JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
44, 2, pp. 351-366, Warsaw 2006

PARALLEL EVOLUTIONARY ALGORITHMS IN SHAPE OPTIMIZATION OF HEAT RADIATORS

Tadeusz Burczyński

Department for Strength of Materials and Computational Mechanics, Silesian University of Technology and Artificial Intelligence Department, Cracow University of Technology
e-mail: Tadeusz.Burczynski@polsl.pl

Adam Długosz
Wacław Kuś

Department for Strength of Materials and Computational Mechanics
Silesian University of Technology
e-mail: Adam.Dlugosz@polsl.pl; Waclaw.Kus@polsl.pl

The paper deals with the application of Parallel Evolutionary Algorithms (PEA) and the Finite Element Method (FEM) in shape optimization of heat radiators. The fitness function is computed with the use of the coupled thermoelsticity modelled by MARC/MENTAT software. The geometry, mesh and boundary conditions are created on the basis of a script language implemented in MENTAT. In order to reduce the number of design parameters in evolutionary algorithms, the shape of the structure is modelled by Bezier curves. Numerical examples for some shape optimization are included.

Key words: coupled thermoelasticity, radiation, finite element method, parallel evolutionary algorithm, evolutionary optimization, shape optimization

1. Introduction

In the present paper, the application of Parallel Evolutionary Algorithms (PEA) and commercial FEM software MARC/MENTAT for shape optimization of heat radiators are presented.

Evolutionary algorithms have had various applications to structural optimization. The main feature of this class of procedures is their randomness. The application of evolutionary algorithms in optimization needs only information about values of an objective (fitness) function. No sensitivity coefficients are required, and the algorithms are able to find the global minimum in the presence of local minima. The main drawback of this techniques is their high computation cost. In order to speed up evolutionary optimization, parallel evolutionary algorithms are proposed instead of sequential evolutionary algorithms.

The fitness function is calculated for each chromosome in each generation by solving a boundary – value problem of thermoelasticity by means of the FEM (Zienkiewicz and Taylor, 2000a,b,c).

Optimized radiators are modelled as structures subjected to mechanical and thermal boundary conditions. The interaction of stress and temperature fields are modelled using the steady-state coupled thermoelasticity formulation. Besides the applied heat and convection, also radiative boundary conditions are taken into account.

In order to create a mesh, boundary conditions and material properties of the model, a preprocessor MENTAT is used. The internal script language implemented in MENTAT allows avoiding the external mesher procedure. Another benefit of this approach is that MENTAT takes into account the shadowing effect in radiation.

This work is an extension of a previous paper in which evolutionary algorithms were used to shape optimization of elastic structures (Burczyński *et al.*, 2002, 2004b; Burczyński and Kuś, 2002, 2004), shape optimization of thermoelastic structures (Burczyński and Długosz, 2001, 2002; Długosz, 2001, 2004) and shape optimization of thermoelastic structures in the presence of radiation (Białecki *et al.*, 2003a,b, 2005; Burczyński and Długosz, 2004a).

2. Fitness function evaluation

2.1. Fitness function evaluation

The fitness function is computed with the use of the coupled thermoelsticity. This problem is solved by commercial FEM software – MARC [22].

In the modeled structure, mechanical as well as thermal boundary conditions are applied. Besides the applied heat and convection, also radiative boundary conditions are taken into account.

In many analyses, the radiative transfer (Modest, 1993; Siegel and Howell, 1992) of heat between surfaces plays a significant role. To model this effect properly, it is necessary to compute the proportion of one surface which is visible from a second surface known as the viewfactor. It is necessary to subdivide the radiative boundary in the heat transfer problem into one or more unconnected cavities. For each cavity, the system defines an outline of the cavity in terms of an ordered sequence of nodes (Fig. 1) [22].

