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MODEL BASED PREDICTIVE CONTROL OF GUYED MAST VIBRATION

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The purpose of this work is to present an algorithm for optimal vibration control of guyed masts and an example of its application to a numerical simulation. The objective of the proposed control system is to minimize amplitudes of transverse vibrations of the top of a mast induced by wind pressure acting on the structure. Control forces are assumed to be physically realized through changes of tension in guy cables, supporting the mast. The only required measurements are velocities of guy cables at the anchor-points. On the basis of those, a complete state of deformation of the structure is obtained by using the Kalman filter. The Davenport spectral density function is adopted as a model of the stochastic action of the wind.

Key words: structural vibrations, vibration control, guyed masts, wind fluctuations

1. Introduction

Guyed masts applied in radio, television and cellular phone industry, belong to the class of vibration-prone structures. Both their height (which can be even a few hundred meters) and their location in open spaces make them exposed to actions of strong windblasts of different velocities. In winter, masts are often covered with icing. It causes an increase, not only of the structure weight, but also of the surfaces of elements exposed to wind pressure. Ice, together with wind pressure, are the most common reasons of mast failures.

Dynamic analysis of a guyed mast is nonlinear, because of nonlinear behaviour of guy cables (McCaffrey and Hartmann, 1972). During recent years, many control methods have been developed, but their practical application to mast-like structures still remains limited. One of the recent approaches was presented by Preumont (2002). Replacing cables with massless strings, he proposed an Integral Force Feedback controller to reduce vibration of a truss structure. Another control technique of cable structures, called Active Stiffness Control, was proposed by Fujino *et al.* (1993) and was applied to cable-stayed bridges. Recently, the problem of damping cable vibrations by semi-active control was investigated by Spencer (2002).

The objective of this paper is the analysis and simulation of the Model Based Predictive Control (Goodwin *et al.*, 2001) of guyed mast vibrations.

2. The model of a mast

In deriving equations of motion, the following assumptions on the dynamics of a mast are made:

- for requirement of proper functioning of the equipment attached to the top of the mast, only vibrations of small amplitudes are taken into account
- guy cables are tightly stretched and do not carry bending loads
- deformations of structural members are linear elastic.

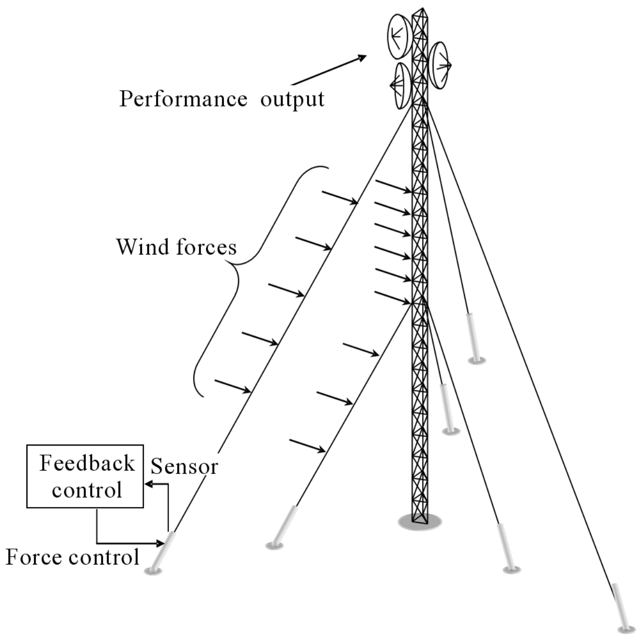

Fig. 1. Guyed mast and its feedback control system

The column of the mast is represented by a prismatic truss of a triangular cross section. One cable end is attached to the mast and the second one to an anchored mechanism, allowing for control of cable tension.

For the purpose of simulation, the mast is discretized according to the Finite Element Method (FEM). Every guy cable is represented by a chain of rods. It is assumed that each chain node has three degrees of freedom. The whole structure, i.e. the mast and its supporting guy cables, is subjected to the action of stochastic wind gusts.

The dynamic analysis of the mast is proceded by its static analysis under its dead load and prestressed forces in the cables. This way, the initial deformation of the structure is obtained, locations of nodes are determined and the global stiffness matrix is updated.

The equations of motion of the N-degree of freedom (DOF) structure can be written as follows

$$\mathbf{M}\ddot{\boldsymbol{q}}(t) + \mathbf{D}\dot{\boldsymbol{q}}(t) + \mathbf{K}\boldsymbol{q}(t) = \mathbf{B}_f\boldsymbol{f}(t) + \mathbf{B}_u\boldsymbol{u}(t) \tag{2.1}$$

where

$\mathbf{M}$ – $N \times N$ mass matrix,
$\mathbf{D}$ – $N \times N$ damping matrix,
$\mathbf{K}$ – $N \times N$ stiffness matrix,
$\boldsymbol{q}$ – N-vector representing displacements of the structure,
$\boldsymbol{f}$ – P-vector of wind velocity fluctuations,
$\boldsymbol{u}$ – R-vector of control forces,
$\mathbf{B}_u$ – control input matrix of proper dimension,
$\mathbf{B}_f$ – wind input matrix of proper dimension

$$\mathbf{B}_u = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \left.\begin{matrix} \\ \\ \\ \\ \\ \\ \end{matrix}\right\} \text{Allocation of the actuators}$$

Nonzero elements in the control input matrix represents the allocation of the actuators. The form of the wind input matrix will be explained in the next section.

