1. Introduction

In the case of automatic steering of self-guided missiles, kinematic equations of spins were related to missile equations of dynamics, using the Boltzmann-Hamel equations (Ładyżyńska-Kozdraś et al., 2008), which were developed in a relative frame of reference $Oxyz$, rigidly connected with the missile (Żyluk, 2009).

At the moment of detecting the target, it was assumed that the missile automatically passes from the flight on the programmed trajectory to tracking flight of the target according to the assumed algorithm, in this case –
the method of proportional navigation. Controlling the motion of the missile is carried out by the deflection of control surfaces, i.e. direction steer and height steer at the angles $\delta_V$ and $\delta_H$ respectively. The control laws constitute kinematic relations of deviations of set and realised flight parameters, stabilising the movement of the missile in channels of inclination and deflection. The realisation of the desired flight path of the missile is carried out by the autopilot, which generates control signals based on the compounds derived for the executive system of steering.

In the final section of navigation, various types of disturbances may affect the missile, such as wind or shock waves from shells exploding nearby. Therefore, additional stabilisation is necessary, in this case performed by the gyroscope. When searching for a ground target, the gyroscope axis, facing down, strictly outlines defined lines on the earth’s surface with its extension. The optic system positioned in the axis of the gyroscope, with a specific angle of view, can thus find the light or infrared signal emitted by a moving object. Therefore, kinematic parameters of reciprocal movement of the missile head and gyroscope axis should be selected to detect the target at the highest probability possible. After locating the target (receiving the signal by the infrared detector), the gyroscope goes into the tracking mode, i.e. from this point its axis takes a specific position in space, being directed onto the target.

2. The general equations of missile dynamics

Dynamic equations of missile motion in flight were derived in quasi-coordinates $\varphi$, $\theta$, $\psi$ and quasi-velocities $U$, $V$, $W$, $P$, $Q$, $R$ (Fig. 1) using the Boltzmann-Hamel equations true for mechanical systems in the system associated with the object.

The following correlation expresses them in a general form

$$\frac{d}{dt} \frac{\partial T^*}{\partial \omega_\mu} - \frac{\partial T^*}{\partial \pi_\mu} + \sum_{r=1}^{k} \sum_{\alpha=1}^{k} \gamma^r_{\mu\alpha} \frac{\partial T^*}{\partial \omega_r} \omega_\alpha = Q^*_\mu$$

(2.1)

where: $\alpha, \mu, r = 1, 2, \ldots, k$, $k$ – number of degrees of freedom, $\omega_\mu$ – quasi-velocities, $T^*$ – kinetic energy expressed in quasi-velocities, $\pi_\mu$ – quasi-coordinates, $Q^*_\mu$ – generalised forces, $\gamma^r_{\alpha\mu}$ – three-index Boltzmann factors, expressed by the following correlation

$$\gamma^r_{\alpha\mu} = \sum_{\delta=1}^{k} \sum_{\lambda=1}^{k} \left( \frac{\partial a_r\delta}{\partial q_\lambda} - \frac{\partial a_r\lambda}{\partial q_\delta} \right) b_{\delta\mu} b_{\lambda\alpha}$$

(2.2)
Relations between quasi-velocities and generalised velocities are

\[
\omega_\delta = \sum_{\alpha=1}^{k} a_{\delta \alpha} \dot{q}_\alpha \dot{q}_\delta = \sum_{\mu=1}^{k} b_{\delta \mu} \omega_\mu
\]  

(2.3)

where: \( \dot{q}_\delta \) – generalised velocities, \( q_k \) – generalised coordinates, \( a_{\delta \alpha} = a_{\delta \alpha}(q_1, q_2, \ldots, q_k) \) and \( b_{\delta \alpha} = b_{\delta \alpha}(q_1, q_2, \ldots, q_k) \) – coordinates being functions of generalised coordinates, while the following matrix correlation exists: \( [a_{\delta \mu}] = [b_{\delta \mu}]^{-1} \).

The Boltzmann-Hamel equations, after calculating the values of Boltzmann factors and indicating kinetic energy in quasi-velocities, a system of ordinary differential equations of the second order was received which describes the behaviour of the missile on the track during guidance.