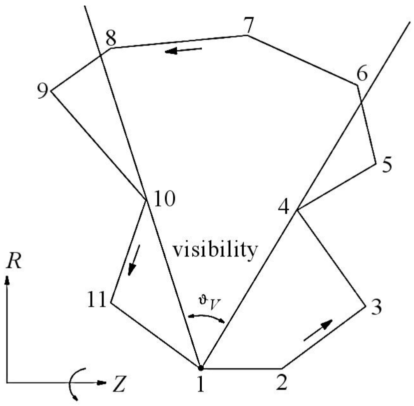

Fig. 1. Radiating cavity

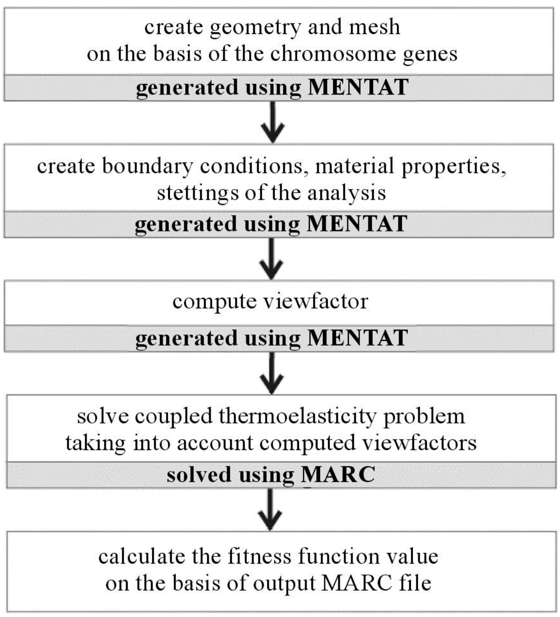

Fig. 2. An algorithm for the evaluation of the fitness function value

The preprocessor MENTAT allows one to calculate the viewfactor which is generally nontrivial. The internal script language implemented in MENTAT also allows one to produce the geometry, mesh, material properties and settings of the analysis. Figure 2 shows steps of the evaluation of the fitness function for each chromosome.

2.2. Geometry modelling

The choice of the geometry modelling method and design variables have great influence on the final solution to the optimisation process. There is a lot of methods for geometry modelling. In the proposed approach, Bezier curves are used to model the geometry of structures. This type of curves is a superset of the more commonly known NURBS (Non-Uniform Rational B-Spline). Using these curves in opitimization, allows one to reduce the number of design parameters.

Manipulation of the control points provides some flexibility to design a large variety of shapes.

The nth-degree Bezier curve is defined by

$$C(u) = \sum_{i=0}^{n} B_{i,n}(u) P_i \tag{2.1}$$

where u is a coordinate ranging in the interval $\langle 0, 1 \rangle$, P_i are control points.

The basis functions $B_{i,n}$ are given by

$$B_{i,n}(u) = \frac{n!}{i!(n-i)!} u^i (1-u)^{n-1} \tag{2.2}$$

The 4th degree Bezier curve is described by the following equation

$$C(u) = (1-u)^4 P_0 + 4u(1-u)^3 P_1 + 6u^2(1-u)^2 P_2 + 4u^3(1-u) P_3 + u^4 P_4 \tag{2.3}$$

An example of the 4th Bezier curves is shown in Fig. 3. Manipulation of the control points gives flexibility to design a large variety of shapes.

By changing the value of t between 0 and 1, we obtain successive points of the curve. For $u = 0$, $C(u) = P_0$ and for $u = 1$, $C(u) = P_4$. Shapes of Bezier curves depend on the position of control points. In order to obtain more complicated shapes, it is necessary to raise up the degree of Bezier curves and introduce more control points.

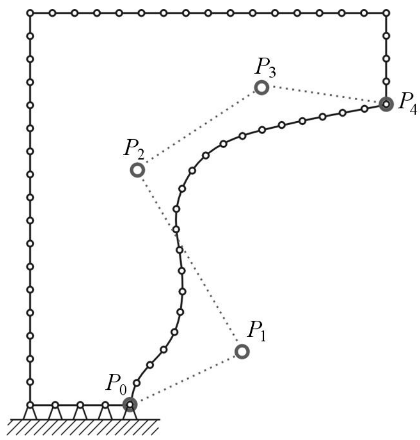

Fig. 3. The modelling of the shape of a boundary by 4th-degree Bezier curves

3. Evolutionary algorithm

Sequential genetic algorithms (Arabas, 2001; Michalewicz, 1996) can be considered as modified and generalized genetic algorithms in which populations are coded by the floating point representation. A solution to this problem is given by the best chromosome whose genes represent design parameters responsible for the shape of the radiator. The evolutionary algorithm starts with a population of chromosomes randomly generated from the feasible solution domain. Figure 4 shows the main steps of the evolutionary algorithm.

The design vector is represented by a chromosome X which consists of genes x_i, $i = 1, \ldots, N$

$$\mathrm{X} = [\mathrm{x}_1, \ldots, \mathrm{x}_i, \ldots, \mathrm{x}_N] \tag{3.1}$$

The genes can be considered as design variables. On each gene, the following constraints are imposed

$$\mathrm{x}_{iL} \leqslant \mathrm{x}_i \leqslant \mathrm{x}_{iR} \qquad i = 1, 2, \ldots, N \tag{3.2}$$

where x_{iL} and x_{iR} are left and right admissible values of x_i.

