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DEVELOPED RELIABILITY-BASED INTERACTION CURVES FOR DESIGN OF REINFORCED CONCRETE COLUMNS

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The provision of a safe structural system is a major object of structural designs. Due to uncertainties of varying magnitudes associated with loadings, the material and geometrical properties, current design methods that are not fully based on the probability concept are not able to yield a constant reliability level. This paper examined the criteria of British Standard Code of Practice (BS 8110, 1997) currently in use in Nigeria, for the design of reinforced concrete columns subjected to axial and bending loads using the probability-based concept. In order to offer designs capable of maintaining a predefined safety level, a computer program in FORTRAN language was developed. The modules in the program were based on BS 8110 (1997) design requirements and the First Order Reliability Method. Individual parameters were considered random with practical probability distributions. The program starts with a preliminary design and iteratively selects values of the design variables that lead to the predefined safety level. Interaction curves were plotted for varying safety indices. On the curves, design decisions relating to ratios of dead to live loads, effective to the gross depth of a section and reinforcement can be made. In contrast to the current method of BS 8110 (1997), the proposed design curves guide the designer to the knowledge of the expected level of safety of the section being designed.

Key words: structural design, reinforced concrete columns, reliability level, interaction curves, safety indices

1. Introduction

There are three main aims of structural engineering design. First, the structure must be safe for the society. Second, the structure must fulfill its intended purpose during its intended life span. Third, the structure must be economical with regard to construction and maintenance cost. Indeed, most design decisions are implicitly or explicitly economic decisions.

The process of structural design begins with the engineer's appreciation of client's requirements. After collecting and assimilating relevant facts, he develops concepts of general structural schemes, appraises them, and then having considered the use of materials and erection methods, he makes an important decision of choosing the final structural scheme. This is followed by a full structural analysis and detailed design (Kong and Evans, 1989).

There are, however, many sources of uncertainties in the structural engineering design. The absolute safety of a structure cannot be guaranteed because of unpredictability of future loading conditions, inability to obtain and express the place-in material properties accurately, use of simplified assumptions in predicting the behaviour of the structure due to loading under consideration, limitations in numerical methods used and the human factor (e.g., errors, omissions, etc.). However, the risk of unacceptable consequences can be limited to an acceptable level in terms of probability, namely the probability of failure. In general, therefore, probabilistic analysis is necessary in the development of design formats (Ayyub and White, 1987).

To account for uncertainty in design, various building codes have been developed, taken into cognizance safety, good performance and cost effectiveness of the structure. Most building codes use a factor of safety, which is usually based on engineering judgment and previous experience with similar structures. Some building codes also apply separate partial safety coefficients to the load and resistance variables in attempt to relate the contribution of each variable towards establishment of the total safety factor (Surahman and Rojiani, 1983).

The purpose of design is to achieve an acceptable probability that a structure will not become unfit for its intended use, that is, that it will not reach the limit state. Thus, any way in which the structure may cease to be fit for use will constitute the limit state, and the design aim is to avoid any such condition being reached during the expected life of the structure (Mosley and Bungey, 1990).

To cater for the this, existing limit state design codes (e.g; BS 8110, 1997) employ partial safety factors to take care of uncertainties inherent in design variables. Moreover, the uncertainties include human error, negligence, poor workmanship or neglected loadings. The factors employed in deterministic operations using constant characteristic values in place of the uncertainties provokes some questions as to the uniformity in reliability of the resultant design, which is the objective the code is supposed to achieve. In order to evaluate the effect of uncertainties, principles of reliability analysis are employed. Under normal situations, the designing carried out using any of the codes to predict the expected performance of structures subjected to a similar design should give a uniform level of reliability or safety. However, according to Afolayan (1990), the realisation of this aim had not been properly accomplished. BOMEL (2001) noted that limit states can be defined as a specified set of states that separate the desired state from the undesirable one which fails to meet the design requirements. More generally, they may be considered without a specific physical interpretation, such that the Limit State is a mathematical criterion that categorizes any set of values of the relevant structural variables (loads, material and geometrical variables) into one of two categories – the *desirable* category (also known as the *safe* set) and the *adverse* category (often referred to as the *failure* set). The word *failure* then means *failure of satisfying the Limit State criterion*, rather than failure in the sense of some dramatic physical event. In Codes of Practice, Limit States are considered to represent various conditions in which a structure would be considered to have failed to fulfill the purposes for which it was built. Normally, limit states relate to material strength, but they are affected by the use, performance, environment, material behaviour, shape, quality, protective measures and maintenance. Columns, as already known, are primarily compression members, although they may also resist bending forces due to continuity of the structure. Columns may be braced or unbraced and either short or slender.