In the frame of reference associated with the moving object \( Oxyz \), they have the following form

\[
M \ddot{V} + KV = Q
\]  

(2.4)

where: \( M \) is the inertia matrix, \( K \) – kinematic relations matrix, \( V \) – velocity vector and
\[
M = \begin{bmatrix}
m & 0 & 0 & 0 & S_z & -S_y \\
0 & m & 0 & -S_z & 0 & S_x \\
0 & 0 & m & S_y & -S_x & 0 \\
0 & -S_z & S_y & I_x & -I_{xy} & -I_{xz} \\
S_z & 0 & -S_x & -I_{yx} & I_y & -I_{yz} \\
-S_y & S_x & 0 & -I_{zx} & -I_{zy} & I_z \\
\end{bmatrix}
\]

\[
K = \begin{bmatrix}
0 & -R & Q & 0 & 0 & 0 \\
R & 0 & -P & 0 & 0 & 0 \\
-Q & P & 0 & 0 & 0 & 0 \\
0 & -W & V & 0 & -R & Q \\
W & 0 & -U & R & 0 & -P \\
-V & U & 0 & -Q & P & 0 \\
\end{bmatrix}
\]

\[V = \text{col}[U, V, W, P, Q, R]\]

The vector of forces and moments of external forces \(\mathbf{Q}\) affecting the moving missile is the sum of interactions of the centre, in which it is moving. This vector consists of forces: aerodynamic \(\mathbf{Q}_a\), gravitational \(\mathbf{Q}_g\), steering \(\mathbf{Q}_\delta\) and thrust \(\mathbf{Q}_T\). The flying missile is steered automatically. The steering is done in two channels: inclination by tilting the height steer by \(\delta_H\) and deflection by tilting the direction steer by \(\delta_V\).

\[
\mathbf{Q} = \mathbf{Q}_g + \mathbf{Q}_a + \mathbf{Q}_\delta + \mathbf{Q}_T = \text{col}[X, Y, Z, L, M, N] \quad (2.5)
\]

where

\[
X = -mg \sin \theta + T - \frac{1}{2} \rho S V_0^2 (C_{xa} \cos \beta \cos \alpha + C_{ya} \sin \beta \cos \alpha - C_{za} \sin \alpha) + X_Q Q + X_{\delta H} \delta_H + X_{\delta V} \delta_V
\]

\[
Y = mg \cos \theta \sin \phi - \frac{1}{2} \rho S V_0^2 (C_{xa} \sin \beta - C_{ya} \sin \beta) + Y_P P + Y_R R + Y_{\delta V} \delta_V
\]

\[
Z = mg \cos \theta \cos \phi - \frac{1}{2} \rho S V_0^2 (C_{xa} \cos \beta \sin \alpha + C_{ya} \sin \beta \sin \alpha + C_{za} \cos \alpha) + Z_Q Q + Z_{\delta H} \delta_H
\]

\[
L = -\frac{1}{2} \rho S V_0^2 l (C_{mxa} \cos \beta \cos \alpha + C_{mya} \sin \beta \sin \alpha - C_{mza} \sin \alpha) + L_P P + L_R R + L_{\delta V} \delta_V
\]

\[
M = -mg x_c \cos \theta \cos \phi - \frac{1}{2} \rho S V_0^2 l (C_{mxa} \sin \beta + C_{mza} \cos \beta) + M_Q Q + M_W W + M_{\delta H} \delta_H
\]

\[
N = mg x_c \cos \theta \sin \phi - \frac{1}{2} \rho S V_0^2 l (C_{mxa} \cos \beta \sin \alpha + C_{mya} \sin \beta \sin \alpha + C_{mza} \cos \alpha) + N_P P + N_R R + N_{\delta V} \delta_V
\]
while: \( m \) – missile mass, \( T \) – missile engine thrust vector (Fig. 2), \( \rho(H) \) – air density at a given flight altitude, \( l \) – characteristic dimension (total length of the missile body), \( S \) – area of reference surface (cross-section of rocket body), \( V_0 = \sqrt{U^2 + V^2 + W^2} \) – velocity of the missile flight, \( C'_{xa}, C'_{ya}, C'_{za}, C'_{mxa}, C'_{mya}, C'_{mza} \) – dimensionless coefficients of aerodynamic component forces, respectively: resistance \( P_{xa} \), lateral \( P_{ya} \) and bearing \( P_{za} \) as well as the moment of tilting \( M_{xa} \), inclination \( M_{ya} \) and deflection \( M_{za} \) (Fig. 2), \( XQ, Y_P, Y_R, Z_Q, L_P, L_R, M_Q, N_P, N_R \) – derivatives of aerodynamic forces and moments with respect to components of linear and angular velocities.

