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FREE VIBRATION OF CLAMPED VISCO-ELASTIC RECTANGULAR PLATE HAVING BI-DIRECTION EXPONENTIALLY THICKNESS VARIATIONS

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Free vibration of a clamped visco-elastic rectangular plate having bi-direction exponentially varying thickness has been analysed on the basis of classical plate theory. For visco-elastic materials, basic elastic and viscous elements are combined. We have assumed the Kelvin model for visco-elasticity, which is a combination of elastic and viscous elements connected in parallel. Here, the elastic element is constituted by a spring and the viscous one is a dashpot. An approximate but quite convenient frequency equation is derived by using the Rayleigh-Ritz technique. Logarithmic decrement, time period and deflection (at two different instant of time) for the first two modes of vibration and for various values of the taper constants and aspect ratio are calculated. Comparison studies have been carried out with bi-linearly thickness variation to establish the accuracy and versatility of the method.

Key words: visco-elasticity, clamped rectangular plate, variable thickness

List of symbols

a	–	length of rectangular plate
b	–	width of rectangular plate
x, y	–	co-ordinates in plane of the plate

h	–	thickness of the plate at point (x, y)
E	–	Young's modulus
G	–	shear modulus
ν	–	Poisson's ratio
$\widetilde{D}$	–	visco-elastic operator
D_1	–	flexural rigidity, $D_1 = Eh^3/[12(1-\nu^2)]$
ρ	–	mass density per unit volume of plate material
t	–	time
η	–	visco-elastic constants
$w(x, y, t)$	–	deflection of the plate
$W(x, y)$	–	deflection function
$T(t)$	–	time function
β_1, β_2	–	taper constants in X- and Y-directions, respectively
Λ	–	logarithmic decrement
K	–	time period

1. Introduction

Plates of uniform and non-uniform thickness are widely used as structural components in various engineering fields such as aerospace industry, missile technology, naval ship design, telephone industry, etc. An extensive review on linear vibration of plates has been given by Leissa (1987) in his monograph and a series of review articles (Leissa, 1969). Several authors (Tomar and Gupta, 1985; Laura *et al.*, 1979) studied the effect of taper constants in two directions on elastic plates, but none of them on visco-elastic plates. Sobotka (1978) considered free vibrations of visco-elastic orthotropic rectangular plates. Gupta and Khanna (2007) studied the effect of linearly varying thickness in both directions on vibration of a visco-elastic rectangular plate.

Young (1950) solved the problem of a rectangular plate by the Ritz method. Free vibrations of rectangular plates whose thickness varies parabolically were studied by Jain and Soni (1973). Bhatnagar and Gupta (1988) studied the effect of thermal gradient on vibration of a visco-elastic circular plate of variable thickness. Kumar (2003) discussed the effect of thermal gradient on some vibration problems of orthotropic visco-elastic plates of variable thickness. Gupta *et al.* (2007a) solved the problem of thermal effect on vibration of a non-homogeneous orthotropic rectangular plate having bi-directional parabolically varying thickness. Gupta *et al.* (2007b) examined vibration of a visco-elastic orthotropic parallelogram plate with linear variation of the thickness.

Visco-elasticity, as its name implies, is a generalisation of elasticity and viscosity. The ideal linear elastic element is the spring. When a tensile force is applied to it, the increase in distance between its two ends is proportional to the force. The ideal linear viscous element is the dashpot.

The main objective of the present investigation is to study the effect of taper constants on vibration of a clamped visco-elastic rectangular plate with bi-direction exponentially thickness variations. It is assumed that the plate is clamped on all four edges. To determine the frequency equation, Rayleigh-Ritz's technique has been applied. It is considered that the visco-elastic properties of the plate are of the Kelvin type.

All material constants, which are used in numerical calculations, have been taken for the alloy DURALIUM, which is commonly used in modern technology.

Logarithmic decrement, time period and deflection (at two different instant of time) for the first two modes of vibration for various values of the aspect ratio a/b and taper constants β_1 and β_2 are calculated. All the results are illustrated with graphs.