The mode of failure of a column can be one of the following:

- Material failure with negligible lateral deflection, which usually occurs with short columns but can also occur when there are large end moments on the column with an intermediate slenderness ratio.
- Material failure intensified by lateral deflection and an additional moment – this type of failure is typical of intermediate columns.
- Instability failure that occurs with slender columns and is liable to be preceded by excessive deflections.

The elastic and the load–factor methods of design both provide indirect controls for the serviceability requirements of a structure. With the limit state design method, the strength requirements are separated from the serviceability requirements. To satisfy the strength requirements, the ultimate load capacity of the structure is assumed to be much greater than the working loads. Two partial safety factors are used, one for material strength and the other for loads and load effects. In almost all instances, the column is the most critical part of a building, bridge or any structural skeletal frame system. Failure of one of such columns could lead to disastrous damage to the structure (Bernardo, 2005).

The complex problem of behaviour of reinforced concrete columns in framed structures has been a subject of considerable analytical and experimental research in the recent years. Non-linearities arising from stress-strain relations of materials, development of cracks in concrete and secondary load-deflection (slenderness) effects lead to a situation in which a column cannot readily be related to the load acting on the structure. Furthermore, whilst the capacity of columns in the practical range of slenderness is usually limited to material strength, the columns at the slender end of the spectrum are susceptible to stability failure before their full cross-sectional strength is attained (Burtler, 1977). In this paper, a probability-based design concept is adopted to provide design charts suitable for reinforced concrete beam-columns.

2. Methodology

In this work, the load S and resistance R were treated as random variables rather than deterministic constants. The safety measure that corresponds to the probability of failure (or the reliability index) was obtained from systematic analysis of uncertainties in all variables. The reliability analysis was based on the First Order Reliability Method as in FORM5 (Golliwitzer *et al.*, 1988).

The variables R and S were functions made up of different basic variables. In order to investigate the effect of variables on the performance of the column as a structural system, a limit state equation in terms of the basic design variables is required. This limit state equation is referred to as the performance or state function and expressed as

$$g(\boldsymbol{X}) = g(X_1, X_2, \ldots, X_n) = R - S \tag{2.1}$$

where X_i for $i = 1, 2, \ldots, n$ represent the basic design variables. The limit state of the system can then be expressed as

$$g(\boldsymbol{X}) = 0 \tag{2.2}$$

Graphically, the line $g(\boldsymbol{X}) = 0$ represents the failure surface, while $g(\boldsymbol{X}) > 0$ represents the safe region, and $g(\boldsymbol{X}) < 0$ corresponds to the failure region. Introduce a set of uncorrelated reduced variates

$$x_i' = \frac{X_i - \mu_{xi}}{\sigma_{xi}} \qquad i = 1, 2, \ldots, n \tag{2.3}$$

In terms of the reduced variates, the limit state equation is given as

$$g(\sigma_{x1}X_1' + \mu_{x1}, \sigma_{x2}X_2' + \mu_{x2}, \ldots, \sigma_{xn}X_n' + \mu_{xn}) = 0 \tag{2.4}$$

The distance from a point $\boldsymbol{X}' = (X_1', X_2', \ldots, X_n')$ on the failure surface $g(\boldsymbol{x}') = 0$ to the origin of X_i space is given as

$$D = \sqrt{{X_1'}^2 + {X_2'}^2 + \ldots + {X_n'}^2} \tag{2.5}$$

In matrix form

$$D = (X_1', X_2', \ldots, X_n') \begin{bmatrix} X_1' \\ X_2' \\ \vdots \\ X_n' \end{bmatrix} = ({\boldsymbol{X}'}^{\top}\boldsymbol{X}')^{1/2} \tag{2.6}$$

The point on the failure surface $({X_1'}^*, {X_2'}^*, \ldots, {X_n'}^*)$, having the minimum distance to the origin, may be determined by minimizing the function D and subjecting equation (2.5) to the constraint $g(\boldsymbol{X}) = 0$. For this purpose, the method of Lagrange's multiplier may be used. Let

$$L = D + \lambda g(\boldsymbol{X}) \tag{2.7}$$

Substituting equation (2.6) into (2.7), gives

$$L = ({\boldsymbol{X}'}^{\top}\boldsymbol{X}')^{1/2} + \lambda g(\boldsymbol{X}) \tag{2.8}$$

where λ is the value of the multiplier.