![Fig. 2. Forces and moments of forces acting on the missile in flight](image)

### 3. Layout of gyroscopic self-guidance of missiles

Figures 3 presents a simplified diagram of the layout of gyroscopic self-guidance of missiles onto a ground target emitting infrared radiation (e.g. a tank or a combat vehicle).

Figure 4 shows the general view of the missile used in the scanning and tracking gyroscope, i.e. one which can perform programmed movements while searching for the target and tracking movements after detecting the ground target through an adequate steering mounted on its frames.

The equations expressing dynamics of this kind of gyroscope steered by omitting the moments of inertia of its frames, have the following form

\[
J_{gk} \frac{d\omega_{gy2}}{dt} \cos \vartheta_g + J_{gk} \omega_{gx2} (\omega_{gz2} + \omega_{gy2} \sin \vartheta_g) + M_k \sin \vartheta_g = M_c
\]

\[
-J_{go} \left( \omega_{gz2} + \frac{d\vartheta_g}{dt} \right) \omega_{gx2} \cos \vartheta_g + \eta_c \frac{d\psi_g}{dt} = M_c
\]
Fig. 3. Diagram of the process of self-guiding a missile on a target

\[ J_{gk}\frac{d\omega_{gz}}{dt} - J_{gk}\omega_{gy}\omega_{gz} + J_{go}\left(\omega_{gz} + \frac{d\Phi_g}{dt}\right)\omega_{gy} + \eta_b\frac{d\varphi_q}{dt} = M_b \]  
\[ J_{go}\frac{d}{dt}\left(\omega_{gz} + \frac{d\Phi_g}{dt}\right) = M_k - M_r \]

where
Model of the final section of navigation...

\[ \omega_{gx_2} = P \cos \psi_g - R \sin \psi_g + \frac{d\psi_g}{dt} \]
\[ \omega_{gy_2} = (P \cos \psi_g + R \sin \psi_g) \sin \vartheta_g + \left( \frac{d\psi_g}{dt} + Q \right) \cos \vartheta_g \]
\[ \omega_{gz_2} = (P \cos \psi_g + R \sin \psi_g) \cos \vartheta_g - \left( \frac{d\psi_g}{dt} + Q \right) \sin \vartheta_g \]

and \( J_{go}, J_{gk} \) – moments of inertia of the gyroscope rotor in terms of its longitudinal axis and precession axis, respectively, \( \vartheta_g, \psi_g \) – angles of rotation of internal and external frames of the gyroscope, respectively, \( M_k, M_{rk} \) – torques driving the rotor of the gyroscope and friction forces in the rotor bearing in the frame, respectively.

The steering moments \( M_b, M_c \) affecting the gyroscope expressed by Eqs. (3.1), found on the PR board, we shall present as follows

\[ M_b = \Pi(t_o, t_w)M_{pb}(t) + \Pi(t_s, t_k)M_{bs} \]
\[ M_c = \Pi(t_o, t_w)M_{pc}(t) + \Pi(t_s, t_k)M_{cs} \]

(3.2)

where: \( \Pi(\cdot) \) are functions of the rectangular impulse, \( t_o \) – time moment of the beginning of spatial scanning, \( t_w \) – moment of detecting the target, \( t_s \) – moment of the beginning of target tracking, \( t_k \) – moment of completing the process of penetration, tracking and laser lighting of the target.
The program steering moments $M^P_b(t)$ and $M^P_c(t)$ put the axis of the gyroscope in the required motion and are found by the method of solving the inverse problem of dynamics (Osiecki and Stefański, 2008)

$$M^P_b(\tau) = \Pi(\tau_o, \tau_w) \left[ \frac{d^2 \vartheta_{gz}}{d\tau^2} + b_b \frac{d\vartheta_{gz}}{d\tau} - \frac{1}{2} \left( \frac{d\psi_{gz}}{d\tau} \right)^2 \sin 2\vartheta_{gz} - \frac{d\psi_{gz}}{d\tau} \cos \vartheta_{gz} \right] \frac{1}{c_b}$$

$$M^P_c(\tau) = \Pi(\tau_o, \tau_w) \left[ \frac{d^2 \psi_{gz}}{d\tau^2} \cos^2 \vartheta_{gz} + b_c \frac{d\psi_{gz}}{d\tau} + \frac{d\psi_{gz}}{d\tau} \frac{d\vartheta_{gz}}{d\tau} \sin 2\vartheta_{gz} \right] \frac{1}{c_c}$$