Additionally, measured and performance outputs are introduced in the following form

$$\boldsymbol{y}(t) = \mathbf{C}_q\boldsymbol{q}(t) + \mathbf{C}_v\dot{\boldsymbol{q}}(t) + \boldsymbol{v}(t) \qquad\qquad \boldsymbol{z}(t) = \mathbf{C}_p\boldsymbol{q}(t) \tag{2.2}$$

where

$\boldsymbol{y}(t)$ – S-vector of measured node displacements and velocities,
$\mathbf{C}_q, \mathbf{C}_v$ – the allocation of the displacement and velocity sensors,
$\mathbf{C}_p$ – degrees of freedom to be controlled

$$\mathbf{C}_q = \mathbf{0} \qquad \mathbf{C}_v = \begin{bmatrix} 0 & \cdots & 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 \end{bmatrix}$$

$$\mathbf{C}_p = \begin{bmatrix} 0 & \cdots & 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & 0 & 1 & \cdots & 0 \end{bmatrix}$$

$\boldsymbol{v}(t)$ is an S-vector characterising the measurement noise which is assumed to be completely random. Moreover, it is assumed that measurement errors do not depend on wind disturbances

$$\begin{aligned} &E(\boldsymbol{v}(t)) = 0 \qquad \forall t \\ &E(\boldsymbol{v}(t)\boldsymbol{v}(\tau)^\top) = \mathbf{R}\delta(t-\tau) \end{aligned}$$

$\boldsymbol{z}(t)$ is a T-vector of the performance output and $\mathbf{C}_p$ is a Boolean matrix of proper dimensions selecting the degrees of freedom significant for the performance output.

The model of the mast, in the configuration space, entails long simulation times of the mast dynamics due the large number of degrees of freedom. To avoid this difficulty, a modal transformation is performed, and the number of dynamic degrees of freedom is reduced to the first N_c mode shapes

$$\boldsymbol{q}(t) = \boldsymbol{\Theta}\boldsymbol{\eta}(t) = \boldsymbol{\Theta}_c\boldsymbol{\eta}_c(t) + \boldsymbol{\Theta}_r\boldsymbol{\eta}_r(t) \approx \boldsymbol{\Theta}_c\boldsymbol{\eta}_c(t) \tag{2.3}$$

where
$\boldsymbol{\Theta}_c$ – $N \times N_c$ matrix of the first N_c mode shapes,
$\boldsymbol{\eta}$ – N-vector of modal amplitudes,
$\boldsymbol{\eta}_c, \boldsymbol{\eta}_r$ – vectors of controlled and residual modal amplitudes of dimensions N_c and N_r, respectively. It is assumed that $N_c \ll N$. N_c depends on the required accuracy of the control.

The mode shapes are normalised with respect to the mass matrix $\mathbf{M}$, that is

$$\boldsymbol{\Theta}_c^\top \mathbf{M} \boldsymbol{\Theta}_c = \mathbf{I}$$

which yields

$$\boldsymbol{\Theta}_c^\top \mathbf{K} \boldsymbol{\Theta}_c = \boldsymbol{\Omega}_c^2$$

where $\boldsymbol{\Omega}_c$ is an $N_c \times N_c$ matrix with the diagonal containing successive eigenfrequencies of the structure.

Next, adopting the model of viscous damping, one obtains

$$\boldsymbol{\Theta}_c^\top \mathbf{D} \boldsymbol{\Theta}_c = 2\boldsymbol{\Xi}_c \boldsymbol{\Omega}_c$$

where $\boldsymbol{\Xi}_c$ is an $N_c \times N_c$ modal damping matrix.

Performing modal reduction and normalising the mass matrix (Thomson, 1981), the equations of motion, measured output $\boldsymbol{y}(t)$ and performance output $\boldsymbol{z}(t)$ take the following forms

$$\begin{aligned} &\ddot{\boldsymbol{\eta}}_c(t) + 2\boldsymbol{\Xi}_c \boldsymbol{\Omega}_c \dot{\boldsymbol{\eta}}_c(t) + \boldsymbol{\Omega}_c^2 \boldsymbol{\eta}_c(t) = \boldsymbol{\Theta}_c^\top \mathbf{B}_f \boldsymbol{f}(t) + \boldsymbol{\Theta}_c^\top \mathbf{B}_u \boldsymbol{u}(t) \\ &\boldsymbol{y}(t) = \mathbf{C}_q \boldsymbol{\Theta}_c \boldsymbol{\eta}_c(t) + \mathbf{C}_v \boldsymbol{\Theta}_c \dot{\boldsymbol{\eta}}_c(t) + \boldsymbol{v}(t) \\ &\boldsymbol{z}(t) = \mathbf{C}_p \boldsymbol{\Theta}_c \boldsymbol{\eta}_c(t) \end{aligned} \tag{2.4}$$

Now, the state vector of the structure is introduced in the form $\boldsymbol{x}_m^\top(t) = [\boldsymbol{\eta}_c^\top(t), \dot{\boldsymbol{\eta}}_c^\top(t)]$ which, after substitution into (2.4), yields

$$\begin{aligned} &\dot{\boldsymbol{x}}_m(t) = \mathbf{A}_m \boldsymbol{x}_m(t) + \mathbf{B}_{m1} \boldsymbol{f}(t) + \mathbf{B}_{m2} \boldsymbol{u}(t) \\ &\boldsymbol{z}(t) = \mathbf{C}_{m1} \boldsymbol{x}_m(t) \\ &\boldsymbol{y}(t) = \mathbf{C}_{m2} \boldsymbol{x}_m(t) + \boldsymbol{v}(t) \end{aligned} \tag{2.5}$$

where

$$\mathbf{A}_m = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\boldsymbol{\Omega}_c^2 & -2\boldsymbol{\Xi}_c \boldsymbol{\Omega}_c \end{bmatrix}$$

$$\mathbf{B}_{m1} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\Theta}_c^\top \mathbf{B}_f \end{bmatrix} \qquad \mathbf{B}_{m2} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\Theta}_c^\top \mathbf{B}_u \end{bmatrix}$$

$$\mathbf{C}_{m1} = \begin{bmatrix} \mathbf{C}_p \boldsymbol{\Theta}_c & \mathbf{0} \end{bmatrix} \qquad \mathbf{C}_{m2} = \begin{bmatrix} \mathbf{C}_q \boldsymbol{\Theta}_c & \mathbf{C}_v \boldsymbol{\Theta}_c \end{bmatrix}$$

Finally, it can be observed that the modal truncation does not affect the dimension of the measured output $\boldsymbol{y}(t)$.

