JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
46, 3, pp. 511-520, Warsaw 2008

BIFURCATIONS IN AN ELECTRO-VIBROIMPACT SYSTEM WITH FRICTION

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Bifurcation phenomena of an electro-vibroimpact system have been investigated by means of numerical analysis. It has been shown that the system undergoes transition from chaotic motion to periodic motion as the control frequency of the solid state relay (one of the system parameters) varies. A close co-relationship with an experimental bifurcation diagram has been observed. Periodic motion has been identified to yield better system performance over chaotic motion. The foundation of implementing an optimal feedback control strategy is established.

Key words: bifurcation, electro-vibroimpact system, numerical analysis

1. Introduction

Studies on vibro-impact systems have revealed very rich system dynamics due to the presence of nonlinearities in the system characteristics (Hinrichs *et al.*, 1997; Pavlovskaia and Wiercigroch, 2003; Peterka, 1996). Construction of Poincaré maps, bifurcation diagrams and basins of attraction are useful to understand the qualitative dynamics of the system.

The considered electro-vibroimpact system is a discontinuous system, both from a mathematical and physical point of view. A detailed approach to describe and solve dynamical systems with motion dependent discontinuities was undertaken by Wiercigroch (2000). An important result from that piece of work was the clarification of accurate mathematical modelling of such systems and the numerical realisation of the analytical solution. While it may be attractive to assume a solution *a priori* be found numerically (Woo *et al.*, 2000), considerable amounts of time are required to program and implement for multi-degree-of-freedom systems. An alternative method to understand the system dynamics is to perform numerical integration as per Pavlovskaia *et al.* (2003). A form of the analytical solution may be obtained from consideration of the coefficient of restitution. Lenci and Rega (2003) treated this problem for a simple inverted impacting problem. Formulation of analytical solutions to systems describing vibro-impact machinery was achieved by Luo *et al.* (2006b). That analysis has also facilitated the description of bifurcation.

A large variety of dynamic responses is known to exist for nonlinear discontinuous systems. For example, systems exhibiting dry friction are known to behave in a chaotic manner, as demonstrated by Stefanski *et al.* (2003). In relation to that, the bifurcational phenomenon for impact systems was scrutinised by Luo *et al.* (2006a,c) and Luo and Xie (1998, 2004). Especially when a very interesting phenomenon such as intermittent chaos is reported for impact systems (Blazejczyk *et al.*, 1994), the associated loss in stability, and possibility of chaos allows for the realisation of an "optimal-harmonic" feedback implementation. This was demonstrated by Lenci and Rega (2000).

Bifurcation analysis involves the study of the change in system topology under the influence of a system parameter. It was originally used by Poincaré (1885) to describe the "splitting" of equilibrium solutions in a family of differential equations. Bifurcations of equilibria usually produce changes in the topological type of a flow (Guckenheimer and Holmes, 1983). Blazejczyk-Okolewska and Kapitaniak (1998) identified co-existing attractors in a mechanical system with impacts by means of bifurcation diagrams and basins of attraction, and concluded that basins of some attractors are so small that random noise prevents trajectories from reaching them.

This paper presents a flavour of bifurcation phenomena from numerical analysis of a new electro-vibroimpact system. The system involves a solenoid driven by a RLC circuit, coupled with a solid state relay, to generate large electro-magnetic forces acting on a metal bar which oscillates within the solenoid. Impacts are generated by placing a stop in the path of bar oscillations. The system was experimentally studied by Nguyen *et al.* (2007). Ho *et al.* (2008) performed numerical analysis of the system and revealed a variety of dynamic responses, ranging from periodic to chaotic. The results established a good correlation with the experimental data. To further confirm the qualitative responses of the system, a bifurcation diagram needs to be constructed, especially to compare with the experimental bifurcation diagram previously observed by Nguyen and Woo (2007).

2. System description

This piece of work scrutinises the bifurcation scenarios of the analysed system. The basis for the physical model of the electro-vibroimpact mechanism was described in detail by Ho *et al.* (2008). Figure 1 shows a schematic of the system, whereas the physical model of the system subject to friction of rails is shown in Fig. 2.

