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ALGORITHM FOR DETERMINATION OF $\widetilde{\sigma}_{ij}(n,\theta)$, $\widetilde{\varepsilon}_{ij}(n,\theta)$, $\widetilde{u}_i(n,\theta)$, $d_n(n)$, $I_n(n)$ FUNCTIONS IN HUTCHINSON-RICE-ROSENGREN SOLUTION AND ITS 3D GENERALIZATION

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In the paper the algorithm to determine $\widetilde{\sigma}_{ij}(n,\theta)$, $\widetilde{\varepsilon}_{ij}(n,\theta)$, $\widetilde{u}_i(n,\theta)$, $d_n(n)$, $I_n(n)$ functions that are necessary to obtain values of stresses, strains and displacements in the crack tip neighborhood according to the Hutchinson, Rice and Rosengren solutions is presented. The algorithm can also be used to determine $\widetilde{\sigma}_{ij}(n,\theta,T_z)$, $\widetilde{\varepsilon}_{ij}(n,\theta,T_z)$, $\widetilde{u}_i(n,\theta,T_z)$, $d_n(n,T_z)$, $I_n(n,T_z)$ functions for a 3D approximate solution of the stress field in front of the crack introduced by Guo Wanlin, where constraint due to the thickness effect is introduced through the T_z function.

Key words: fracture mechanics, HRR fields, 3DHRR field approximation

1. Introduction

In 1968, Hutchinson (1968) published a fundamental work, which characterised stress field in front of a crack for the non-linear Ramberg-Osgood (R-O) material. Following Williams (1952), Hutchinson proposed the Airy function for non-linear materials in the form of a series

$$\phi = r^s\widetilde{\phi}_1(\theta) + r^t\widetilde{\phi}_2(\theta) + \dots \tag{1.1}$$

where r and θ are polar coordinates of the coordinate system located at the crack tip. The functions $\widetilde{\phi}_i(\theta)$ describe angular changes of components of the stress tensor.

Hutchinson limited his considerations to the first dominant element of this series. Using the compatibility equation and the R-O relationship, Hutchison obtained formula for the stress field in front of a crack in the form

$$\begin{aligned} \sigma_e &= \widetilde{K} r^{s-2} \widetilde{\sigma}_e(\theta, s) \qquad & \sigma_\theta &= \widetilde{K} r^{s-2} \widetilde{\sigma}_\theta(\theta, s) \\ \sigma_r &= \widetilde{K} r^{s-2} \widetilde{\sigma}_r(\theta, s) \qquad & \sigma_{r\theta} &= \widetilde{K} r^{s-2} \widetilde{\sigma}_{r\theta}(\theta, s) \end{aligned} \tag{1.2}$$

where $s = (2n+1)/(n+1)$, n is the R-O exponent, σ_e is the equivalent stress, σ_r, σ_θ, $\sigma_{r\theta}$ are the stress tensor components in the polar coordinate system, $\widetilde{K}$ is the plastic stress intensity factor, which can be related to the J-integral through the relationship (McClintock, 1971)

$$\widetilde{K} = \Big(\frac{J}{\alpha\sigma_0\varepsilon_0 I_n}\Big)^{\frac{1}{1+n}} \tag{1.3}$$

where: α is the R-O constant, E is Young's modulus, σ_0 is the yield stress, ε_0 is the strain related to σ_0 through the relation $\varepsilon_0 = \sigma_0/E$.