In scalar notation

$$L = \sqrt{{X_1'}^2 + {X_2'}^2 + \ldots + {X_n'}^2} + \lambda g(x_1, x_2, \ldots, x_n) \tag{2.9}$$

in which $X_i = \sigma_{xi}X_i' + \mu_{xi}$.

Minimizing L, we obtain $n+1$ equations with $n+1$ unknowns as

$$\frac{\partial L}{\partial \boldsymbol{X}'} = \frac{\boldsymbol{X}'}{\sqrt{{X_1'}^2 + {X_2'}^2 + \ldots + {X_n'}^2}} + \frac{\lambda \partial g}{\partial \boldsymbol{X}'} = \boldsymbol{0} \tag{2.10}$$

and

$$\frac{\partial L}{\partial \lambda} = g(X_1, X_2, \ldots, X_n) = 0 \tag{2.11}$$

The solution to equations (2.10) and (2.11) would yield the most probable failure point $(X_1'^{*}, X_2'^{*}, \ldots, X_n'^{*})$. Introducing the gradient vector

$$\boldsymbol{G} = \Big[\frac{\partial g}{\partial X_1'}, \frac{\partial g}{\partial X_2'}, \ldots, \frac{\partial g}{\partial X_n'}\Big] \tag{2.12}$$

in which

$$\frac{\partial g}{\partial X_i} = \frac{\partial g}{\partial x_i}\frac{\partial x_i}{\partial X_i} = \sigma_{xi}\frac{\partial g}{\partial X_i} \tag{2.13}$$

therefore in vector form, we have

$$\frac{\boldsymbol{X'}}{(\boldsymbol{X'}^{\top}\boldsymbol{X'})^{1/2}} + \lambda\boldsymbol{G} = \boldsymbol{0} \tag{2.14}$$

from which

$$\boldsymbol{X'} = -\lambda D\boldsymbol{G} \tag{2.15}$$

from equation (2.6)

$$D = \Big[(\lambda D\boldsymbol{G}^{\top})(\lambda D\boldsymbol{G})\Big]^{1/2} = \lambda D(\boldsymbol{G}^{\top}\boldsymbol{G})^{1/2} \tag{2.16}$$

and

$$\lambda = (\boldsymbol{G}^{\top}\boldsymbol{G})^{-1/2} \tag{2.17}$$

Substituting equation (2.17) into equation (2.15), gives

$$\boldsymbol{X'} = \frac{-\boldsymbol{G}D}{(\boldsymbol{G}^{\top}\boldsymbol{G})^{1/2}} \tag{2.18}$$

Multiplying both sides of equation (2.18) by $\boldsymbol{G}^{\top}$, we have

$$\boldsymbol{G}^{\top}\boldsymbol{X'} = \frac{-\boldsymbol{G}^{\top}\boldsymbol{G}D}{(\boldsymbol{G}^{\top}\boldsymbol{G})^{1/2}} = -(\boldsymbol{G}^{\top}\boldsymbol{G})^{1/2}D \tag{2.19}$$

Thus

$$D = \frac{-\boldsymbol{G}^{\top}\boldsymbol{X'}}{(\boldsymbol{G}^{\top}\boldsymbol{G})^{1/2}} \tag{2.20}$$

If the minimum distance from the origin to the line representing the failure surface is denoted by β, then

$$\beta = D = \frac{-\boldsymbol{G}^{*\top}\boldsymbol{X}'^{*}}{(\boldsymbol{G}^{*\top}\boldsymbol{G}^{*})^{1/2}} \tag{2.21}$$

where $\boldsymbol{G}^*$ is the gradient vector at the most probable failure point $(X_1'^*, X_2'^*, \ldots, X_n'^*)$. In scalar form equation (2.21) is

$$\beta = -\frac{\sum\limits_i X_i'^* \Big[\frac{\partial g}{\partial X_i'^*}\Big]_*}{\sqrt{\sum\limits_i \Big[\frac{\partial g}{\partial X_i'^*}\Big]_*^2}} \tag{2.22}$$

where the derivatives are performed at $(X_1'^*, X_2'^*, \ldots, X_n'^*)$ and the most probable point on the failure surface for the minimum β from equation (2.18)

$$X_i'^* = -\alpha_i^* \beta \qquad\qquad i = 1, 2, \ldots, n \tag{2.23}$$

where

$$\alpha_i = \frac{\Big[\frac{\partial g}{\partial X_i'^*}\Big]_*}{\sqrt{\sum\limits_i \Big[\frac{\partial g}{\partial X_i'^*}\Big]_*^2}} \tag{2.24}$$

are direction cosines along the axes X_i'.