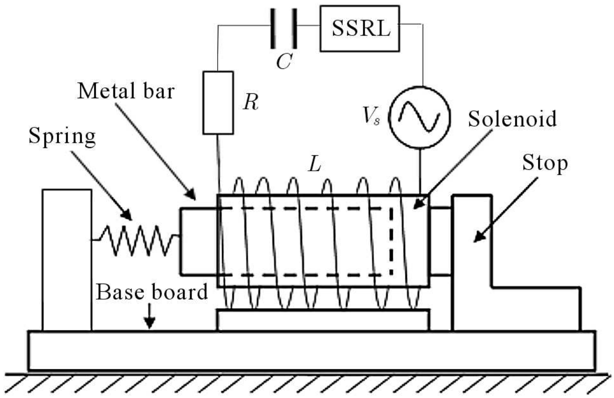

Fig. 1. Schematic diagram of the prototype of an electro-vibroimpact device (Nguyen, 2007)

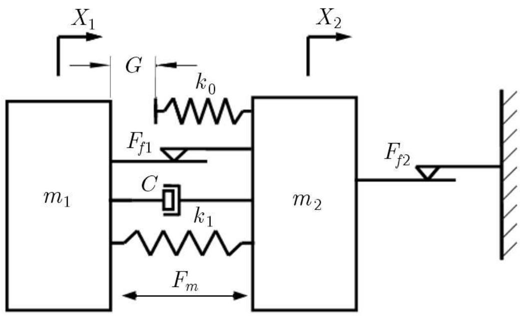

Fig. 2. Physical model of the electro-vibroimpact system (Ho *et al.*, 2008)

The governing equations of motion for the system can be expressed as (Ho *et al.*, 2008)

$$
\begin{aligned}
u' &= v \\
v' &= \Big(\frac{1}{m_1}+\frac{1}{m_2}\Big)\Big[\frac{1}{2}y^2\frac{\partial L}{\partial u} - \mu_1 m_1 g\,\mathrm{sgn}\,(v) - k_0(u-G)H(u-G) - cv + \\
&\quad -k_1(u-X_0)\Big]
\end{aligned}
$$

$$
\begin{aligned}
w' &= x \\
x' &= \frac{1}{m_2}\Big[-\frac{1}{2}y^2\frac{\partial L}{\partial u} + \mu_1 m_1 g\,\mathrm{sgn}\,(v) + k_0(u-G)H(u-G) + cv + \\
&\quad + k_1(u-X_0) - \mu_2(m_1+m_2)g\,\mathrm{sgn}\,(\dot{X}_2)\Big] \\
y' &= z \\
z' &= \frac{1}{L}\Big[\omega P_{ctr}V_s\cos(\omega t) - \Big(R + 2\frac{\partial L}{\partial u}v\Big)z - \Big(\frac{1}{C} + \frac{\partial^2 L}{\partial u^2}v^2 + \frac{\partial L}{\partial u}v'\Big)y\Big]
\end{aligned}
\tag{2.1}
$$

where u is the relative displacement of the metal bar with respect to the moling mechanism, v is the velocity of the metal bar with respect to the base board, w is the displacement of the mole, x is the velocity of the mole, y is the current, z is the first derivative of the current, m_1 is the mass of the metal bar, m_2 is the mass of the mole, μ_1 and μ_2 are frictional coefficients of the dry friction forces F_{f1} and F_{f2}, g is the acceleration of gravity, k_0 is the stiffness of the obstacle block, c is the damping coefficient, k_1 is the spring stiffness, G is the gap position, X_0 is the initial displacement of the metal bar, ω is the frequency of power supply, V_s is the voltage amplitude, R is the resistance, C is the capacitance, L is the inductance function, P_{ctr} is the factor of the control frequency and $H(\cdot)$ is the Heaveside step function defined as

$$
H(x) = \begin{cases} 1 & \text{if} \quad x > 0 \\ 0 & \text{if} \quad x \leqslant 0 \end{cases}
\tag{2.2}
$$

In this way, the discontinuous mechanical characteristics of the abrupt change in the stiffness which reflects an impact, and the velocity-dependent friction are described in this mathematical model. Since points of bifurcation are associated with the loss of stability, a scan of the system dynamics while varying a system parameter can confirm a range of system parameter values for which a operation is optimum. In particular, the switching frequency of the solid state relay is varied. The bar displacement relative to the base board is observed to identify the range of the frequency for which the achieved forward progression is maximum.