(3.3)

where

$$\tau = t\Omega \quad \Omega = \frac{J_{go}n_g}{J_{gk}} \quad c_b = c_c = \frac{1}{J_{gk}\Omega^2} \quad b_b = b_c = \frac{\eta_b}{J_{gk}\Omega}$$

and $\vartheta_{gz}, \psi_{gz}$ are the angles determining the position of the gyroscope axis in space.

For the target tracking status, values of angles determining the given position of the gyroscope axis are equal to

$$\vartheta_{gz} = \varepsilon \quad \psi_{gz} = \sigma$$

(3.4)

where: $\varepsilon, \sigma$ are the angles determining a given position of the target observation line (TOL).

The angles $\varepsilon, \sigma$ are defined by the following relationships constituting kinematic equations TOL (Mishin, 1990)

$$\frac{dr_e}{dt} = V_{pxe} - V_{exe} \quad -\frac{d\varepsilon}{dt} r_e \cos \varepsilon = V_{pye} - V_{cye}$$

$$\frac{d\sigma}{dt} r_e = V_{pze} - V_{cze}$$

(3.5)

where

$$V_{pxe} = V_0 \left[ \cos(\varepsilon - \chi_p) \cos \varepsilon \cos \gamma_p - \sin \varepsilon \sin \gamma_p \right]$$
$$V_{pye} = -V_0 \sin(\varepsilon - \chi_p) \cos \gamma_p$$
$$V_{pze} = V_0 \left[ \cos(\varepsilon - \chi_p) \sin \varepsilon \cos \gamma_p - \cos \varepsilon \sin \gamma_p \right]$$
$$V_{exe} = V_c \left[ \cos(\varepsilon - \chi_c) \cos \varepsilon \cos \gamma_c - \sin \varepsilon \sin \gamma_c \right]$$
$$V_{cye} = -V_c \sin(\varepsilon - \chi_c) \cos \gamma_c$$
$$V_{cze} = V_c \left[ \cos(\varepsilon - \chi_c) \sin \varepsilon \cos \gamma_c - \cos \varepsilon \sin \gamma_c \right]$$
and \( r_e \) – distance between the centre of gravity mass of PR and the ground target, \( V_0, V_c \) – velocities of PR and the ground target, \( \gamma_p = \theta - \alpha, \chi_p = \psi - \beta \) – position angles of the PR velocity vector, \( \gamma_c, \chi_c \) – position angles of the velocity vector of the ground target.

If angular deviations between the real angles \( \vartheta_g \) and \( \psi_g \) and required angles \( \vartheta_{gz} \) and \( \psi_{gz} \) are denoted as follows

\[
e_\psi = \psi_g - \psi_{gz} \quad e_\vartheta = \vartheta_g - \vartheta_{gz}
\] (3.6)

then the tracking steering moments of the gyroscope shall be expressed as follows

\[
M_b^s(\tau) = \Pi(\tau_s, \tau_k) \left( T_b e_\vartheta - T_c e_\psi + T_g \frac{d e_\vartheta}{d\tau} \right)
\]

\[
M_c^s(\tau) = \Pi(\tau_s, \tau_k) \left( T_b e_\psi + T_c e_\vartheta + T_g \frac{d e_\vartheta}{d\tau} \right)
\] (3.7)

where

\[
T_b = \frac{k_b}{J g k \Omega^2} \quad T_c = \frac{k_c}{J g k \Omega^2} \quad T_g = \frac{h_g}{J g k \Omega}
\]