Two kinds of mutation are applied: a uniform mutation and a Gaussian mutation. The operator of the uniform mutation replaces a randomly chosen gene of the chromosome with a new random value x_i', (Fig. 5a). This value corresponds to the design parameter with its constrains. Figure 5b shows the method of working of the uniform mutation.

For the Gaussian mutation a new value of the gene is created with the use of Gaussian distribution. The searching for the Gaussian mutation is local

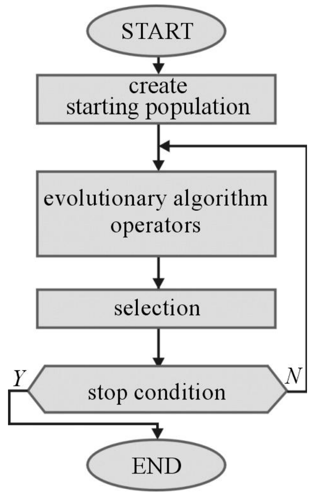

Fig. 4. A flowchart of the sequential evolutionary algorithm

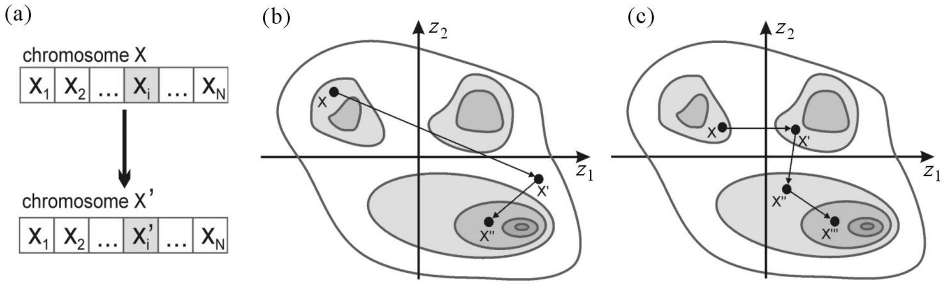

Fig. 5. (a) The scheme of the mutation, (b) searching the uniform mutation, (c) searching the Gaussian mutation

(Fig. 5c). The probability of the mutation decides how many genes will be modified in each population.

The operator of the simple crossover creates two new chromosomes x' and y' from two randomly selected chromosomes x and y. Both chromosomes are cut in a random position and coupled together (Fig. 6a). Figure 6b shows the method of working of the simple crossover.

The ranking selection allows chromosomes with a great value of the fitness function to survive. The first step of the ranking selection is sorting all the chromosomes according to the value of the fitness function. Then, on the basis of the position in the population, the probability of surviving is attributed to each chromosome by the expression

$$prob(rank) = q(1-q)^{rank-1} \tag{3.3}$$

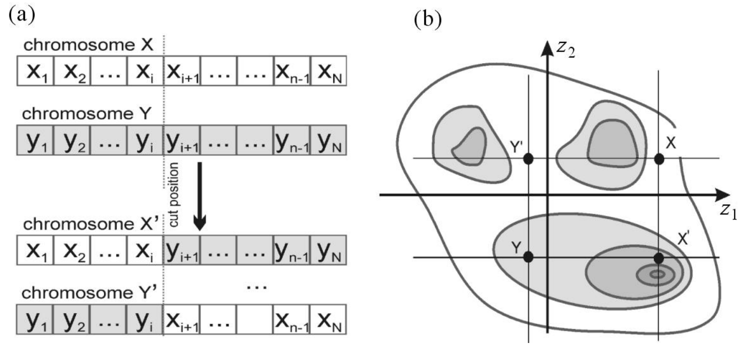

Fig. 6. (a) The scheme of the simple crossover, (b) simple crossover (method of working)

where: *rank* is the position of the chromosome after sorting (for the best chromosome $rank = 1$), $prob(rank)$ is the probability of the chromosome survival, q is a selection coefficient.

Sequential genetic and evolutionary algorithms are well known and applied in many areas of optimization. The main disadvantage of these algorithms is their long CPU time. The parallel evolutionary algorithms (Cantû-Paz, 1998; 1999, 2000; Gordon and Whitley, 1993; Tanese, 1989) perform evolutionary processes in the same manner as their sequential counterparts. The difference is in the fitness function evaluation. While for sequential processes all members of the population are processed by the same CPU, in the case of parallel algorithms the valuates of the fitness function are found concurrently.