3. Bifurcation diagrams

Both experimental observation and numerical integration have revealed that both periodic and chaotic trajectories exist by varying the control frequency of

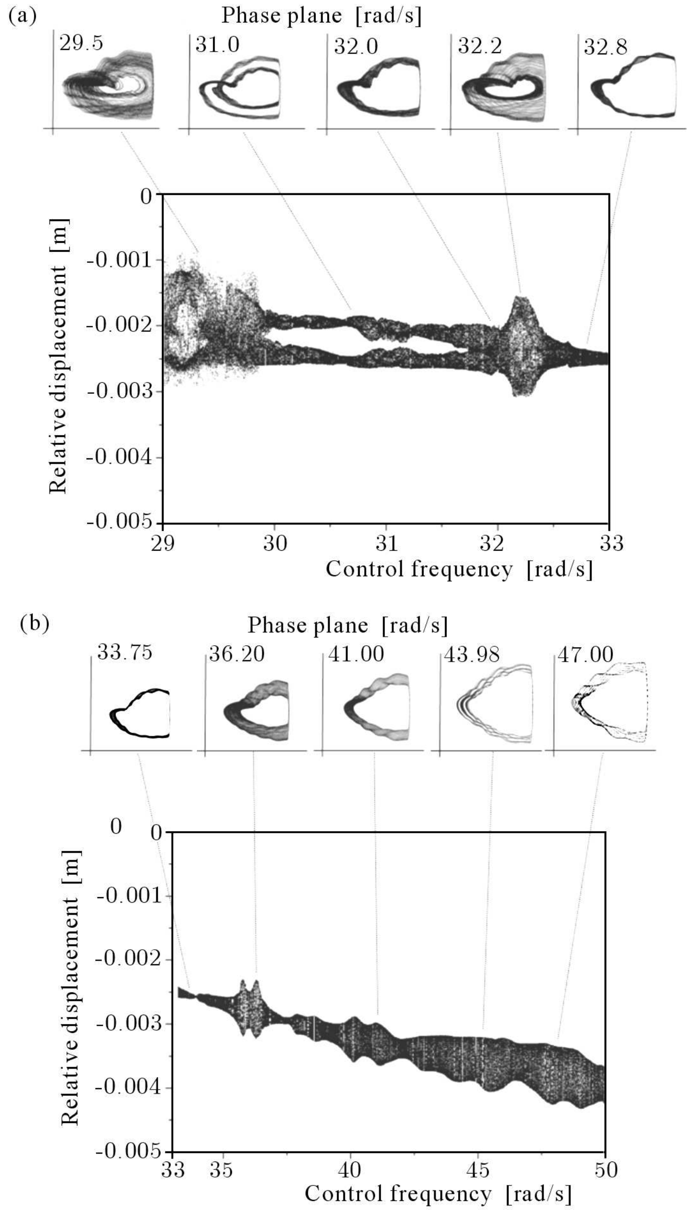

Fig. 3. Bifurcation diagram of relative motion of the bar for the same system parameters as in Nguyen (2007). The variable is the control frequency which ranges from (a) 29 rad/s to 33 rad/s (4.62 Hz to 5.25 Hz) and (b) 33 rad/s to 50 rad/s (5.25 Hz to 7.96 Hz). System parameters are $V_s = 82.02\,\mathrm{V}$, $\mu_1 = 0.295$, $\mu_2 = 0.235$, $C = 32\,\mu\mathrm{F}$, $c = 0.155\,\mathrm{kg/s}$, $R = 27.5\,\Omega$, $m_1 = 0.297\,\mathrm{kg}$, $m_2 = 2.94\,\mathrm{kg}$, $G = -0.002\,\mathrm{m}$, $k_0 = 1.24 \cdot 10^5\,\mathrm{N/m}$, $k_1 = 200\,\mathrm{N/m}$ and $X_0 = 0.022\,\mathrm{m}$