2. Equation of motion and its analysis

The equations of motion of a visco-elastic rectangular plate of variable thickness are (Gupta and Khanna, 2007)

$$\begin{aligned}
&[D_1(W,_{xxxx} + 2W,_{xxyy} + W,_{yyyy}) + 2D_1,_x (W,_{xxx} + W,_{xyy}) + \\
&\quad + 2D_1,_y (W,_{yyy} + W,_{yxx}) + D_1,_{xx} (W,_{xx} + \nu W,_{yy}) + \\
&\quad + D_1,_{yy} (W,_{yy} + \nu W,_{xx}) + 2(1-\nu) D_1,_{xy} W,_{xy}] - \rho h p^2 W = 0 \\
&\ddot{T} + p^2 \widetilde{D} T = 0
\end{aligned} \tag{2.1}$$

where (2.1) are differential equations of motion for an isotropic plate of variable thickness made of a visco-elastic material describing lateral deflection and free vibration, respectively.

Here p^2 is a constant.

The expressions for kinetic energy T_1 and strain energy V_1 are (Leissa, 1969)

$$
\begin{aligned}
T_1 &= \frac{1}{2}\rho p^2 \int\limits_0^a \int\limits_0^b hW^2 \, dydx \\
V_1 &= \frac{1}{2} \int\limits_0^a \int\limits_0^b D_1[(W_{,xx})^2 + (W_{,yy})^2 + 2\nu W_{,xx} W_{,yy} + 2(1-\nu)(W_{,xy})^2] \, dydx
\end{aligned}
\tag{2.2}
$$

Assuming the thickness variation of the plate in both directions as

$$
h = h_0 \mathrm{e}^{\beta_1 \frac{x}{a}} \mathrm{e}^{\beta_2 \frac{y}{b}} \tag{2.3}
$$

where β_1 and β_2 are the taper constants in the x- and y-directions, respectively, and $h_0 = h$ at $x = y = 0$.

The flexural rigidity of the plate can now be written as (assuming the Poisson's ratio ν is constant)

$$
D_1 = \frac{Eh_0^3}{12(1-\nu^2)} \Big(\mathrm{e}^{\beta_1 \frac{x}{a}}\Big)^3 \Big(\mathrm{e}^{\beta_2 \frac{y}{b}}\Big)^3 \tag{2.4}
$$

3. Solutions and frequency equation

To find a solution, we use the Rayleigh-Ritz technique. This method requires that the maximum strain energy must be equal to the maximum kinetic energy. So, it is necessary for the problem under consideration that

$$
\delta(V_1 - T_1) = 0 \tag{3.1}
$$

for arbitrary variations of W satisfying relevant geometrical boundary conditions.

For a rectangular plate clamped (c) along all the four edges, the boundary conditions are

$$
W = \begin{cases} W_{,x} = 0 & \text{at} \quad x = 0 \ \wedge \ x = a \\ W_{,y} = 0 & \text{at} \quad y = 0 \ \wedge \ y = b \end{cases} \tag{3.2}
$$

and the corresponding two-term deflection function is taken as (Gupta and Khanna, 2007)

$$
W = \Big[\frac{x}{a}\frac{y}{b}\Big(1-\frac{x}{a}\Big)\Big(1-\frac{y}{b}\Big)\Big]^2 \Big[A_1 + A_2 \frac{x}{a}\frac{y}{b}\Big(1-\frac{x}{a}\Big)\Big(1-\frac{y}{b}\Big)\Big] \tag{3.3}
$$

which satisfies equations (3.2).

Assuming non-dimensional variables as

$$X = \frac{x}{a} \qquad Y = \frac{y}{a} \qquad \overline{W} = \frac{W}{a} \qquad \overline{h} = \frac{h}{a} \tag{3.4}$$

and using equations (2.4) and (3.4) in equations (2.2), one obtains

$$\begin{aligned}
T_1 &= \frac{1}{2}\rho p^2 \overline{h}_0 a^5 \int\limits_0^1 \int\limits_0^{b/a} e^{\beta_1 X} e^{\beta_2 Y \frac{a}{b}} \overline{W}^2 \, dY dX \\
V_1 &= Q \int\limits_0^1 \int\limits_0^{b/a} \Big(e^{\beta_1 X} e^{\beta_2 Y \frac{a}{b}}\Big)^3 \cdot \\
&\quad \cdot [(\overline{W},_{XX})^2 + (\overline{W},_{YY})^2 + 2\nu \overline{W},_{XX} \overline{W},_{YY} + 2(1-\nu)(\overline{W},_{XY})^2] \, dY dX
\end{aligned} \tag{3.5}$$