The approach used in this study was to allot the entire task of computing the fitness function corresponding to one chromosome to one processor unit. In this case, the maximum (wall clock) computing time is shorter than its sequential counterpart by nearly N times, where N denotes the number of involved CPUs. Small overhead comes from mutual communication between the units and evaluation of new populations.

The flowchart of the parallel evolutionary algorithm is shown in Fig. 7. As has been already mentioned, the starting population is created randomly. The evolutionary operators change values of the genes in the chromosomes, and the fitness function value for each chromosome is computed.

The server/master transfers the chromosomes to clients/slaves. The slaves compute values of the fitness function and transmit them to the master. The generation of a new population is carried out by the server after the values of the fitness functions corresponding to each member of the old population are available. The creation of the new population is a random process. The

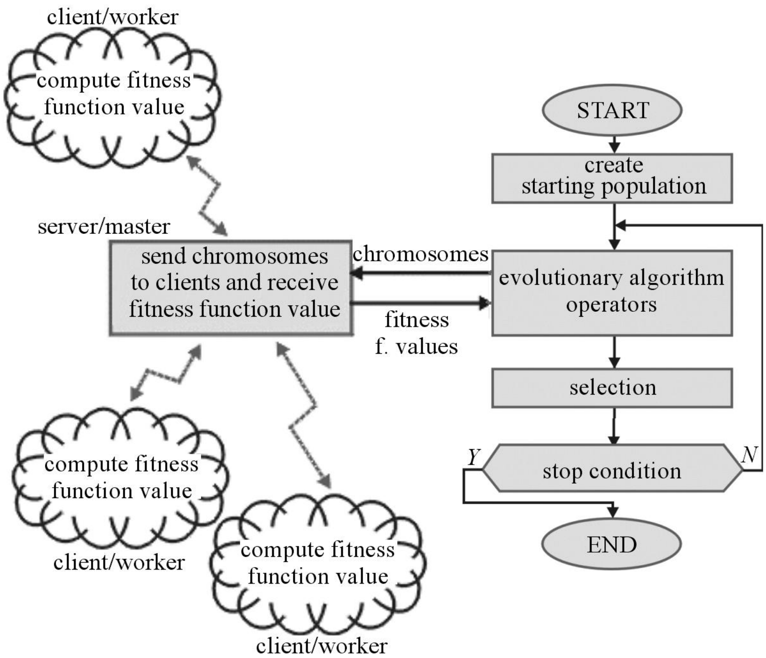

Fig. 7. A flowchart of the parallel evolutionary algorithm

probability for including fitter chromosomes in the new population is higher. The process of generation of new populations is terminated when the stop criterion is fulfilled. The latter condition was defined by defining the maximum number of iterations.

4. Optimization problem

The aim of the optimization is to find the optimal shape of the heat radiator shown in Fig. 8. The objective function is defined as the minimum volume of the structure

$$\min_{\mathrm{X}} V(\mathrm{X}) \tag{4.1}$$

with constraints imposed on the maximum temperature $(T - T^{ad} \leqslant 0)$ and the maximum equivalent stress $(\sigma_{eq} - \sigma_{eq}^{ad} \leqslant 0)$.

X is a vector of design parameters which is represented by a chromosome with the floating point representation.

The fitness function is created by the method of the penalty function which takes into account the volume of the structure and imposed constraints.

The invariable dimensions and values of boundary conditions along the Z axis are assumed. Due to the above reasons, the problem is modelled as two-dimensional (2D).

All computations were carried out on two-processors ($N = 2$) of a Pentium 4 1.4 GHz computer.

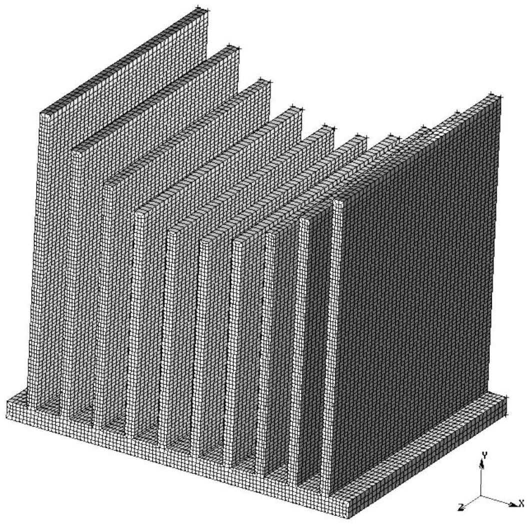

Fig. 8. The heat radiator

5. Numerical examples

The problem of the optimal shape of a heat radiator of the type used to dissipate heat from electrical devices is considered. The radiator is made of copper whose material properties are shown in Table 1.