3. Wind modelling

The mast is exposed to action of wind, and therefore experiences time-varying loads. Initially, wind pressure was only modelled as a static loading. Later, (Iannuzzi and Spinelli, 1987) more realistic wind models were introduced in the form of trigonometric series. Currently, in the modelling of stochastic wind pressure, linear filters are frequently used (Gawronski *et al.*, 1994).

The overall wind velocity can be decomposed into its average value $\overline{f}$ and velocity fluctuation $f(t)$

$$F(t) = \overline{f} + f(t) \tag{3.1}$$

It is assumed that the average value $\overline{f}$ exerts a static load. The fluctuation component $f(t)$ is a random with the zero mean, which in the frequency domain is characterized by the spectral density function called the Davenport spectrum

$$S_f(n) = \frac{4\overline{f}_{10}^2\kappa}{n}\frac{X^2}{(1+X^2)^{4/3}} \qquad\qquad X = \frac{1200n}{\overline{f}_{10}} \tag{3.2}$$

where n is the frequency [Hz], $\overline{f}_{10}$ is the average velocity at the 10 meter altitude [m/s] and κ – terrain roughness coefficient.

The along-wind force acting on both the mast and its guy cables can be decomposed into a static and dynamic part

$$P(t) = \frac{1}{2}\rho_a C_d A_e \overline{f}^2 + \rho_a C_d A_e \overline{f} f(t) \tag{3.3}$$

where ρ_a is the air density, C_d – drag coefficient, A_e – exposition area, $\overline{f}$ denotes the average wind velocity and f is the wind velocity fluctuation.

The above relation allows determination of elements of the matrix $\mathbf{B}_f$ in $(2.4)_1$

$$\mathbf{B}_f = \begin{bmatrix} \rho_a C_d A_{e,1}\overline{f}\cos\alpha \\ \rho_a C_d A_{e,1}\overline{f}\sin\alpha \\ 0 \\ \rho_a C_d A_{e,2}\overline{f}\cos\alpha \\ \rho_a C_d A_{e,2}\overline{f}\sin\alpha \\ 0 \\ \vdots \end{bmatrix} \tag{3.4}$$

where α is the wind direction.

The wind load is obtained by applying as an input a purely random process to a filter that approximates the Davenport spectrum (Davenport, 1961) within a desired bandwidth.

The Davenport filter is presented as a linear dynamical system of the form

$$\begin{aligned} \dot{\boldsymbol{x}}_w(t) &= \mathbf{A}_w \boldsymbol{x}_w(t) + \mathbf{B}_w \boldsymbol{w}(t) \\ \boldsymbol{f}(t) &= \mathbf{C}_w \boldsymbol{x}_w(t) \end{aligned} \tag{3.5}$$

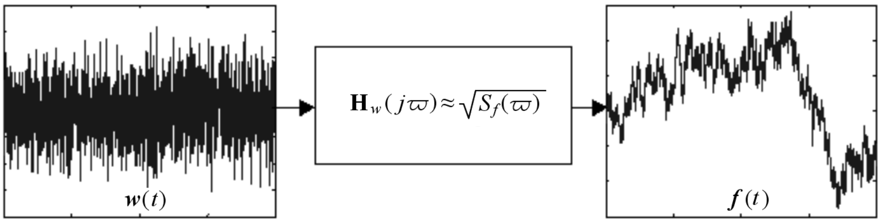

Fig. 2. Davenport filter

where $\boldsymbol{w}(t)$ is a purely random sequence of data,

$$\begin{aligned} E(\boldsymbol{w}(t)) &= 0 \qquad \forall t \\ E(\boldsymbol{w}(t)\boldsymbol{w}(\tau)^\top) &= \mathbf{Q}_w\delta(t-\tau) \end{aligned} \tag{3.6}$$

and the matrices $\mathbf{A}_w$, $\mathbf{B}_w$, $\mathbf{C}_w$ are chosen such that the following formula is satisfied

$$\mathbf{H}_w(j\varpi) = \mathbf{C}_w(s\mathbf{I} - \mathbf{A}_w)^{-1}\mathbf{B}_w \tag{3.7}$$

The structure of the Davenport filter ensures that the input $\boldsymbol{w}(t)$ is uncorrelated with the measurement noise $\boldsymbol{v}(t)$

$$E\Big(\boldsymbol{v}(t)\boldsymbol{w}(\tau)^\top\Big) = \mathbf{0} \tag{3.8}$$

4. Controllability

Before designing a control system, it is important to verify the controllability. The classical criterion for the controllability tells that a dynamical system is controllable if its controllability matrix has the rank $2N_c$

$$\text{rank}\Big[\mathbf{B}_{m2} \quad \mathbf{A}_m\mathbf{B}_{m2} \quad \mathbf{A}_m^2\mathbf{B}_{m2} \quad \dots \quad \mathbf{A}_m^{2N_c-1}\mathbf{B}_{m2}\Big] = 2N_c \tag{4.1}$$

where N_c is the number of mode shapes.

However, in large systems there are numerical difficulties involved in calculating this rank. Hence, from the computational point of view, it is much more convenient to use an alternative method. One of them is the concept of modal controllability index. Information on the controllability is here obtained for particular mode shapes, through calculating the lengths of successive rows of the matrix $\mathbf{\Theta}_c^\top\mathbf{B}_u$

$$\mu_j = \mathbf{\Theta}_c^{(j)\top}\mathbf{B}_u\mathbf{B}_u^\top\mathbf{\Theta}_c^{(j)} \qquad j = 1,2,\dots,N_c \tag{4.2}$$

where $\mathbf{\Theta}_c^{(j)}$ is the jth column of the matrix of the mode shapes.