where

$$Q = \frac{E\overline{h}_0^3 a^3}{24(1-\nu^2)}$$

Substituting the expressions for T_1 and V_1 from (3.5) into equation (3.1), one obtains

$$V_2 - \lambda^2 p^2 T_2 = 0 \tag{3.6}$$

where

$$\begin{aligned}
V_2 &= \int\limits_0^1 \int\limits_0^{b/a} \Big(e^{\beta_1 X} e^{\beta_2 Y \frac{a}{b}}\Big)^3 \cdot \\
&\quad \cdot [(\overline{W},_{XX})^2 + (\overline{W},_{YY})^2 + 2\nu \overline{W},_{XX} \overline{W},_{YY} + 2(1-\nu)(\overline{W},_{XY})^2] \, dY dX \\
T_2 &= \int\limits_0^1 \int\limits_0^{b/a} e^{\beta_1 X} e^{\beta_2 Y \frac{a}{b}} \overline{W}^2 \, dY dX \\
\lambda^2 &= \frac{12\rho(1-\nu^2)a^2}{E\overline{h}_0^2}
\end{aligned} \tag{3.7}$$

and λ is a frequency parameter.

For better accuracy of the results, the exponents are taken up to the fifth degree of X and Y.

Equation (3.6) has the unknowns A_1 and A_2 due to substitution of W from equation (3.3). These two constants are to be determined from equation (3.6) as

$$\frac{\partial(V_2 - \lambda^2 p^2 T_2)}{\partial An} = 0 \qquad n = 1, 2 \tag{3.8}$$

After simplifying equation (3.8), one gets

$$bn_1A_1 + bn_2A_2 = 0 \qquad n = 1,2 \tag{3.9}$$

where bn_1, bn_2 $(n = 1,2)$ involve the parametric constant and frequency parameter.

For the non-trivial solution, the determinant of coefficients of equation (3.9) must be zero. So one gets the frequency equation as

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} = 0 \tag{3.10}$$

From equation (3.10), one can obtain a quadratic equation in p^2 from which two values of p^2 can be found. After determining A_1 and A_2 from (3.9), one can obtain the deflection function W in form

$$W = \Big[XY\frac{a}{b}(1-X)\Big(1-Y\frac{a}{b}\Big)\Big]^2\Big[1+\frac{-b_{11}}{b_{12}}XY\frac{a}{b}(1-X)\Big(1-Y\frac{a}{b}\Big)\Big] \tag{3.11}$$

if one chooses $A_1 = 1$.

4. Time functions of visco-elastic plates

Equation $(2.1)_2$ is defined as a general differential equation of time functions for free vibrations of visco-elastic plates. It depends on the visco-elastic operator $\widetilde{D}$, which is

$$\widetilde{D} \equiv 1 + \frac{\eta}{G}\frac{d}{dt} \tag{4.1}$$

for Kelvin's model (Gupta and Khanna, 2007).

After substituting equation (4.1) into $(2.1)_2$, one obtains

$$\ddot{T} + \frac{p^2\eta}{G}\dot{T} + p^2T = 0 \tag{4.2}$$

Expression (4.2) is a differential equation of the second order for the time function T.

Solution to equation (4.2) will be

$$T(t) = \mathrm{e}^{a_1t}(C_1\cos b_1t + C_2\sin b_1t) \tag{4.3}$$

where

$$a_1 = -\frac{p^2\eta}{2G} \qquad\qquad b_1 = p\sqrt{1-\Big(\frac{p\eta}{2G}\Big)^2} \tag{4.4}$$

and C_1, C_2 are constants which can be determined easily from the initial conditions of the plate. Assuming the initial conditions as

$$T = 1 \quad \wedge \quad \dot{T} = 0 \qquad\quad \text{at} \quad t = 0 \tag{4.5}$$

and using equation (4.5) in (4.3), one obtains

$$C_1 = 1 \qquad\qquad C_2 = -\frac{a_1}{b_1} \tag{4.6}$$

Finally, one gets

$$T(t) = \mathrm{e}^{a_1 t}\Big(\cos b_1 t - \frac{a_1}{b_1}\sin b_1 t\Big) \tag{4.7}$$

after substituting (4.6) into (4.3).