Thus, relationships (1.2) are usually known in the form

$$\begin{aligned} \sigma_{ij} &= \sigma_0\Big(\frac{J}{\alpha\sigma_0\varepsilon_0 I_n r}\Big)^{\frac{1}{1+n}} \widetilde{\sigma}_{ij}(\theta, n) + \ldots \\ \varepsilon_{ij} &= \alpha\varepsilon_0\Big(\frac{J}{\alpha\sigma_0\varepsilon_0 I_n r}\Big)^{\frac{n}{1+n}} \widetilde{\varepsilon}_{ij}(\theta, n) + \ldots \\ u_i - \widehat{u}_i &= \alpha\sigma_0 r\Big(\frac{J}{\alpha\sigma_0\varepsilon_0 I_n r}\Big)^{\frac{n}{1+n}} \widetilde{u}_i(\theta, n) \end{aligned} \tag{1.4}$$

Functions $\widetilde{\sigma}_{ij}(n,\theta)$, $\widetilde{\varepsilon}_{ij}(n,\theta)$, $\widetilde{u}_i(n,\theta)$, $I_n(n)$ must be found by solving the fourth order non-linear homogenous differential equation for the plane stress and plane strain independently (Hutchinson, 1968). In the literature, these functions are presented for limited values of the strain hardening exponent n. However, it is very often required to use values of these functions for other values of n than given in the literature. The program proposed in this paper allows one to obtain all functions in the HRR solution for an arbitrary exponent n.

2. Generalization of the HRR solution to the 3D-case

Values of the functions $\widetilde{\sigma}_{ij}$, $\widetilde{\varepsilon}_{ij}$, $\widetilde{u}_i$ and I_n for a predetermined stain hardening exponent n depend on selection of the plane stress or plane strain model of an element. In a real specimen, the plane strain or stress states can be found in the vicinity of the symmetry axis of the specimen close to the crack edge or near the free surface of the specimen, respectively. The remaining part of the specimen along the crack front is dominated by three-dimensional stress and strain fields.

Guo (1993a) defined the T_z-function as

$$T_z = \frac{\sigma_{33}}{\sigma_{11} + \sigma_{22}} \tag{2.1}$$

For the non-linear plastic materials, T_z is equal to 0 for plane stress and 0.5 for plane strain. Thus, T_z changes from 0.5 to 0 along the crack front from the specimen axis to the specimen surface.

Using function (2.1), Guo postulated the Maxwell stress function in the form

$$\phi_i = \widetilde{K} r^{s(T_z)} \widetilde{\phi}_i(\theta, T_z) \tag{2.2}$$

where the functions $\widetilde{\phi}_i(\theta, T_z)$ describe the angular changes of stress tensor components. In the functions ϕ_i both the s exponent and $\widetilde{\phi}_i$ functions were assumed to be dependent on the function T_z. The ϕ_i functions were used to obtain a solution analogous to the HRR field. However, Guo's solution is not limited to the plane strain or plane stress cases. Since the T_z function changes along the crack front, Guo's solution covers also these layers of the material along the crack front which are in the 3D state of the stress and strain field. The only requirement is to know the $T_z(x_1, x_3)$ function.

Guo showed that the $s(T_z)$ function is equal to s in the HRR solution for the plane strain $(T_z = 0.5)$ or plane stress $(T_z = 0)$ cases only. However, between the specimen axis and the specimen surface $s(0 < T_z < 0.5) \neq s_{HRR}$. Moreover, for all these points along the crack front, the Jintegral is not path independent.

Because at the above problems, Guo postulated a general approximate formula for a quasi-three-dimensional case in the form

$$\sigma_{ij} = \sigma_0 \Big(\frac{J_{far}}{\alpha \sigma_0 \varepsilon_0 I_n(n, T_z) r} \Big)^{\frac{1}{1+n}} \widetilde{\sigma}_{ij}(\theta, n, T_z) \tag{2.3}$$

where: J_{far} is the J-integral computed for the contour of integration drawn over the domain dominated by the plane stress $(r > B$ where B is specimen thickness). In Eq. (2.3), the out of plane effect was taken into account by the value of T_z. Guo demonstrated, by comparison with numerical results, that using Eq. (2.3) the error in stress values along the crack edge was not greater than 7% for $n = 10$ and decreased with n. Equation (2.3) can be used for each point along the crack front as well as for the mean value of T_z through the specimen thickness. In that case, one obtains an intermediate case with respect to the plane strain and plane stress models. In the polar coordinate system, the Guo solution is