The index equal to zero signifies that the corresponding mode shape is uncontrollable. Additional attention has to be paid to those indices which correspond to multiple eigenfrequencies (Joshi, 1989).

For the guyed mast under consideration it was observed that the control forces very weakly affect the symmetrical mode shapes of cables (Fig. 4), but they have significant influence on the antisymmetrical ones (Fig. 5), i.e. those involving transverse vibrations of the column. It is advantageous because those modes are responsible for transverse vibration of the top of the mast, which is going to be damped.

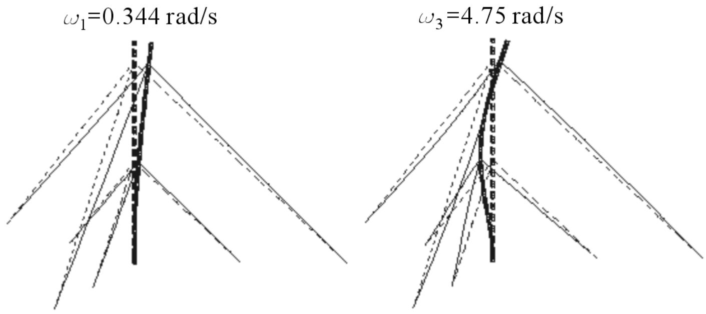

Fig. 3. Dominant mode shapes of column

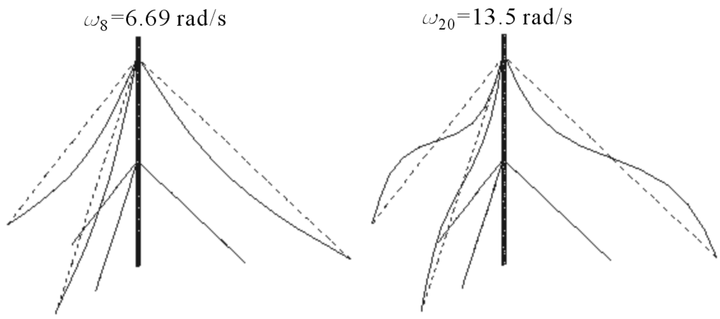

Fig. 4. Dominant axi-symmertical mode shapes of cables

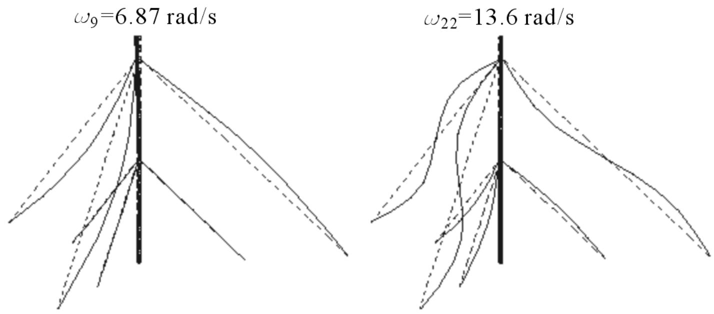

Fig. 5. Dominant antisymmetrical mode shapes of cables

5. Control system

One of the possible implementations of this idea is a control system which consists of: a velocity sensor, digital controller and hydraulic actuator. The actuator is driven by the controller which gets information from the velocity sensor.

As a result, the control forces change the tension in guy cables. Moreover, it is assumed that the control system is collocated (Fig. 6), which means that sensor placements coincide with that of actuators. Aditionally, the hydraulic actuator and velocity sensor are supported in such a way that the control forces do not affect measurements.

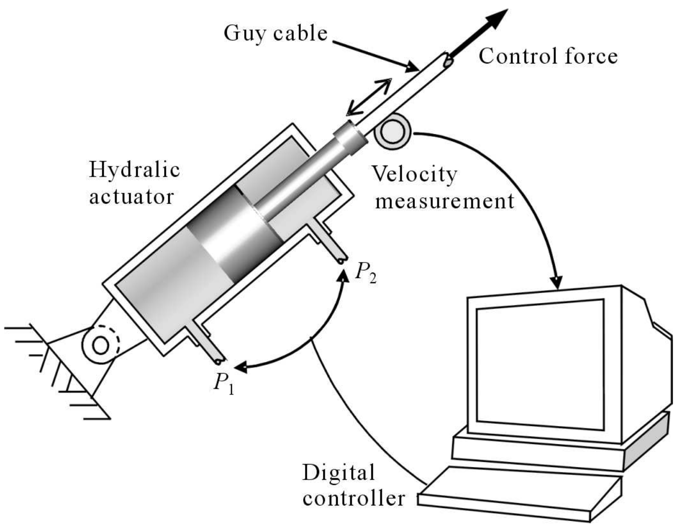

Fig. 6. Control system scheme

6. Control strategy

After answering the question of the controllability, one can proceed to choose a suitable control strategy. In this paper, the idea of Model Based Predictive Control (Goodwin *et al.*, 2001) is used. It is an algorithm, based on on-line solving an optimal control problem (Fig. 7), which can be summarized in the following steps:

(i) at each time instant, using past and current measurements, estimate the current state vector

(ii) solve on-line the optimal control problem over some future interval, taking into account the current and future constraints

(iii) use, as the current control signal, the first step in the computed optimal control sequence.

When dealing with an uncertain structure, the control system has to be augmented with a model updating procedure. The objective of that procedure is to identify current dynamics of the structure which operates in varying environmental conditions.