the solid state relay (SSR). An experimental bifurcation diagram constructed by Nguyen and Woo (2007) reflected the dynamics of the metal bar observed in the laboratory. Period-2 motion was observed for frequencies lower than 5 Hz, beyond which period-1 solution exists, valid up to a frequency of 8.3 Hz. At control frequencies greater than 8.3 Hz, the amplitude of the bar displacement decreased abruptly and significantly. The progression rate of the moling rig also dropped correspondingly. Those observations were checked against a bifurcation diagram constructed by numerical integration of the mathematical model (using the Dynamics software, see Yorke and Nusse (1998)), and shown in Fig. 3. 960 different values of the control frequency were used in the iteration. In Fig. 3a, the frequency increases from 4.62 Hz to 5.25 Hz in increments of $6.5625 \cdot 10^{-4}$ Hz. In Fig. 3b, the frequency increases from 5.25 Hz to 7.96 Hz with an increment of $2.8229 \cdot 10^{-3}$ Hz. Besides that, for each control frequency, 60 cycles were allowed to elapse to allow transients to subside and sample points from a steady state trajectory. Data pertaining to 200 cycles of steady state motion were captured to identify the main features of the system dynamics. In Fig. 3a, when the frequency increases from 4.62 Hz to 5.25 Hz, chaotic motion changes to period-2 motion before settling to period-1 motion.

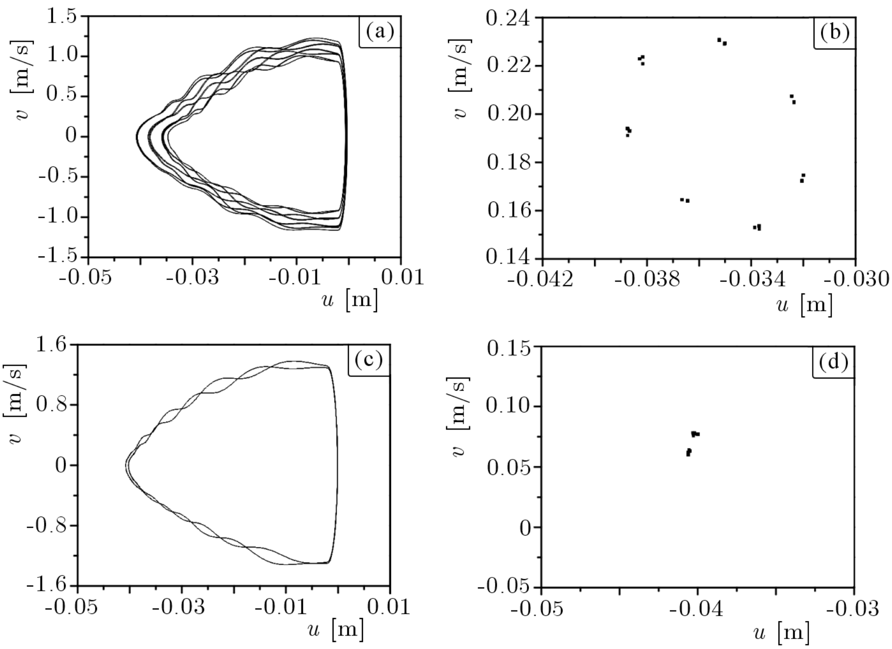

Fig. 4. Phase planes of relative motion of the bar at a control frequency of (a) 7 Hz and (c) 8 Hz. Poincaré maps are plotted for (b) 7 Hz and (d) 8 Hz

This synchronous trajectory becomes more apparent in Fig. 3b for higher frequencies. The phase portraits and Poincaré maps for 7 Hz and 8 Hz confirm that the amplitude fluctuation decreases with the increasing frequency. This is shown in Fig. 4. At a frequency of 7 Hz, period-1 motion is shown in Fig. 4a. Due to the fluctuation in the amplitude about a mean value, Poincaré sampling results in 7 distinct points on the map, as shown in Fig. 4b. On inspection of the corresponding time history in Ho *et al.* (2008), period-1 orbit is confirmed. Hence, the amplitude fluctuation here caused scatter in the data points. These may be then considered as belonging to one average amplitude of motion. To construct this, 35 cycles of the displacement were allowed to elapse before 734 data points (i.e. 734 cycles) were taken, so as to ensure that all transients had to be subsided, and a steady state had been reached. A similar situation is observed in Fig. 4c, when the control frequency is 8 Hz. Here, the amplitude fluctuation is less than the previous case, and period-1 motion is even more apparent. Due to variation between two amplitudes of motion very close to each other, the Poincaré map of Fig. 4d shows plots of two amplitudes very similar in magnitude. For this frequency, data points were taken after 40 transient cycles.