$$\begin{aligned} &\sigma_r = Kr^{s-2}\widetilde{\sigma}_r(n,\theta,T_z) \qquad && \sigma_\theta = Kr^{s-2}\widetilde{\sigma}_\theta(n,\theta,T_z) \\ &\sigma_{r\theta} = Kr^{s-2}\widetilde{\sigma}_{r\theta}(n,\theta,T_z) \end{aligned} \tag{2.4}$$

where

$$\begin{aligned} &\widetilde{\sigma}_r(n,\theta,T_z) = s\widetilde{\phi} + \frac{\partial^2\widetilde{\phi}}{\partial\theta^2} \qquad && \widetilde{\sigma}_\theta(n,\theta,T_z) = s(s-1)\widetilde{\phi} \\ &\widetilde{\sigma}_{r\theta}(n,\theta,T_z) = (1-s)\frac{\partial\widetilde{\phi}}{\partial\theta} \end{aligned} \tag{2.5}$$

and

$$\begin{aligned} \sigma_e &= Kr^{s-2}\widetilde{\sigma}_e(n,\theta,T_z) \\ \sigma_z &= T_z Kr^{s-2}[\widetilde{\sigma}_r(n,\theta,T_z) + \widetilde{\sigma}_\theta(n,\theta,T_z)] \end{aligned} \tag{2.6}$$

In the following section the functions in Eqs (2.5) will be determined.

3. Calculation of values of $\widetilde{\sigma}_{ij}(n,\theta,T_z)$, $\widetilde{\varepsilon}_{ij}(n,\theta,T_z)$, $\widetilde{u}_i(n,\theta,T_z)$, $d_n(n,T_z)$, $I_n(n,T_z)$

The constitutive relation of a homogeneous isotropic elastoplastic continuum can be expressed by

$$\varepsilon_{ij} = (1+\nu)S_{ij} + \frac{1-2\nu}{3}\sigma_{kk}\delta_{ij} + \frac{3}{2}\alpha\sigma_e^{n-1}S_{ij} \tag{3.1}$$

where S_{ij} is the stress deviator, δ_{ij} is the Kronecker delta, ν is the Poisson ratio and σ_e is the von Misses equivalent stress.

The strain components in the cylindrical coordinate system can be written in the form

$$\begin{aligned} \varepsilon_{rr} &= (1+\nu)\sigma_{rr} - (1+T_z)\nu(\sigma_{rr}+\sigma_{\theta\theta}) + \frac{3}{2}\alpha\sigma_{eff}^{n-1}\Big[\sigma_{rr} - \frac{1+T_z}{2}(\sigma_{rr}+\sigma_{\theta\theta})\Big] \\ \varepsilon_{\theta\theta} &= (1+\nu)\sigma_{\theta\theta} - (1+T_z)\nu(\sigma_{rr}+\sigma_{\theta\theta}) + \frac{3}{2}\alpha\sigma_{eff}^{n-1}\Big[\sigma_{\theta\theta} - \frac{1+T_z}{2}(\sigma_{rr}+\sigma_{\theta\theta})\Big] \\ \varepsilon_{r\theta} &= (1+\nu)\sigma_{r\theta} + \frac{3}{2}\alpha\sigma_{eff}^{n-1}\sigma_{r\theta} \end{aligned} \tag{3.2}$$

Making use of Eq. (2.4) and Eq. (3.2) in the compatibility equation (3.3)

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\varepsilon_{\theta\theta}) + \frac{1}{r^2}\frac{\partial^2 \varepsilon_{rr}}{\partial \theta^2} - \frac{1}{r}\frac{\partial \varepsilon_{rr}}{\partial r} - \frac{2}{r^2}\frac{\partial}{\partial r}\Big(r\frac{\partial \varepsilon_{r\theta}}{\partial \theta}\Big) = 0 \tag{3.3}$$

one can obtain forth order non-linear homogeneous differential equation (3.4) with two unknowns: the functions $\widetilde{\phi}(\theta, T_z)$ and the singularity parameter $s(T_z)$