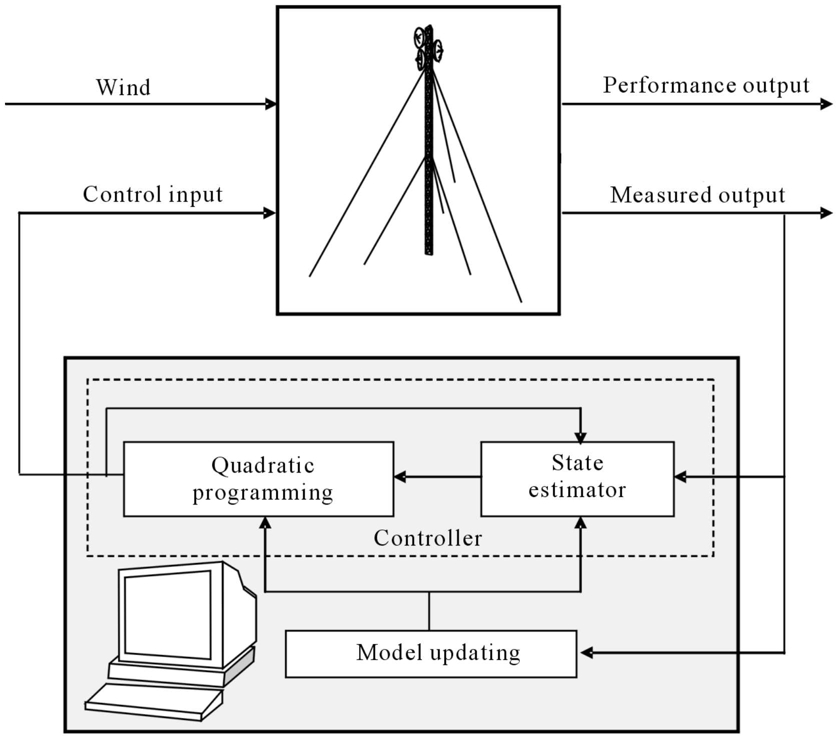

Fig. 7. Model predictive control of guyed mast

7. Optimal estimator

Optimal control of a dynamical system requires knowledge of the state of that system. In practice, individual state variables cannot be determined exactly by direct measurements. Instead, measurements that can be made are functions of the state variables, and these measurements contain random errors. A guyed mast itself is also subjected to random wind disturbances. If the structure is completely observable, and, in collocated systems, this property holds simultaneously with the controllability, a virtual dynamical system called the estimator can be constructed.

The goal of the estimator is to assess, on the basis of the measured quantities, the current deformation of the structure and the value of the wind force (Fig. 8). For that purpose, an augmented dynamical system is introduced which consists of a model of the guyed mast and a model of the wind force represented by the Davenport filter. It takes the form

$$\begin{aligned} \dot{\boldsymbol{x}}_m(t) &= \mathbf{A}_m\boldsymbol{x}_m(t) + \mathbf{B}_{m1}\mathbf{C}_w\boldsymbol{x}_w(t) + \mathbf{B}_{m2}\boldsymbol{u}(t) \\ \dot{\boldsymbol{x}}_w(t) &= \mathbf{A}_w\boldsymbol{x}_w(t) + \mathbf{B}_w\boldsymbol{w}(t) \\ \boldsymbol{z}(t) &= \mathbf{C}_{m1}\boldsymbol{x}_m(t) \\ \boldsymbol{y}(t) &= \mathbf{C}_{m2}\boldsymbol{x}_m(t) + \boldsymbol{v}(t) \end{aligned} \tag{7.1}$$

The above equations can be written uniformly as

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \mathbf{A}_c\boldsymbol{x}(t) + \mathbf{B}_{c1}\boldsymbol{w}(t) + \mathbf{B}_{c2}\boldsymbol{u}(t) \\ \boldsymbol{z}(t) &= \mathbf{C}_{c1}\boldsymbol{x}(t) \\ \boldsymbol{y}(t) &= \mathbf{C}_{c2}\boldsymbol{x}(t) + \boldsymbol{v}(t) \end{aligned} \tag{7.2}$$

Finally, equations (7.2) are translated into a discrete time state-space formulation (Franklin *et al.*, 1990)

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \mathbf{A}\boldsymbol{x}_k + \mathbf{B}_1\boldsymbol{w}_k + \mathbf{B}_2\boldsymbol{u}_k \\ \boldsymbol{z}_k &= \mathbf{C}_1\boldsymbol{x}_k && \text{(performance output)} \\ \boldsymbol{y}_k &= \mathbf{C}_2\boldsymbol{x}_k + \boldsymbol{v}_k && \text{(measured output)} \end{aligned} \tag{7.3}$$

The design of an optimal estimator depends on probabilistic data, concerning the initial condition of the system, disturbances and measurement errors

$$\begin{aligned} & E(\boldsymbol{x}_0\boldsymbol{x}_0^\top) = \mathbf{M}_0 && E(\boldsymbol{w}_k\boldsymbol{w}_l^\top) = \frac{1}{\Delta t}\mathbf{Q}_w\delta_{kl} = \mathbf{Q}\delta_{kl} \\ & E(\boldsymbol{v}_k\boldsymbol{v}_l^\top) = \mathbf{R}\delta_{kl} \end{aligned} \tag{7.4}$$

where Δt is the sampling time.