Table 1. Material properties

Parameter	Value
Young modulus	120 000 MPa
Poisson ratio	0.3
Thermal expansion coef.	$16.5 \cdot 10^{-6}$ 1/K
Heat conductivity	400 W/(mK)
Emissivity	0.8

The geometry (cross section), fixed dimensions (in mm) and boundary conditions are shown in Fig. 9.

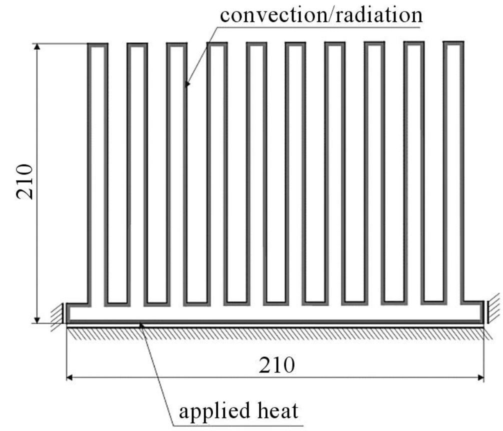

Fig. 9. Geometry and boundary conditions of the analysed heat radiator

Several tests have been performed for the following cases:

- applied heat flux $q = 500\,\mathrm{W/m^2}$ and $q = 1000\,\mathrm{W/m^2}$ on the bottom side of the structure,
- applied convection on the edge of the fins for convective heat transfer $\alpha = 2\,\mathrm{W/(m^2K)}$ and ambient temperature $T^{\infty} = 25°\mathrm{C}$,
- applied convection ($\alpha = 2\mathrm{W/(m^2K}$, $T^{\infty} = 25°\mathrm{C}$) and radiation on the edge of the fins for emissivity $e = 0.8$.

A constraint on the maximum equivalent stress $\sigma_{eq}^{ad} = 200\,\mathrm{MPa}$ is applied. The maximum temperature in the structure $T^{ad} = 50°\mathrm{C}$ for heat flux $q = 500\,\mathrm{W/m^2}$ is assumed. For heat flux $q = 1000\,\mathrm{W/m^2}$ it is $T^{ad} = 100°\mathrm{C}$.

The constant number of fins, equal to 10 is assumed. The height and width of the fins can vary during the optimization process. They are modelled using Bezier curves consisting of 6 control points. The control polygon of the height (P^0-P^5) and the control polygon of the width (P^0-P^5) are shown in Fig. 10. The height of the bottom part of the structure can vary as well.

Due to symmetry ($P^0 \xleftrightarrow{sym} P^5$, $P^1 \xleftrightarrow{sym} P^4$, $P^2 \xleftrightarrow{sym} P^3$, $N^0 \xleftrightarrow{sym} N^5$, $N^1 \xleftrightarrow{sym} N^4$, $N^2 \xleftrightarrow{sym} N^3$), the total number of the design parameters is equal to 7. The admissible values of the design parameters are given in Table 2. Table 3 contains values of the parameters of the parallel evolutionary algorithm.

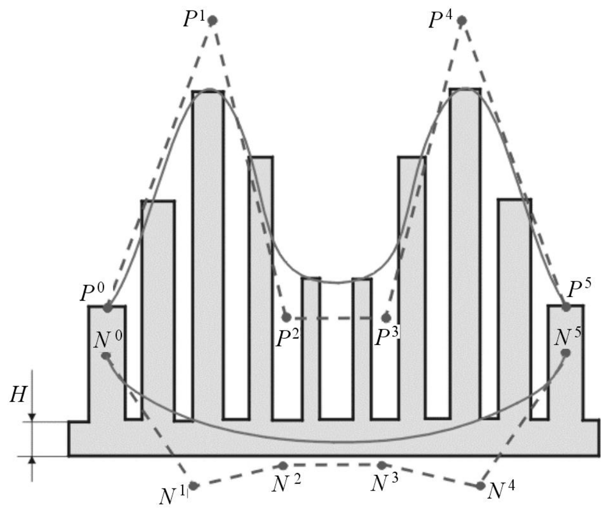

Fig. 10. The method of modelling the shape of the radiator

Table 2. Admissible values of the design parameters

Parameter	Heat flux $q = 500\,\mathrm{W/m^2}$	Heat flux $q = 1000\,\mathrm{W/m^2}$
$P^0 = P^5$	141.48 mm	90.98 mm
$P^1 = P^4$	31.17 mm	84.24 mm
$P^2 = P^3$	133.82 mm	111.02 mm
$N^0 = N^5$	5 mm	5 mm
$N^1 = N^4$	5 mm	5 mm
$N^2 = N^3$	5 mm	5 mm
H	7 mm	7 mm
Volume	126 mm^3	114 mm^3