Thus, deflection of the vibrating mode $w(x,y,t)$, which is equal to $W(x,y)T(t)$, may be expressed as

$$\begin{aligned} w = \Big[XY\frac{a}{b}(1-X)\Big(1-Y\frac{a}{b}\Big)\Big]^2\Big[1-\frac{b_{11}}{b_{12}}XY\frac{a}{b}(1-X)\Big(1-Y\frac{a}{b}\Big)\Big]\cdot \\ \cdot\mathrm{e}^{a_1 t}\Big(\cos b_1 t - \frac{a_1}{b_1}\sin b_1 t\Big) \end{aligned} \tag{4.8}$$

by making use of equations (4.7) and (3.11).

The vibration period of the plate is

$$K = \frac{2\pi}{p} \tag{4.9}$$

where p is the frequency given by equation (3.10).

The logarithmic decrement of vibrations, defined by the standard formula, is

$$\Lambda = \ln\frac{w_2}{w_1} \tag{4.10}$$

where w_1 is the deflection at any point on the plate at the time period $K = K_1$, and w_2 is the deflection at the same point and the time period succeeding K_1.

5. Numerical evaluations

The values of logarithmic decrement Λ, time period K and deflection w (at two differenting instants) for a clamped visco-elastic rectangular plate for different values of taper constants β_1, β_2 and aspect ratio a/b at different points for the first two modes of vibrations are calculated.

The following material parameters are used: $E = 7.08 \cdot 10^{10}\,\mathrm{N/m^2}$, $G = 2.632{\cdot}10^{10}\,\mathrm{N/m^2}$, $\eta = 14.612{\cdot}10^5\,\mathrm{N\,s/m^2}$, $\rho = 2.80{\cdot}10^3\,\mathrm{kg/m^3}$, $\nu = 0.345$. The data corresponds tu DURALIUM reported in Gupta and Khanna (2007).

The thickness of the plate at the centre is $h_0 = 0.01\,\mathrm{m}$.

6. Results and discussion

Numerical results for a visco-elastic isotropic clamped rectangular plate of exponentially varying thickness in both directions have been accurately computed by using the latest computer technology. Computations have been made for the logarithmic decrement Λ, time period K and deflection w (for two time instants) for different values of the taper constants β_1, β_2 and aspect ratio a/b for the first two modes of vibrations. All results are presented in Fig. 1 to Fig. 7. The comparison is made with the author's paper (Gupta and Khanna, 2007) on a plate with linearly variable thickness.

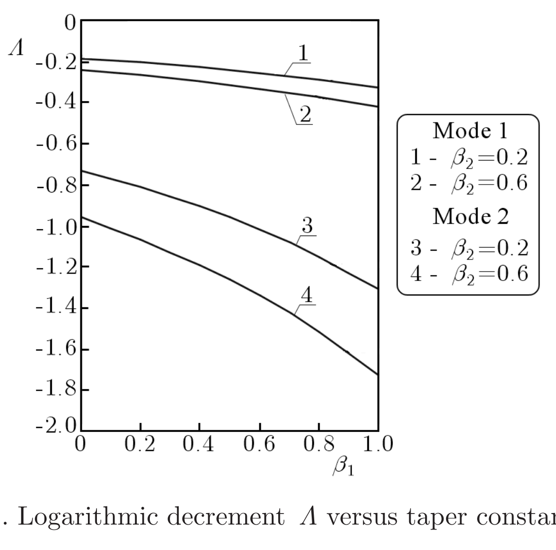

Fig. 1. Logarithmic decrement Λ versus taper constant β_1

In Fig. 1, it can be easily seen that for a fixed value of the aspect ratio $a/b = 1.5$ as the taper constant β_1 increases, the logarithmic decrement Λ decreases continuously for both modes of vibration for two values of β_2.

Figure 2 shows a steady decrease in the time period K with an increase of the taper constant β_1 for a fixed aspect ratio $a/b = 1.5$ and two values of β_2. It is simply seen that the time period K decreases as the taper constants increase for both modes of vibration.