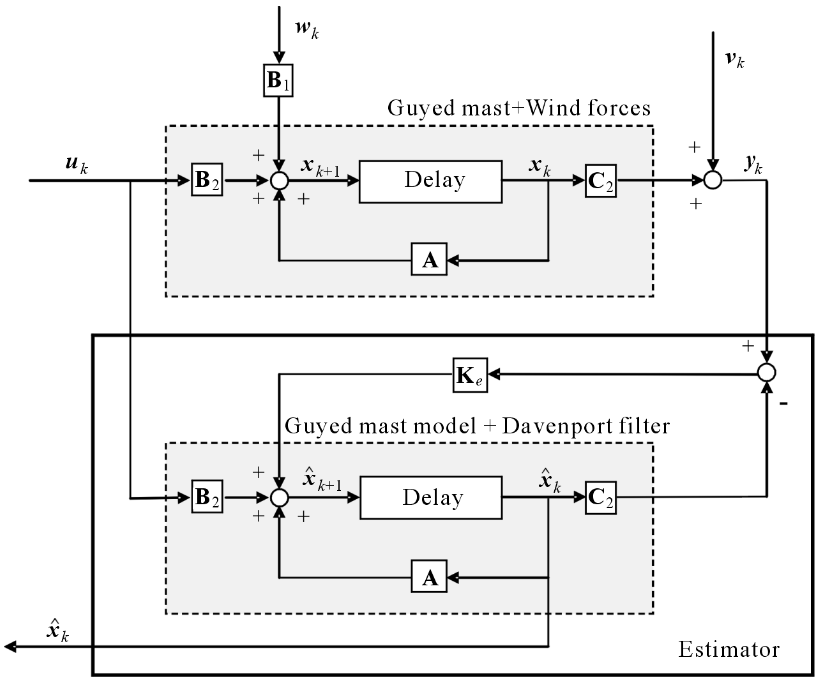

Fig. 8. Kalman filter concept

Taking into account the above facts, the maximum likelihood estimate of the state $\boldsymbol{x}_m$ is given by sequential use of the following procedure (Bryson and Ho, 1975)

$$
\begin{aligned}
&\widehat{\boldsymbol{x}}_{k+1} = \mathbf{A}\widehat{\boldsymbol{x}}_k + \mathbf{B}_2\boldsymbol{u}_k + \mathbf{K}_e(\boldsymbol{y}_k - \mathbf{C}_2\widehat{\boldsymbol{x}}_k) \qquad \widehat{\boldsymbol{x}}_0 \ \text{ given}\\
&\mathbf{K}_e = \mathbf{A}\mathbf{P}_k\mathbf{C}_2^\top\mathbf{R}^{-1} \qquad k = 0, 1, \dots, m\\
&\mathbf{P}_k = \mathbf{M}_k - \mathbf{M}_k\mathbf{C}_2^\top(\mathbf{C}_2\mathbf{M}_k\mathbf{C}_2^\top + \mathbf{R})^{-1}\mathbf{C}_2\mathbf{M}_k\\
&\mathbf{M}_{k+1} = \mathbf{A}\mathbf{P}_k\mathbf{A}^\top + \mathbf{B}_1\mathbf{Q}\mathbf{B}_1^\top
\end{aligned}
\tag{7.5}
$$

To implement MPC, it is required to know not only the current, but also future states of the system. Theory of optimal prediction tells that the best estimate of the future state vector can be obtained by taking the expected value of the forward solution to discrete state equations, calculated on the basis of the current estimate:

$$
\widehat{\boldsymbol{x}}_k = \mathbf{A}^k\widehat{\boldsymbol{x}}_0 + \sum_{m=0}^{k-1}\mathbf{A}^{k-1-m}\mathbf{B}_2\boldsymbol{u}_m \qquad k = 1, 2, \dots, n \tag{7.6}
$$

8. Optimal control

Knowing the current state estimate, one can propose an optimization procedure for the discussed control system. The following cost function is proposed by wishing to drive the current state vector to the smallest possible value over a specified time interval, but without spending too much control effort to achieve this goal

$$J = \boldsymbol{z}_n^\top \boldsymbol{\Phi}_{zn}\boldsymbol{z}_n + \sum_{k=0}^{n-1} \boldsymbol{z}_k^\top \boldsymbol{z}_k + \boldsymbol{u}_k^\top \boldsymbol{\Psi}\boldsymbol{u}_k = \boldsymbol{x}_n^\top \boldsymbol{\Phi}_n \boldsymbol{x}_n + \sum_{k=0}^{n-1} \boldsymbol{x}_k^\top \boldsymbol{\Phi}\boldsymbol{x}_k + \boldsymbol{u}_k^\top \boldsymbol{\Psi}\boldsymbol{u}_k \tag{8.1}$$

where $\boldsymbol{\Phi}_n$, $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$ are weighting matrices of proper dimensions.

It is assumed that the control forces $\boldsymbol{u}_k$ are several times smaller than the static tensile force in cables. This allows one to use a quadratic cost function as the performance index.

Assuming a random wind fluctuation to be equal to its mean value, which is zero, and using the solution to the discrete state equation, one can arrive at the following quadratic programming problem. Additionally, the constraints on the control force amplitude can be expressed in the form of linear inequalities. Finally, one obtains a quadratic programming problem with the constraints

$$J = J_0 + \mathbf{U}^\top \boldsymbol{\Gamma}\mathbf{U} + \mathbf{H}\mathbf{U} \qquad\qquad \mathbf{U}_{min} \leqslant \mathbf{U} \leqslant \mathbf{U}_{max} \tag{8.2}$$

where $\mathbf{H} = 2\boldsymbol{x}_0^\top \boldsymbol{\Omega}^\top \overline{\boldsymbol{\Phi}\boldsymbol{\Lambda}}$, $\boldsymbol{\Gamma} = \overline{\boldsymbol{\Lambda}}^\top \overline{\boldsymbol{\Phi}\boldsymbol{\Lambda}} + \overline{\boldsymbol{\Psi}}$ and

$$\overline{\boldsymbol{\Lambda}} = \begin{bmatrix} \mathbf{B}_2 & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{A}\mathbf{B}_2 & \mathbf{B}_2 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{A}^2\mathbf{B}_2 & \mathbf{A}\mathbf{B}_2 & \mathbf{B}_2 & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}^{n-1}\mathbf{B}_2 & \mathbf{A}^{n-2}\mathbf{B}_2 & \mathbf{A}^{n-3}\mathbf{B}_2 & \dots & \mathbf{B}_2 \end{bmatrix} \qquad \boldsymbol{\Omega} = \begin{bmatrix} \mathbf{I} \\ \mathbf{A} \\ \vdots \\ \mathbf{A}^{n-1} \\ \mathbf{A}^n \end{bmatrix}$$

$$\overline{\boldsymbol{\Psi}} = \begin{bmatrix} \boldsymbol{\Psi} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Psi} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\Psi} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \boldsymbol{\Psi} \end{bmatrix} \qquad \overline{\boldsymbol{\Phi}} = \begin{bmatrix} \boldsymbol{\Phi} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Phi} & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \boldsymbol{\Phi} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \boldsymbol{\Phi}_n \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} \boldsymbol{u}_0 & \boldsymbol{u}_1 & \boldsymbol{u}_2 & \dots & \boldsymbol{u}_{n-1} \end{bmatrix}^\top$$

Constraints can be also imposed on the rate of change of the control forces. This can be particularly important in the case of using hydraulic actuators.