FORM5 (Golliwitzer *et al.*, 1988), a computer package which uses the algorithm was used to compute the implied safety levels for different limit states, outlining the design criteria for reinforced concrete columns using BS 8110 (1997). In this work, the design is considered satisfactory if the following condition is satisfied

$$(\beta - \beta_t)^2 \leqslant \varepsilon \tag{2.25}$$

where β is the calculated safety index obtained from the reliability program on the basis of the input variables, β_t is the target safety index, and this is chosen depending on the specification that satisfies the class of the structure, ε is the level of acceptance of the design points.

If the above condition does not hold, the design of the column is repeated until it is satisfied. When the condition is satisfied for varying values of the design variables, the parameters so obtained are considered to provide a uniform reliability level.

3. Design values at uniform reliability level and discussion

FORTRAN program for designing reinforced concrete columns under axial and end moments was developed to BS 8110 (1997) at a constant reliability level. The program starts by reading the values of height, breadth, length of the column on both x and y planes, cover to reinforcement on far and near sides, bar size, minimum eccentricity, moments on both x and y planes (for biaxial columns), characteristic strength of concrete and reinforcing steel, as applicable to a particular design. These values are read from a data file.

Design parameters at varying safety indices ranging from 3.0 to 4.5 were generated. Some of the variables such as load, moment, characteristic strengths of concrete and reinforcement were kept constant, and the load ratio was varied from 0.1 to 1.5. For each load ratio, the depth and breadth of sections were varied at one point or another to generate various reinforcement ratios corresponding to a particular target safety index. Sections were designed in accordance with the British Standard BS 8110 (1997) and the First Order Reliability Method (FORM). Samples of the resulting curves are shown in Figs. 1 to 5 for the target reliability index of 4.5. The values of N/bh and M/bh^2 used for plotting the interaction curves were obtained from the limiting equations

$$N = 0.45 f_{cu} b d_c + f_{s1} A_{s1} + f_{s2} A_{s2} \tag{3.1}$$

and

$$M = 0.45 f_{cu} b d_c \Big[\frac{h}{2} - \frac{d_c}{2}\Big] + f_{s1} A_{s1} \Big[\frac{h}{2} - d_p\Big] + f_{s2} A_{s2} \Big[d - \frac{h}{2}\Big] \tag{3.2}$$

in which

N – axial load

M – moment

f_{s1} – compressive force in the reinforcement area A_{s1} and acting through its centroid

A_{s1} – area of reinforcement near the more highly compressed face of the column section

f_{s2} – tensile or compressive force in the reinforcement area A_{s2} and acting through its centroid

A_{s2} – area of reinforcement in the less compressed face of the column section

d_c – depth of the stress block

d_p – effective depth of top reinforcement

d – effective depth of bottom reinforcement.

The plots in Figs. 1 to 5 were done for different reinforcement ratios ρ and load ratios α when $f_{cu} = 30\,\mathrm{N/mm^2}$ and $f_y = 460\,\mathrm{N/mm^2}$. It is clear that as the depth ratio increases, there is a corresponding decrease in the amount of reinforcement required to maintain a predefined safety level. Thus, the results confirm the fact that the larger the sections, the higher the safety levels. At a constant safety level but increasing depth ratio, considering the interaction curves, it is shown that it is better to design columns to carry more moments than the axial load. However, when compared to conventional curves in BS 8110 (1997), it is seen that for increasing depth ratio the columns take more moment at the same axial load, though without information on the explicit safety level. Several curves at varying levels of safety showing similar trends are reported in the work of Akindahunsi (2003).