Thus, the steering law for the autopilot, taking into account the dynamics of inclination of steers, shall be expressed as follows

\[
\frac{d^2 \delta_m}{dt^2} + h_m \frac{d \delta_m}{dt} + k_m \delta_m = k_m (\gamma_p - \gamma^*_p) + h_m \left( \frac{d \gamma_p}{dt} - \frac{d \gamma^*_p}{dt} \right)
\]

\[
\frac{d^2 \delta_n}{dt^2} + h_n \frac{d \delta_n}{dt} + k_n \delta_n = k_n (\chi_p - \chi^*_p) + h_n \left( \frac{d \chi_p}{dt} - \frac{d \chi^*_p}{dt} \right)
\] (3.8)

where: \( b_m, b_n \) are the coefficients of stabilising steers, \( k_m, k_n \) – coefficients of amplifications of steer drives, \( h_m, h_n \) – coefficients of suppressions of steer drives, \( k_m, k_n \) – coefficients of amplifications of the autopilot regulator, \( h_m, h_n \) – coefficients of suppressions of the autopilot regulator.

The required angles of position \( \gamma^*_p, \chi^*_p \) of the PR velocity vector are determined by the method of proportional navigation (Koruba, 2001)

\[
\frac{d \gamma^*_p}{dt} = a_\gamma \frac{d \vartheta_g}{dt} \quad \frac{d \chi^*_p}{dt} = a_\chi \frac{d \psi_g}{dt}
\] (3.9)

where: \( a_\gamma, a_\chi \) are the required coefficients of proportional navigation.
4. Obtained results and final conclusions

The tested model of navigation and the steering of the self-guiding missile describes the fully autonomous motion of the Maverick combat vessel, which is to directly attack and destroy a ground target after being detected and identified.

Figures 5-8 show selected results of digital simulation of the dynamics of the missile during self-guidance on a detected ground target. It was assumed that the missile was launched from an aircraft-carrier moving at a speed of 200 m/s at a height of 400 m. The target was moving along an arc of a circle at the speed of 10m/s. The parameters of the steered gyroscope were as follows

\[
J_{gk} = 2.5 \cdot 10^{-4} \text{kgm}^2 \quad J_{go} = 5.0 \cdot 10^{-4} \text{kgm}^2
\]
\[
n_g = 600 \frac{\text{rad}}{\text{s}} \quad \eta_b = \eta_c = 0.01 \frac{\text{Nms}}{\text{rad}}
\]

while the coefficients of its regulator had the values

\[
k_b = 31.480 \frac{\text{Nm}}{\text{rad}} \quad k_c = 2.986 \frac{\text{Nm}}{\text{rad}} \quad h_g = 31.525 \frac{\text{Nms}}{\text{rad}}
\]

The coefficients of proportional navigation and parameters of the regulator of PR autopilot were as follows

\[
a_\gamma = 3.5 \quad a_\chi = 3.5 \quad k_m = 2.703 \frac{\text{Nm}}{\text{rad}}
\]
\[
k_n = 11.439 \frac{\text{Nm}}{\text{rad}} \quad h_m = 9.887 \frac{\text{Nms}}{\text{rad}}
\]

Fig. 5. Spatial trajectory of a self-guiding missile
Fig. 6. Change of angles of attack and glide angles in function of time

Fig. 7. Angular position of the missile in function of time during guidance

Fig. 8. Angular position of the gyroscope axis in function of time
With the parameters selected as in the above, the target was destroyed in 6 s of the flight.

It should be emphasised that the gyroscope scanning and tracking layout proposed in this paper improves the stability of the system of missile self-guidance and increases resistance to vibrations born from the board of the missile itself.

References


Model końcowego odcinka nawigacji samonaprowadzającego pocisku rakietowego sterowanego giroskopem

Streszczenie

W pracy zaprezentowano modelowanie dynamiki samonaprowadzającego pocisku rakietowego sterowanego przy użyciu dźwigówek. W tego rodzaju pociskach rakietowych atakujących samodzielnie wykryte cele głównym elementem jest samonaprowa-
wadząca głowica, której napęd stanowi giroskop sterowany. W pracy przedstawiona została dynamika i sposób sterowania takiego pocisku. Poprawność opracowanego modelu matematycznego potwierdziła symulacja numeryczna przeprowadzona dla pocisku klasy „Maverick” wyposażonego w giroskop będący elementem wykonawczym skanowania powierzchni ziemi i śledzenia wykrytego na niej celu. Analizie numerycznej poddana została zarówno dynamika giroskopu, jak i pocisku podczas procesu skanowania i śledzenia wykrytego celu. Wyniki przedstawione zostały w postaci graficznej.

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