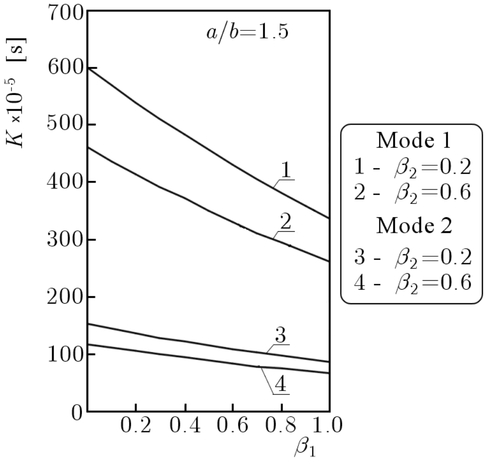

Fig. 2. Time period K versus taper constant β_1

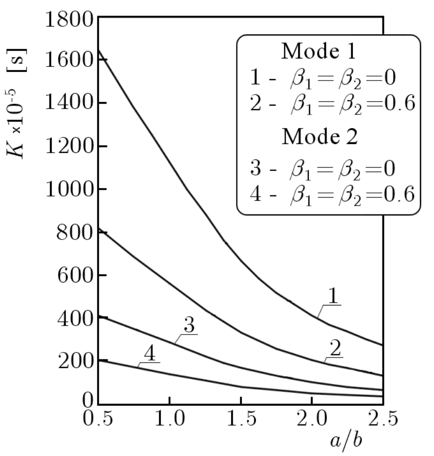

Fig. 3. Time period K versus aspect ratio a/b

Figure 3 shows the time period K for different values of the aspect ratio a/b for both modes of vibration for uniform and non-uniform thickness with the following constants:

(i) $\beta_1 = \beta_2 = 0.0$

(ii) $\beta_1 = \beta_2 = 0.6$

In both cases, one can note that the time period K decreases as the aspect ratio a/b increases for both modes of vibration.

Figures 4, 5, 6 and 7, respectively, depict numerical values of the deflection w for a fixed aspect ratio $a/b = 1.5$ for the first two modes of vibration and different values of X and Y for the following cases:

(i) Fig. 4 – $\beta_1 = \beta_2 = 0$ and time is $0\,\mathrm{K}$

(ii) Fig. 5 – $\beta_1 = \beta_2 = 0$ and time is $5\,\mathrm{K}$

(iii) Fig. 6 – $\beta_1 = \beta_2 = 0.6$ and time is $0\,\mathrm{K}$

(iv) Fig. 7 – $\beta_1 = \beta_2 = 0.6$ and time is $5\,\mathrm{K}$

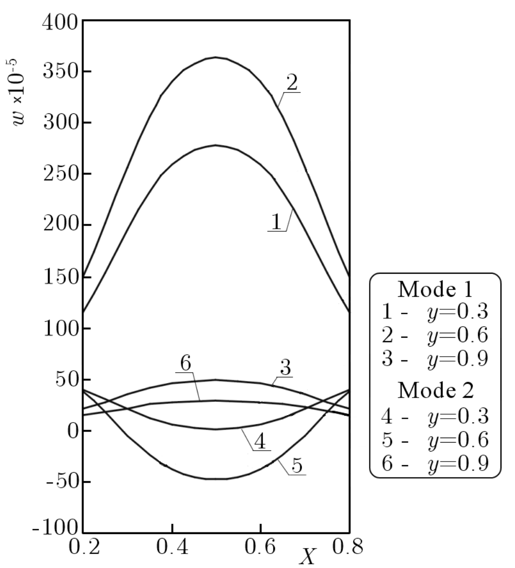

Fig. 4. Deflection w versus X; $T = 0\,\mathrm{K}$, $\beta_1 = \beta_2 = 0$, $a/b = 1.5$

Separate figures are given for the first and second mode of vibration (Figs. 5-7). One can conclude from all the four figures that the deflection w for the first mode of vibration initially increases and then decreases as X grows for different values of Y,.

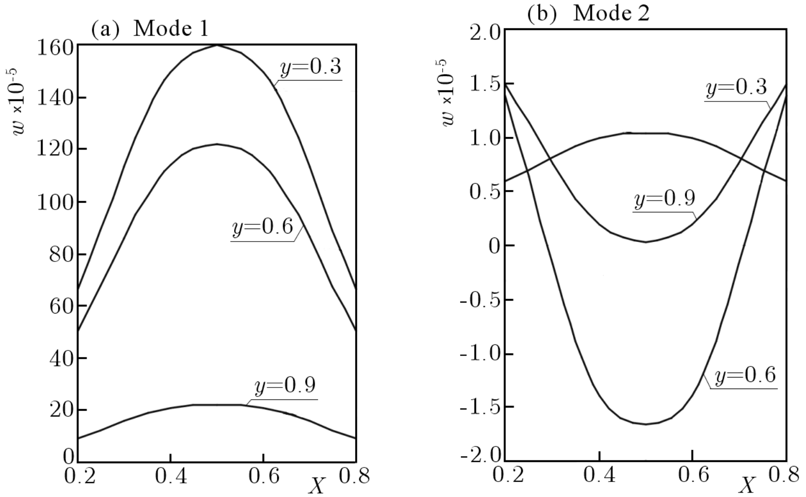

Fig. 5. Deflection w versus X; $T = 5\,\mathrm{K}$, $\beta_1 = \beta_2 = 0$, $a/b = 1.5$