Table 3. Parameters of the parallel evolutionary algorithm

Parameter	Range
P^0, P^1, P^2, P^3, P^4, P^5	20 mm - 200 mm
N^0, N^1, N^2, N^3, N^4, N^5	5 mm - 15 mm
H	7 mm - 20 mm

Several numerical tests have been performed for each case. The best results of the optimization are presented in:

- Figure 11 and Table 4 – for applied convection on the edge of the fins only

- Figure 12 and Table 5 – for applied convection and radiation on the edge of the fins.

The temperature distribution and equivalent for Misses stress distribution in the radiator is presented in Fig. 13.

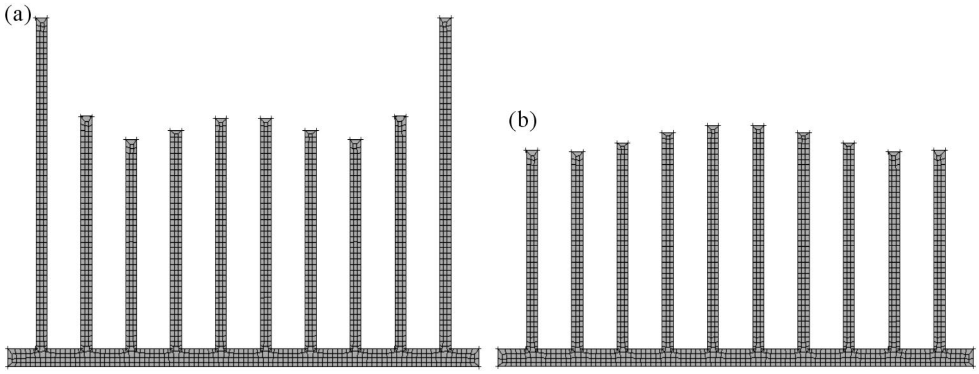

Fig. 11. The optimal shape of the radiator (convection only) for heat flux: (a) $q = 500\,\mathrm{W/m^2}$, (b) $q = 1000\,\mathrm{W/m^2}$

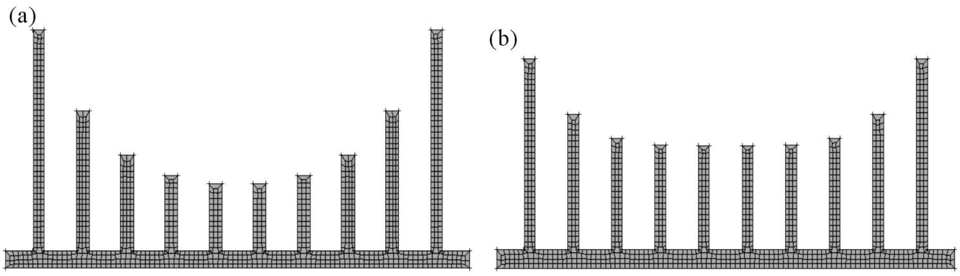

Fig. 12. The optimal shape of the radiator (convection and radiation) for heat flux: (a) $q = 500\,\mathrm{W/m^2}$, (b) $q = 1000\,\mathrm{W/m^2}$

Table 4. Values of the design parameters for optimization (convection only)

Parameter	Value
Number of genes in chromosome	7
Number of chromosomes in each population	10
Number of generations	500
Probability of Gaussian mutation	1
Probability of simple crossover	0.5
Rank selection pressure	0.8

Table 5. Values of the design parameters for optimization (convection and radiation)

Parameter	Heat flux $q = 500\,\mathrm{W/m^2}$	Heat flux $q = 1000\,\mathrm{W/m^2}$
$P^0 = P^5$	97.73 mm	84.74 mm
$P^1 = P^4$	22.00 mm	29.49 mm
$P^2 = P^3$	36.93 mm	57.72 mm
$N^0 = N^5$	5 mm	5 mm
$N^1 = N^4$	7.48 mm	5 mm
$N^2 = N^3$	5 mm	5.05 mm
H	7 mm	7.32 mm
Volume	82 mm^3	81 mm^3