$$\begin{aligned}
&[n(s-2)+1][n(s-2)]\widetilde{\sigma}_e^{n-1}\Big[\widetilde{\phi}\Big(s\Big(\frac{3(s-1)-s(1+T_z)}{2}\Big)\Big) - \frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big(\frac{1+T_z}{2}\Big)\Big] + \\
&+\frac{\partial^2}{\partial\theta^2}\Big\{\widetilde{\sigma}_e^{n-1}\Big[\widetilde{\phi}\Big(s\Big(\frac{3-s(1+T_z)}{2}\Big)\Big) + \frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big(\frac{2-T_z}{2}\Big)\Big]\Big\} + \\
&-n(s-2)\widetilde{\sigma}_e^{n-1}\Big[\widetilde{\phi}\Big(s\Big(\frac{3-s(1+T_z)}{2}\Big)\Big) + \frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big(\frac{2-T_z}{2}\Big)\Big] + \\
&-2[n(s-2)+1]\frac{\partial}{\partial\theta}\Big\{\frac{3}{2}(1-s)\widetilde{\sigma}_e^{n-1}\frac{\partial\widetilde{\phi}}{\partial\theta}\Big\} = 0
\end{aligned} \tag{3.4}$$

In order to find values of $\widetilde{\sigma}_{ij}(n,\theta,T_z)$, $\widetilde{\varepsilon}_{ij}(n,\theta,T_z)$, $\widetilde{u}_i(n,\theta,T_z)$, $d_n(n,T_z)$, $I_n(n,T_z)$, Eq. (3.4) should be solved first. After transformation of Eq. (3.4) one can obtain

$$\frac{\partial^4\widetilde{\phi}}{\partial\theta^4} = \frac{NUMERATOR}{DENOMINATOR} \tag{3.5}$$

where

$$\begin{aligned}
&NUMERATOR = \\
&-\Big[Z_{16}\widetilde{\phi}^2 + Z_{15}\Big(\frac{\partial\widetilde{\phi}}{\partial\theta}\Big)^2 + \Big(Z_{17}\widetilde{\phi} + Z_{12}\frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big)\frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big]^{\frac{n-1}{2}}\Big[Z_9\widetilde{\phi} - Z_{10}\frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big] + \\
&-\frac{n-1}{2}\Big[Z_{16}\widetilde{\phi}^2 + Z_{15}\Big(\frac{\partial\widetilde{\phi}}{\partial\theta}\Big)^2 + \Big(Z_{17}\widetilde{\phi} + Z_{12}\frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big)\frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big]^{\frac{n-3}{2}}\cdot \\
&\cdot\Big[\frac{\partial\widetilde{\phi}}{\partial\theta}\Big(2Z_{16}\widetilde{\phi} + \frac{\partial^2\widetilde{\phi}}{\partial\theta^2}(2Z_{15}+Z_{17})\Big) + \frac{\partial^3\widetilde{\phi}}{\partial\theta^3}\Big(Z_{17}\widetilde{\phi} + 2Z_{12}\frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big)\Big]\Big[Z_{11}\frac{\partial\widetilde{\phi}}{\partial\theta} + 2Z_5\frac{\partial^3\widetilde{\phi}}{\partial^3\theta}\Big] + \\
&-\frac{n-1}{2}\frac{n-3}{2}\Big[Z_{16}\widetilde{\phi}^2 + Z_{15}\Big(\frac{\partial\widetilde{\phi}}{\partial\theta}\Big)^2 + \Big(Z_{17}\widetilde{\phi} + z_{12}\frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big)\frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big]^{\frac{n-5}{2}}\cdot \\
&\cdot\Big[\frac{\partial\widetilde{\phi}}{\partial\theta}\Big(2Z_{16}\widetilde{\phi} + \frac{\partial^2\widetilde{\phi}}{\partial\theta^2}(2Z_{15}+Z_{17})\Big) + \frac{\partial^3\widetilde{\phi}}{\partial\theta^3}\Big(Z_{17}\widetilde{\phi} + 2Z_{12}\frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big)\Big]^2\Big(Z_4\widetilde{\phi} + Z_5\frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big) + \\
&-\frac{n-1}{2}\Big[Z_{16}\widetilde{\phi}^2 + Z_{15}\Big(\frac{\partial\widetilde{\phi}}{\partial\theta}\Big)^2 + \Big(Z_{17}\widetilde{\phi} + Z_{12}\frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big)\frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big]^{\frac{n-3}{2}}\cdot
\end{aligned} \tag{3.6}$$