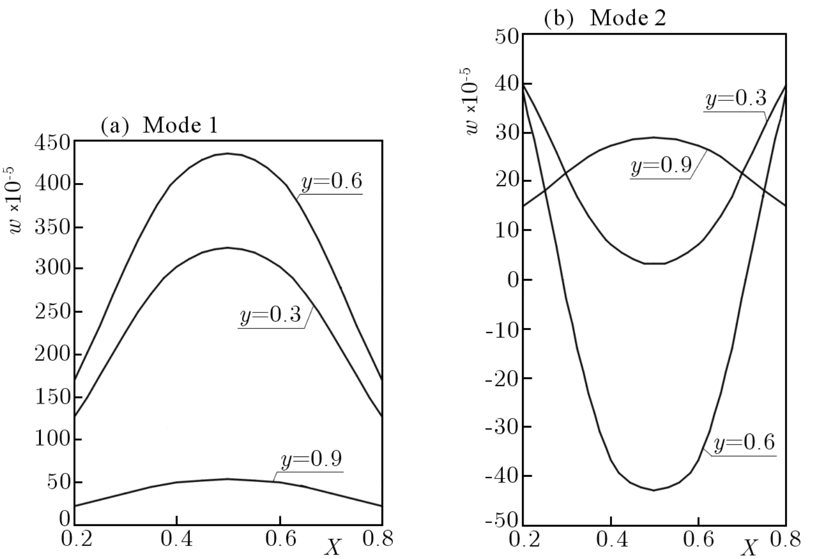

Fig. 6. Deflection w versus X; $T = 0\,\mathrm{K}$, $\beta_1 = \beta_2 = 0.6$, $a/b = 1.5$

Also, one can see that the deflection w for the second mode of vibration for $Y = 0.3$ and $Y = 0.6$ first decreases and then increases, while for $Y = 0.9$, the deflection increases first and then decreases as X grows.

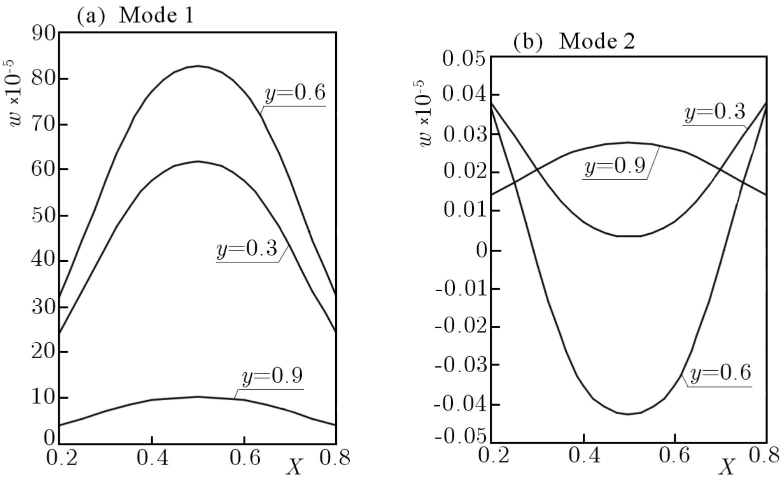

Fig. 7. Deflection w versus X; $T = 5\,\mathrm{K}$, $\beta_1 = \beta_2 = 0.6$, $a/b = 1.5$

One can get results for higher modes of vibration by introducing more terms to equation (3.3).

In the figures, Mode 1 and Mode 2 means the first and second mode of vibration, respectively.

7. Conclusion

The results for a uniform isotropic clamped visco-elastic rectangular plate are compared with the results published by the authors (Gupta and Khanna, 2007) and found to be in close agreement. The results of the present paper, shown in Fig. 1 to Fig. 3, are given in Table 1 to Table 3 together with results obtained in Gupta and Khanna (2007), which are placed in brackets in these tables.