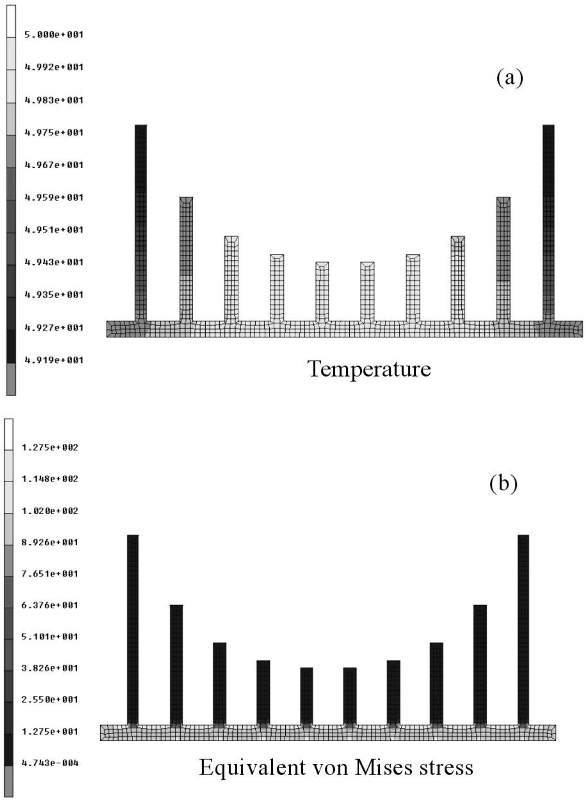

Fig. 13. (a) Temperature distribution in the radiator, (b) equivalent von Mises stress distribution in the radiator (convection and radiation, heat flux $q = 500\,\mathrm{W/m^2}$)

6. Conclusions

An effective intelligent technique of evolutionary design based on parallel computation has been presented. The important feature of this approach is its great flexibility and strong probability of finding the global optimal solution. The parallel evolutionary algorithm yields short optimization time. Bezier curves allow reducing the number of design parameters. In the tests with applied convection and radiation on the edge of the fins, the maximum temperature is bigger than in the tests without radiation. The optimal shape of the radiator for the case without radiation has a better value of the fitness function.

Acknowledgement

This research is financed from the Foundation for Polish Science (2005-2008) and budget resources of the Ministry of Education and Science (2006-2007) as the research project.

References

1. Arabas J., 2001, *Wykłady z algorytmów ewolucyjnych*, WNT

2. Beer G., 1983, Finite element, boundary element and coupled analysis of unbounded problems in elastostastics, *Int. J. Numer. Meth. Eng.*, **19**, 567-580

3. Białecki R.A., Burczyński T., Długosz A., Kuś W., Ostrowski Z., 2003a, Parallel evolutionary algorithms in optimization of thermoelastic structures in the presence of radiation, *Symposium on Methods of Artificial Intelligence AI-METH 2003*, Gliwice

4. Białecki R.A., Burczyński T., Długosz A., Kuś W., Ostrowski Z., 2003b, Shape optimization of thermoelastic heat radiating bodies using evolutionary algorithms, *15th International Conference on Computer Methods in Mechanics CMM-2003*, Wisła

5. Białecki R.A., Burczyński T., Długosz A., Kuś W., Ostrowski Z., 2005, Evolutionary shape optimization of thermolastic bodies exchanging heat by convection and radiation, *Computer Methods in Applied Mechanics and Engineering*, **194**, 1839-1859

6. Burczyński T., Beluch W., Długosz A., Kuś W., Orantek P., 2002, Evolutionary computation in structural optimization, *4th GRACM Congress on Computational Mechanics*, Patras, Greece

7. BURCZYŃSKI T., DŁUGOSZ A., 2001, Application of boundary element method in identification problems of thermoelasticity, *Electronic Journal of Boundary Elements*, **BETEQ**, 3, http://tabula.rutgers.edu/EJBE/proceedings/2001/no3/

8. BURCZYŃSKI T., DŁUGOSZ A., 2002, Evolutionary optimization in thermoelastic problems using boundary element method, *Computational Mechanics*, **28**, 3-4, Springer, Berlin

9. BURCZYŃSKI T., DŁUGOSZ A., KUŚ W., 2004, Shape optimization of heat radiators using parallel evolutionary algorithms, *Short papers of the Symposium on Methods of Artificial Intelligence AI-METH 2004*, Gliwice

10. BURCZYŃSKI T., KUŚ W. 2002, Distributed evolutionary algorithms in shape optimization of nonlinear structures, *Lectures Notes on Computational Science*, Springer