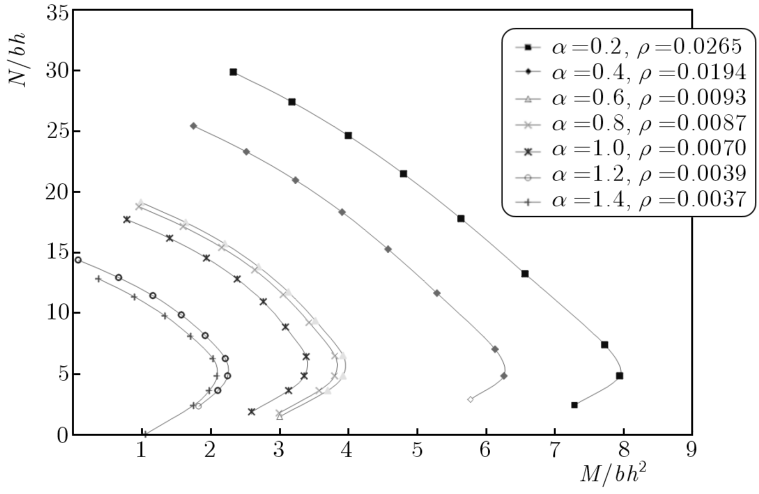

Fig. 1. Interaction curves at $d/h = 0.75$ and $\beta = 4.5$

4. Conclusion

The column interaction curves predicated on predefined safety levels have been developed considering uncertainties in all the variables relating to geometry, loading and material properties. The BS 8110 (1997) design requirements were used as the basis for necessary transformation. It has been shown that as some factors remained constant, while varying the breadth of the section, a small decrease in the reinforcement ratio was noted, but a substantial increase in the depth is responsible for a significant decrease in the value of the reinforcement ratio. This confirms the fact that the larger the sections, the higher the safety level and the position of BS 8110 (1997) that reinforced concrete columns are adequate in compression since they are primarily compression members is af-

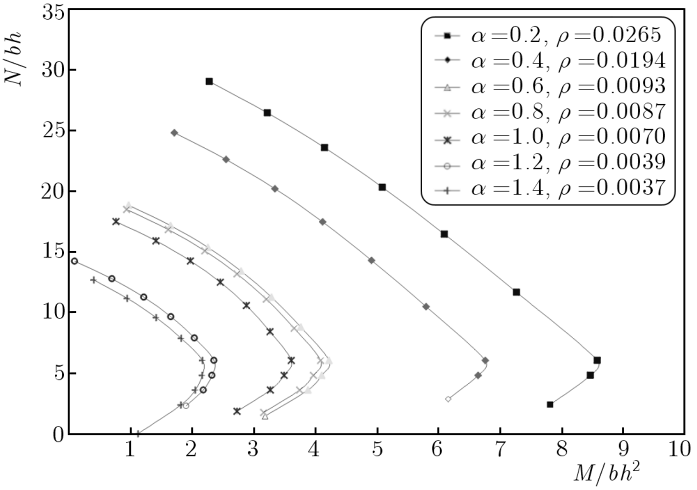

Fig. 2. Interaction curves at $d/h = 0.8$ and $\beta = 4.5$

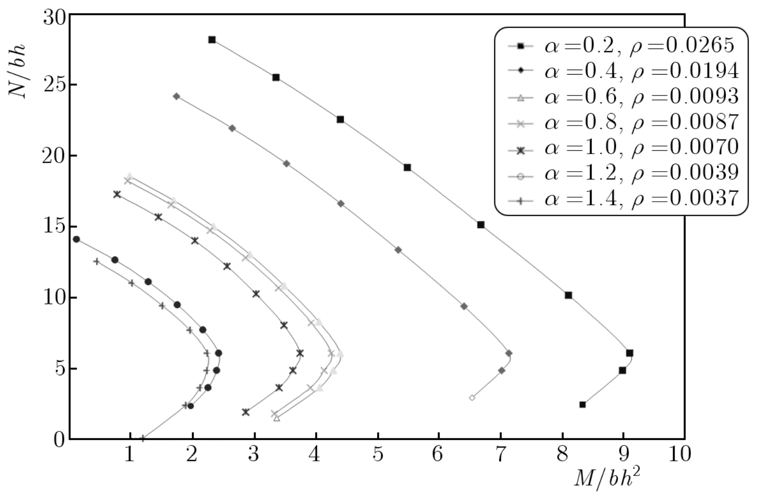

Fig. 3. Interaction curves at $d/h = 0.85$ and $\beta = 4.5$

firmed. It is also established within the context of this study that at a constant safety level but increasing depth ratio, it is better to design columns to carry more moments than axial loads.