After comparing, the authors conclude that as the taper constant increases for exponentially varying thickness, the time period and logarithmic decrement decrease in comparison to the increasing taper constant for linearly varying thickness. Therefore, engineers are provided with a method to develop plates in a manner so that they can fulfill the requirements.

Table 1. Logarithmic decrement for various parameters β_1 and β_2

β_1	$\beta_2 = 0.2$ First mode	$\beta_2 = 0.2$ Second mode	$\beta_2 = 0.6$ First mode	$\beta_2 = 0.6$ Second mode
0.0	−0.183075 (−0.181522)	−0.729066 (−0.722762)	−0.238421 (−0.220184)	−0.959711 (−0.881917)
0.2	−0.203070 (−0.200110)	−0.809745 (−0.798014)	−0.264408 (−0.242727)	−1.066887 (−0.974714)
0.4	−0.226834 (−0.219396)	−0.905293 (−0.876352)	−0.295142 (−0.266093)	−1.192726 (−1.071635)
0.6	−0.254990 (−0.239209)	−1.018582 (−0.957267)	−0.331403 (−0.290085)	−1.341444 (−1.172157)
0.8	−0.288020 (−0.259428)	−1.152337 (−1.040427)	−0.373808 (−0.314561)	−1.517993 (−1.275984)
1.0	−0.326156 (−0.279965)	−1.308829 (−1.125632)	−0.422684 (−0.339418)	−1.727861 (−1.382989)

Table 2. Time period for various parameters β_1 and β_2

β_1	$\beta_2 = 0.2$ First mode	$\beta_2 = 0.2$ Second mode	$\beta_2 = 0.6$ First mode	$\beta_2 = 0.6$ Second mode
0.0	599.1 (604.2)	151.5 (152.8)	460.1 (498.2)	115.8 (125.8)
0.2	540.1 (548.1)	136.7 (138.7)	415.0 (452.0)	104.6 (114.1)
0.4	483.6 (500.0)	122.6 (126.5)	371.9 (412.4)	94.0 (104.1)
0.6	430.3 (458.6)	109.4 (116.1)	331.3 (378.3)	84.1 (95.6)
0.8	381.0 (422.9)	97.1 (107.1)	293.8 (349.0)	75.0 (88.2)
1.0	336.6 (392.0)	86.1 (99.3)	260.0 (323.5)	66.7 (81.8)

Table 3. Time period for various parameters a/b, β_1 and β_2

a/b	$\beta_1 = \beta_2 = 0.0$		$\beta_1\beta_2 = 0.6$	
	First mode	Second mode	First mode	Second mode
0.5	1650.2 (1650.1)	412.5 (412.6)	818.8 (934.9)	204.9 (233.2)
1.0	1129.0 (1129.0)	288.5 (288.5)	559.7 (639.4)	143.8 (163.3)
1.5	667.9 (667.9)	169.0 (169.0)	331.3 (378.3)	84.1 (95.6)
2.0	412.6 (412.5)	103.3 (103.2)	204.7 (233.7)	51.3 (58.3)
2.5	274.5 (274.4)	68.0 (68.1)	136.2 (155.5)	33.8 (38.5)

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Drgania swobodne utwierdzonej lepkosprężystej prostokątnej płyty o dwukierunkowo wykładniczo zmiennej grubości

Streszczenie

W pracy rozważono problem swobodnych drgań utwierdzonej lepkosprężystej prostokątnej płyty o dwukierunkowo wykładniczo zmiennej grubości na podstawie klasycznej teorii płyt. Uwzględniono lepkosprężyste właściwości materiału płyty, bazując na podstawowych elementach reologicznych. Przyjęto model Kelvina, tj. równoległą kombinację elementu sprężystego i wiskotycznego. Równanie ruchu płyty rozwiązano metodą Rayleigha-Ritza, otrzymując przybliżoną, ale wygodną do analizy postać wyrażenia w dziedzinie częstości. Następnie wyznaczono wartość logarytmicznego dekrementu tłumienia, okresu drgań i ugięcia płyty dla dwóch pierwszych funkcji własnych dla różnych parametrów opisujących zmienną grubość i wymiary zewnętrzne płyty. Wyniki obliczeń przy uwzględnieniu zmiany grubości płyty porównano z dotychczasowymi rezultatami badań w celu potwierdzenia dokładności i uniwersalności metody.

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