After solution, the first signal in the optimal control sequence is applied, and the whole procedure is repeated at the next time instant

$$\mathbf{U}^{OPT} = \arg \min_{\mathbf{L}\mathbf{U} \leqslant \mathbf{b}} \mathbf{H}\mathbf{U} + \mathbf{U}^{\top}\mathbf{\Gamma}\mathbf{U} \tag{8.3}$$

where

$$\mathbf{L} = \begin{bmatrix} \mathbf{I} \\ -\mathbf{I} \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} \mathbf{U}_{max} \\ \mathbf{U}_{min} \end{bmatrix}$$

$\mathbf{I}$ is identity matrix.

Closed-loop stability of the system can be verified by Lyapunov theory where the quadratic cost index is chosen as the Lyapunov function. As it was mentioned earlier, the identification of the mast parameters, like its mass and stiffness, is performed by an updating procedure of the model. Similarly to the case of the state estimator, the assessment of the mast parameters could be done by a Kalman filter whose state is defined by the uncertain parameters of the mast. To realize the above idea, it is necessary to assume a certain measure for the system of parameter estimation, like covariance of uncertainty of the parameters.

9. Numerical simulation

In this section some results of numerical simulations are shown. A 100-meter-high mast is supported by 6 active guy cables. The guy cables are anchored at an angle of 45°. The column of the mast is a spatial truss structure with a cross-section of shape of an equilateral triangle of 1.5 meter side length. The constitutent elements of the column are circular tubes with following cross-section areas: $0.0074\,\mathrm{m}^2$ for vertical members, $0.001\,\mathrm{m}^2$ for horizontal and diagonal members. It is assumed that the structure is made from steel, which has following properties: Young's modulus $E = 205\,\mathrm{GPa}$, mass density $\rho = 7500\,\mathrm{kg/m}^3$. The diameter of guy cables is chosen as $d = 30\,\mathrm{mm}$. The guy cables are prestressed by a force of $T = 400\,\mathrm{kN}$, which correspond to internal stresses of 570 MPa.

Dynamics of the mast is modelled by 32 mode shapes of frequencies ranging from 0 to 20 rad/s. For the first 10 modes, damping equal to 1% of the critical damping is assumed, i.e.

$$\mathbf{D} = 0.006\mathbf{M} + 0.0001\mathbf{K}$$

In order to verify the controllability of the structure, modal controllability indices are calculated:

Mode no.	Frequency ω [rad/s]	Modal controllability index μ
1.	0.3438	9.8361E-2
2.	0.3440	9.8499E-2
3.	4.7474	9.0610E-2
4.	4.7492	9.0650E-2
5.	6.6279	7.9389E-9
6.	6.6892	4.2767E-6
7.	6.6892	3.9231E-6
8.	6.6899	1.8798E-7
9.	6.8727	8.8209E-3
10.	6.8747	8.9116E-3

The data characterising wind gusts are following:

- air density $1.22\,\text{kg/m}^3$
- average wind velocity $25\,\text{m/s}$
- drag coefficient $C_d = 1$

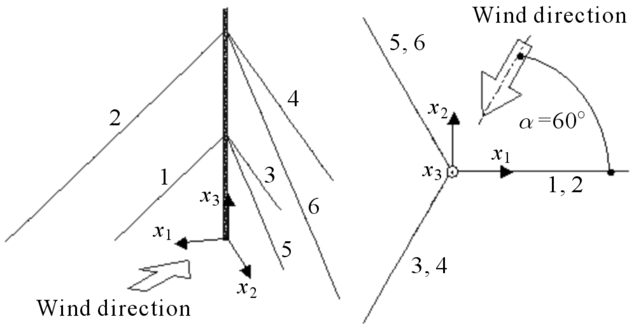

Fig. 9. Scheme of guyed mast and direction of applied wind forces

Davenport filter (3.7) has the following transfer function

$$\mathbf{H}_w(s) = \frac{a_0 + a_1 s + a_2 s^2 + a_3 s^3}{b_0 + b_1 s + b_2 s^2 + b_3 s^3 + b_4 s^4}$$

where the filter parameters are

$$a_0 = 3.4197 \qquad b_0 = 0.3538$$
$$a_1 = -686.3151 \qquad b_1 = 22.7788$$
$$a_2 = 230.1426 \qquad b_2 = 224.7118$$
$$a_3 = 3.9021 \qquad b_3 = 38.2997$$
$$b_4 = 0.331$$

Full spatial correlation of velocity fluctuation is assumed.

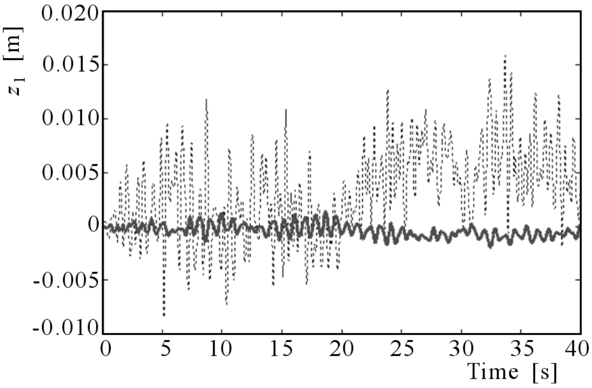

Fig. 10. Displacement in x_1 direction (dashed line-without control, solid line-with MP control)