4. Discussion

The experimental observation that periodic relative motion of the metal bar with respect to the base board is most beneficial to the overall progression rate achieved by the mechanism was mentioned in the previous section. This is confirmed by the experimental bifurcation diagram shown by Nguyen and Woo (2007). Meanwhile, numerical integration has also revealed a similar phenomenon. This is shown in Fig. 5.

Experimental results revealed that the progression achieved by the mechanism for 5 seconds peaked at a control frequency of 8.3 Hz (Nguyen and Woo, 2007). Predictions of the achieved forward progression were obtained from numerical integration and are shown in Figure 5. Close correlations with the experimental data from Nguyen and Woo (2007) are observed. For example, the maximum progression achieved at a control frequency of 8.1 Hz compares favourably to 8.3 Hz in the experimental result. Besides that, local maxima were found in both cases, corresponding to 4 Hz in the numerical integration and 3.3 Hz in the experiment. In the experiment, there was almost no progression right after the local maxima and the forward progression increased gradually after that until the peak value (i.e. in the region of 3.4 Hz to 8.3 Hz).

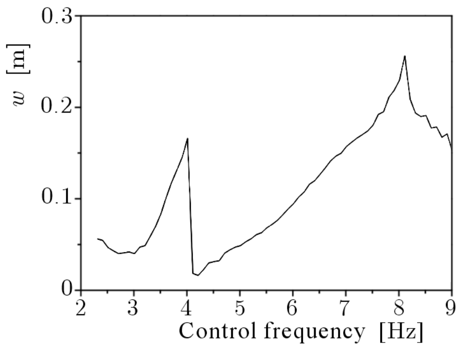

Fig. 5. The achieved mole progression for 5 seconds with respect to the control frequency obtained from numerical integration. The same set of parameters was used as in Fig. 3

The numerical integration showed a similar trend but with a different range of the control frequency (from 4 Hz to 8.1 Hz). The achieved progression dropped significantly after the peak value for both cases. However, there are some differences in the absolute magnitudes of the achieved progression for the simulation and the experiment. For example, the maximum achieved progression was found to be approximately 0.17 m at 8.3 Hz in the experiment, whereas the numerical study predicted the maximum value of 0.256 m at 8.1 Hz and 0.194 m at 8.3 Hz. This difference might have been caused by the impact energy loss, which is not accounted for in the mathematical model.

5. Conclusions

Qualitative responses of an electro-vibroimpact system have been revealed through bifurcation analysis. When the control frequency of the solid state relay increases from 4.62 Hz to 7.96 Hz, of motion the system varies from chaotic to period-2 before settling to period-1 motion. This has been observed both numerically and experimentally. The results further confirm that periodic motions are most beneficial to the mole progression rate. On gaining more insight to the system bifurcation phenomena, coupled with an approximate analytical solution, an optimal feedback control system can be then designed to achieve a better forward progression rate.

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Bifurkacje w układzie maszyny elektro-wibracyjnej z uderzeniami przy uwzględnieniu tarcia

Streszczenie

W artykule przedstawiono zjawiska bifurkacyjne zachodzące w układzie maszyny elektro-wibracyjnej z uderzeniami za pomocą analizy numerycznej. Pokazano, że układ wykazuje przejście z ruchu chaotycznego do periodycznego przy zmianach częstości sterującej (jednego z parametrów układu) pracą bezstykowego przekaźnika mocy. Zaobserwowano bliską współzależność otrzymanych diagramów bifurkacyjnych z wynikami doświadczalnymi. Stwierdzoną lepszą wydajność urządzenia dla zakresu parametrów zapewniających ruch periodyczny. Sformułowano podstawy do określenia i wdrożenia optymalnej strategii sterowania układu opartej na sprzężeniu zwrotnym.

Manuscript received December 17, 2008; accepted for print March 2, 2008