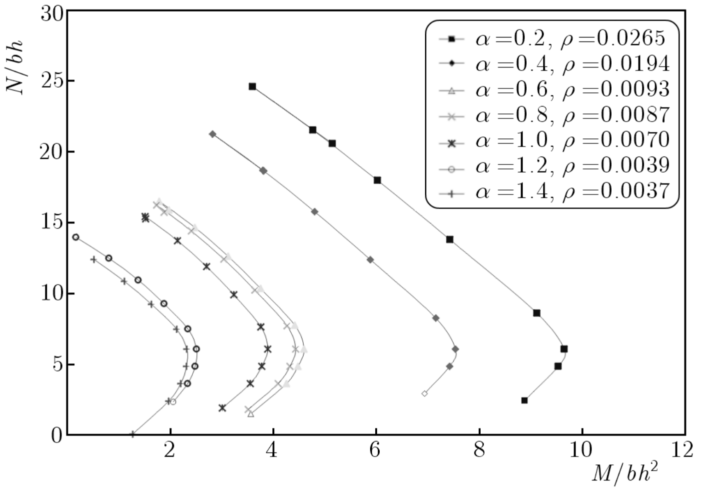

Fig. 4. Interaction curves at $d/h = 0.9$ and $\beta = 4.5$

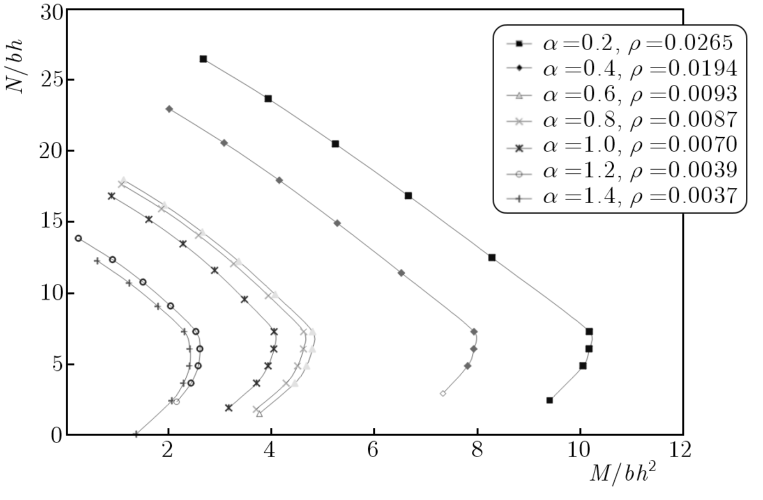

Fig. 5. Interaction curves at $d/h = 0.95$ and $\beta = 4.5$

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Zmodyfikowane, niezawodnościowo zorientowane krzywe obciążeniowe w projektowaniu wzmacnianych kolumn betonowych

Streszczenie

Dostarczenie zamawiającemu konstrukcji spełniającej wymogi bezpieczeństwa jest jednym z głównych celów projektowania. Z powodu niepewności co do wartości spodziewanych obciążeń oraz rozkładu materiałowych i geometrycznych parametrów danego obiektu, konstruowanie nie oparte całkowicie na rachunku prawdopodobieństwa nie jest w stanie zapewnić jednoznacznego wyniku ze stałym poziomem niezawodności. W niniejszej pracy dokonano analizy brytyjskiej normy zarządzania ryzykiem, tzw. British Standard Code of Practice BS 8110 z 1997 r., która opiera się na rachunku prawdopodobieństwa i obecnie jest stosowana w Nigerii przy projektowaniu wzmacnianych kolumn betonowych przenoszących obciążenie osiowe i gnące. Do

tworzenia konstrukcji mogących zapewnić utrzymanie założonego poziomu bezpieczeństwa ułożono program komputerowy w języku FORTRAN. Moduły tego programu oparto właśnie na zaleceniach BS 8110 oraz procedurze iteracyjnej analizy niezawodności pierwszego rzędu (tzw. FORM). Poszczególne parametry potraktowano jako wartości losowe z obserwowanymi w praktyce rozkładami prawdopodobieństwa. Opracowany program rozpoczyna działanie od sformułowania konstrukcji wstępnej i iteracyjnie dobiera wartości zmiennych parametrów konstrukcyjnych, zmierzając do uzyskania założonego poziomu bezpieczeństwa. W artykule, otrzymane krzywe obciążeniowe narysowano dla różnych wskaźników bezpieczeństwa. Na ich podstawie można podjąć decyzje projektowe dotyczące stosunku ciężaru własnego do użytkowego oraz w konsekwencji parametrów przekroju i wzmocnienia. W odróżnieniu od procedury według dotychczasowo stosowanej normy BS 8110, krzywe obciążeniowe interakcji moment gnący – siła osiowa zaprezentowane w tej pracy dają konstruktorowi jednoznaczną znajomość oczekiwanego poziomu bezpieczeństwa projektowanego przekroju kolumny.

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