11. BURCZYŃSKI T., KUŚ W., 2004, Optimization of structures using distributed and parallel evolutionary algorithms, *Parallel Procesing and Applied Mathematics, PPAM2003, Revised papers, Lecture Notes on Computational Sciences*, **3019**, Springer, 572-579

12. BURCZYŃSKI T., KUŚ W., DŁUGOSZ A., POTERALSKI A., SZCZEPANIK M., 2004, Sequential and distributed evolutionary computations in structural optimization, *ICAISC International Conference on Artificial Intelligence and Soft Computing, Lecture Notes on Artificial Intelligence*, **3070**, Springer

13. CANTÛ-PAZ E., 1998, A survey of parallel genetic algorithms, *Calculaterus Parallelus, Reseaux et Systems Reparits*, **10**, 2, 141-171, Herems Paris

14. CANTÛ-PAZ E., 1999, Migration policies, selection pressure, and parallel evolutionary algorithms, In: *Late Breaking Papers at the Genetic and Evolutionary Computation Conference*, Orlando FL, Brave S., Wu. A. (Edit.)

15. CANTÛ-PAZ E., 2000, On the effect of migration on the fitness distribution of parallel evolutionary algorithms, In: *Workshop on Evolutionary Computation and Parallel Processing at GECCO-2000*, Las Vegas, NV, 3-6

16. DŁUGOSZ A., 2001, Boundary element method in sensitivity analysis and optimisation of thermoelastic structures, PhD Thesis, Silesian University of Technology, Gliwice (in Polish)

17. DŁUGOSZ A., 2004, Evolutionary computation in thermoelastic problems, *IUTAM Symposium on Evolutionary Methods in Mechanics*, T. Burczyński, A. Osyczka (Edit.), Kluwer, Dordrecht, 69-80

18. GORDON V.S., WHITLEY D., 1993, Serial and parallel genetic algorithms as function optimizers, *International Conference on Genetic Algorithms*, S. Forrest (Edit.), Morgan Kaufmann

19. HOLLAND J.H., 1975, *Adaptation in Natural and Artificial Systems*, The Univ. of. Michigan Press, Ann Arbor

20. Michalewicz Z. 1996, *Genetic Algorithms + Data Structures = Evolutionary Programs*, Springer Verlag, Berlin, New York

21. Modest M., 1993, *Radiative Heat Transfer*, McGraw Hill, New York

22. MSC.MARC Theory and user information, Vol. A-D, MSC Software Corporation, 2001

23. Siegel H., Howell J.R., 1992, *Thermal Radiation Heat Transfer*, 3rd ed. Hemisphere, Washington DC

24. Tanese R., 1989, Distributed genetic algorithms, *Proc. 3rd ICGA*, 434-439, J.D. Schaffer (Edit.), San Mateo, USA

25. Zienkiewicz O.C., Taylor R.L., 2000a, *The Finite Element Method*, **1-2**, Butterworth, Oxford

26. Zienkiewicz O.C., Taylor R.L., 2000b, *The Finite Element Method. Nonlinear*, **2**, Butterworth, Oxford

27. Zienkiewicz O.C., Taylor R.L., 2000c, *The Finite Element Method. The Fluid Mechanics*, **3**, Butterworth, Oxford

Zastosowanie równoległego algorytmu ewolucyjnego do optymalizacji kształtu radiatorów

Streszczenie

W pracy przedstawiono zastosowanie algorytmów ewolucyjnych oraz metody elementów skończonych (MES) w optymalizacji kształtu radiatorów. Zastosowano algorytm ewolucyjny, w którym funkcja celu wyznaczana jest w sposób równoległy, więc obliczenia przeprowadzane mogą być na wielu komputerach wieloprocesorowych. Tego typu podejście znacznie skraca czas obliczeń w porównaniu do sekwencyjnego algorytmu ewolucyjnego. Wartość funkcji celu wyznaczana jest na podstawie rozwiązania zagadnienia termosprężystości z wykorzystaniem oprogramowania MES MARC/MENTAT. Przy rozwiązywania zagadnienia bezpośredniego uwzględniany jest radiacyjny strumień ciepła. Wyznaczenie stref zacieniania, niezbędnych do jego wyznaczenia, realizowane jest również za pomocą procesora MENTAT. W celu zmniejszenia liczby zmiennych projektowych przy modelowaniu geometrii radiatora wykorzystano krzywe Béziera. Ponadto praca zawiera przykłady numeryczne optymalizacji dla różnych konfiguracji warunków brzegowych.

Manuscript received December 14, 2006; accepted for print January 23, 2006