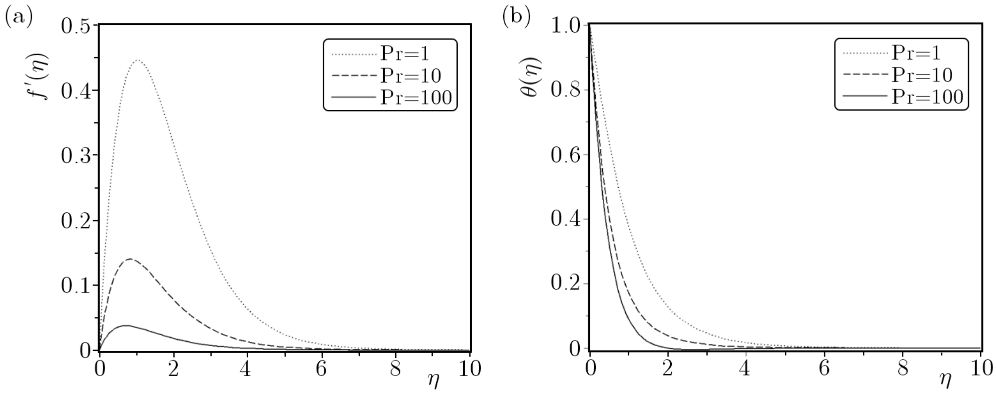

Fig. 3. Velocity profiles $f'(\eta)$ (a) and temperature profiles $\theta(\eta)$ (b) for $M = 2$ and different Prandtl numbers

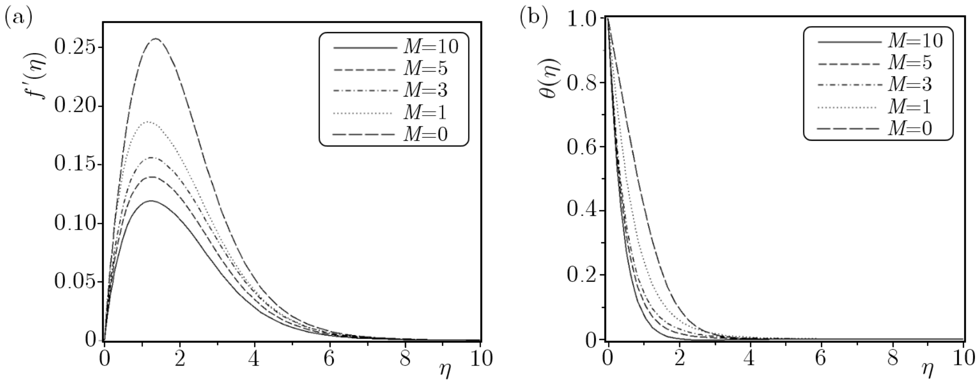

Fig. 4. Velocity profiles $f'(\eta)$ (a) and temperature profiles $\theta(\eta)$ (b) for $\mathrm{Pr} = 7$ and different values of M

Figure 3b also admits Table 3, and it portrays that when Pr diminishes, the temperature gradient at the surface increases. It is clear that when it is talked about a higher Prandtl number fluid, it has high viscosity and this makes the thermal boundary layer thinner and heat transfer rate higher at the surface. Figure 4a shows the velocity profiles $f'(\eta)$ when $\mathrm{Pr} = 7$ and M gets different amounts. A large skin friction coefficient is obtained by a large velocity gradient at the surface. This is the reality for large M and is depicted in Fig. 4a correspondent to Table 4. The last effort in Fig. 4b presents the direct relationship between M and the temperature gradient at the surface in a way that increasing M increases the temperature gradient, too. And this is said previously in Table 4. Totally, it can be mentioned that for validating these achievements the satisfaction of boundary conditions is adequate, and by reviewing Figs. 1 to 4, this is approved here.

6. Conclusion

In this effort, steady laminar free convection boundary layer flow on a horizontal plate with variable wall temperature has been investigated analytically. The subject is scrutinized using the Homotopy Analysis Method. The issue is based on the variation of two parameters, the Prandtl number and M, and in this paper it is shown how these two parameters influence the temperature profile, velocity profile, heat transfer coefficient and skin friction. Also the results are in good agreement with other works and the gratification of the boundary layers is witnessed. The results show:

- The homotopy analysis method is an applicable method to solve nonlinear fluid dynamic problems.
- The velocity gradient at the surface and skin friction increase with increasing values of M and the Prandtl number.
- The temperature gradient at the surface and M values have direct relationship.
- The temperature gradient at the surface increases by decreasing the Prandtl value.

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Nowa analiza naturalnego opływu konwekcyjnej warstwy przyściennej na poziomej płycie o zróżnicowanej temperaturze powierzchni

Streszczenie

W pracy zajęto się swobodnym, laminarnym opływem konwekcyjnej warstwy przyściennej na poziomej płycie w ujęciu czysto analitycznym. Bezwymiarowe równania warstwy przekształcono do postaci różniczkowej zwyczajnej, co pozwoliło na zastosowanie metody homotopii. Otrzymano analityczne wyniki określające wykładnik potęgowy opływu dla różnych wartości liczby Prandtla oraz stałej M. Badania rozszerzono na $\gg 1$, a rezultaty porównano z opracowaniami innych badaczy. Zweryfikowano również wpływ liczby Prandtla oraz stałej M na rozkład temperatury i prędkości opływu w rozważanej warstwie.

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