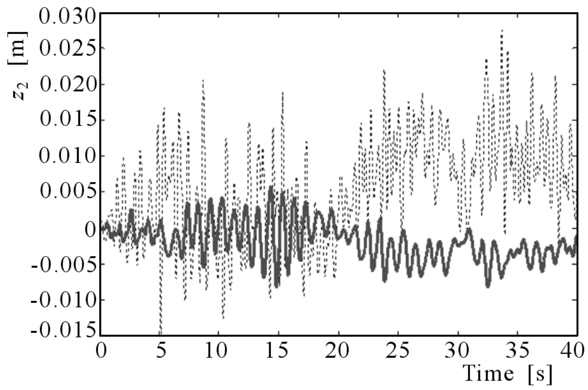

Fig. 11. Displacement in x_2 direciton (dashed line-without control, solid line-with MP control)

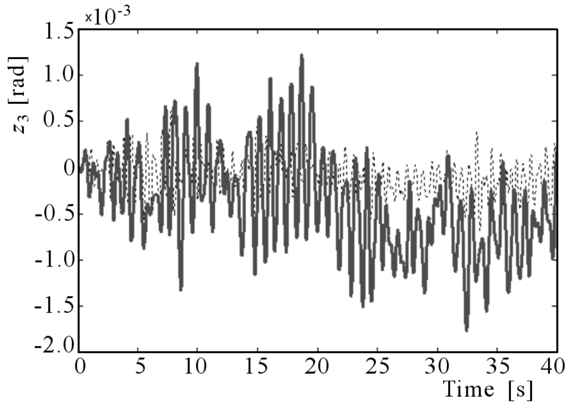

Fig. 12. Inclination angle in x_1x_3 plane (dashed line-without control, solid line-with MP control)

In Fig. 10 - Fig. 13, dynamic behaviour of the mast is presented. The dashed line corresponds to the open loop system without control and the solid line to the closed loop system with Model Based Predictive Control. The disturbing forces are applied in the form of random fluctuations with the Davenport spectrum. In the case of the transverse displacement of the top of the mast, a significant reduction of vibration amplitudes can be observed. The control strategy is effective regardless of the wind direction.

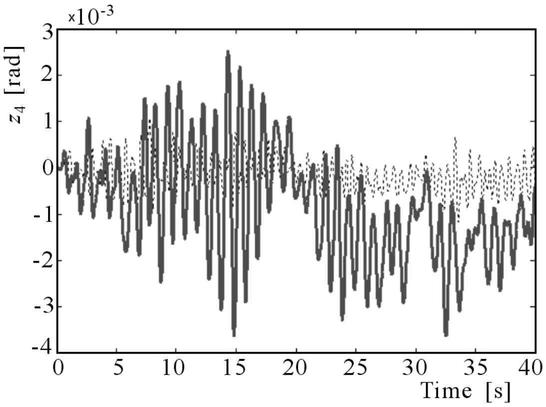

Fig. 13. Inclination angle in x_2x_3 plane (dashed line-without control, solid line-with MP control)

10. Conclusions

- A 3D FEM model of a guyed mast under control forces is proposed.
- The external loading is included in the form of a stochastic wind model with the Davenport spectrum.
- Controllability of individual mode shapes of the mast is determined.
- A model based estimator for the mast is constructed.
- It has been demonstrated that a combination of the Kalman and Davenport filters enables prediction of wind forces acting on the guyed mast.
- A numerical simulation of the control process is presented, displaying a significant reduction in vibration amplitudes of the top of the mast.

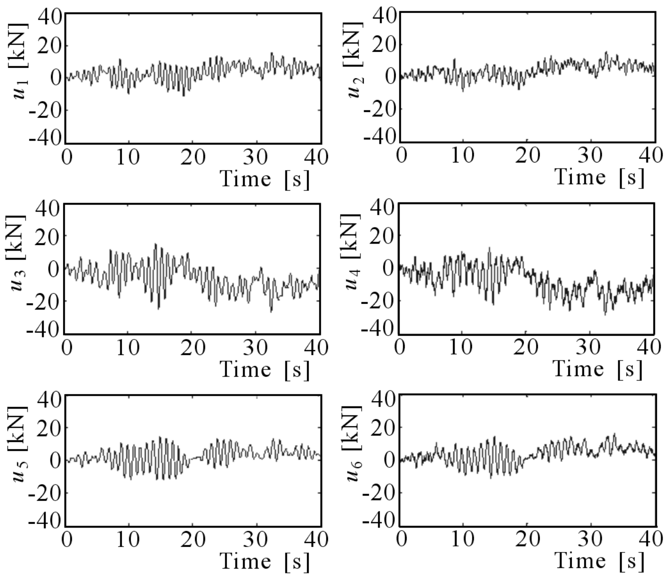

Fig. 14. Control forces in guy cables (Fig. 9)

11. Future work

The linear dynamical model of a guyed mast the present work is based on does not reflect such phenomena as e.g. parametric resonance. On the other hand, it was shown by Preumont (2002) that Active Damping strategies are effective even in the presence of loads capable of inducing parametric resonance. To verify in this respect the performance of the proposed algorithm based on Model Based Predictive Control, further research in this field is required.

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Predykcyjne sterowanie drganiami masztów z odciągami

Streszczenie

Celem pracy jest algorytm i jego przykładowe zastosowanie do symulacji numerycznej optymalnego sterowania drganiami masztów z odciągami. Zadaniem układu sterowania jest minimalizacja amplitud poprzecznych drgań wierzchołka masztu wywołanych oddziaływaniem wiatru. Realizacja sił sterowania odbywa się poprzez zmianę naciągu w odciągach masztu. Estymacja pełnego stanu deformacji konstrukcji na podstawie pomiaru jedynie prędkości w punktach zakotwienia odciągów uzyskana jest poprzez wykorzystanie filtru Kalmana. Do zamodelowania losowego oddziaływania wiatru użyto funkcję gęstości widmowej Davenporta.

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