$$\cdot\Big[\frac{\partial\widetilde{\phi}}{\partial\theta}\Big(2Z_{16}\frac{\partial\widetilde{\phi}}{\partial\theta}+\frac{\partial^3\widetilde{\phi}}{\partial\theta^3}(2Z_{15}+2Z_{17})\Big)+\frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big(2Z_{16}\widetilde{\phi}+\frac{\partial^2\widetilde{\phi}}{\partial\theta^2}(2Z_{15}+Z_{17})\Big)+ +2Z_{12}\Big(\frac{\partial^3\widetilde{\phi}}{\partial\theta^3}\Big)^2\Big]\Big(Z_4\widetilde{\phi}+Z_5\frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big)$$

$$DENOMINATOR = Z_5\Big[Z_{16}\widetilde{\phi}^2+Z_{15}\Big(\frac{\partial\widetilde{\phi}}{\partial\theta}\Big)^2+\Big(Z_{17}\widetilde{\phi}+z_{12}\frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big)\frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big]^{\frac{n-1}{2}}+ +\frac{n-1}{2}\Big[Z_{16}\widetilde{\phi}^2+Z_{15}\Big(\frac{\partial\widetilde{\phi}}{\partial\theta}\Big)^2+\Big(Z_{17}\widetilde{\phi}+Z_{12}\frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big)\frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big]^{\frac{n-3}{2}}\cdot \cdot\Big(Z_{17}\widetilde{\phi}+2Z_{12}\frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big)\Big(Z_4\widetilde{\phi}+Z_5\frac{\partial^2\widetilde{\phi}}{\partial\theta^2}\Big) \tag{3.7}$$

and

$$\begin{aligned}
Z_1 &= [n(s-2)+1][n(s-2)] & Z_{10} &= Z_1Z_3+Z_8Z_5-Z_4+Z_6Z_7\\
Z_2 &= s\Big(\frac{3(s-1)-s(1+T_z)}{2}\Big) & Z_{16} &= s^2Z_{12}+Z_{12}Z_{13}-sZ_{14}\\
Z_3 &= \frac{1+T_z}{2} & Z_{12} &= 1-T_z+T_z^2\\
Z_4 &= s\Big(\frac{3-s(1+T_z)}{2}\Big) & Z_{13} &= s^2(s-1)^2\\
Z_5 &= \frac{2-T_z}{2} & Z_{14} &= (1+2T_z-2T_z^2)s(s-1)\\
Z_6 &= 2[n(s-2)+1] & Z_{15} &= 3(s-1)^2\\
Z_7 &= \frac{3}{2}(1-s) & Z_{11} &= 2Z_4-Z_6Z_7\\
Z_8 &= n(s-2) & Z_{17} &= 2sZ_{12}-Z_{14}\\
Z_9 &= Z_1Z_2-Z_8Z_4
\end{aligned}$$

To solve Eq. (3.5), a combination of numerical methods was used (see Fig. 1). The function $\widetilde{\phi}(\theta)$ was found by the forth order Runge-Kutta method (Burden and Faires, 1985), and in order to find the initial value of $\partial^2\widetilde{\phi}(0)/\partial\theta^2$ and the value of the power exponent $s(T_z)$, the shooting method (Burden and Faires, 1985; Marciniak *et al.*, 2000) was used.

It turns out that during the process of numerical computations, more iterations are required when the strain hardening exponent n increases. A solution to the plane strain problem is more easily obtained. A satisfactory convergence

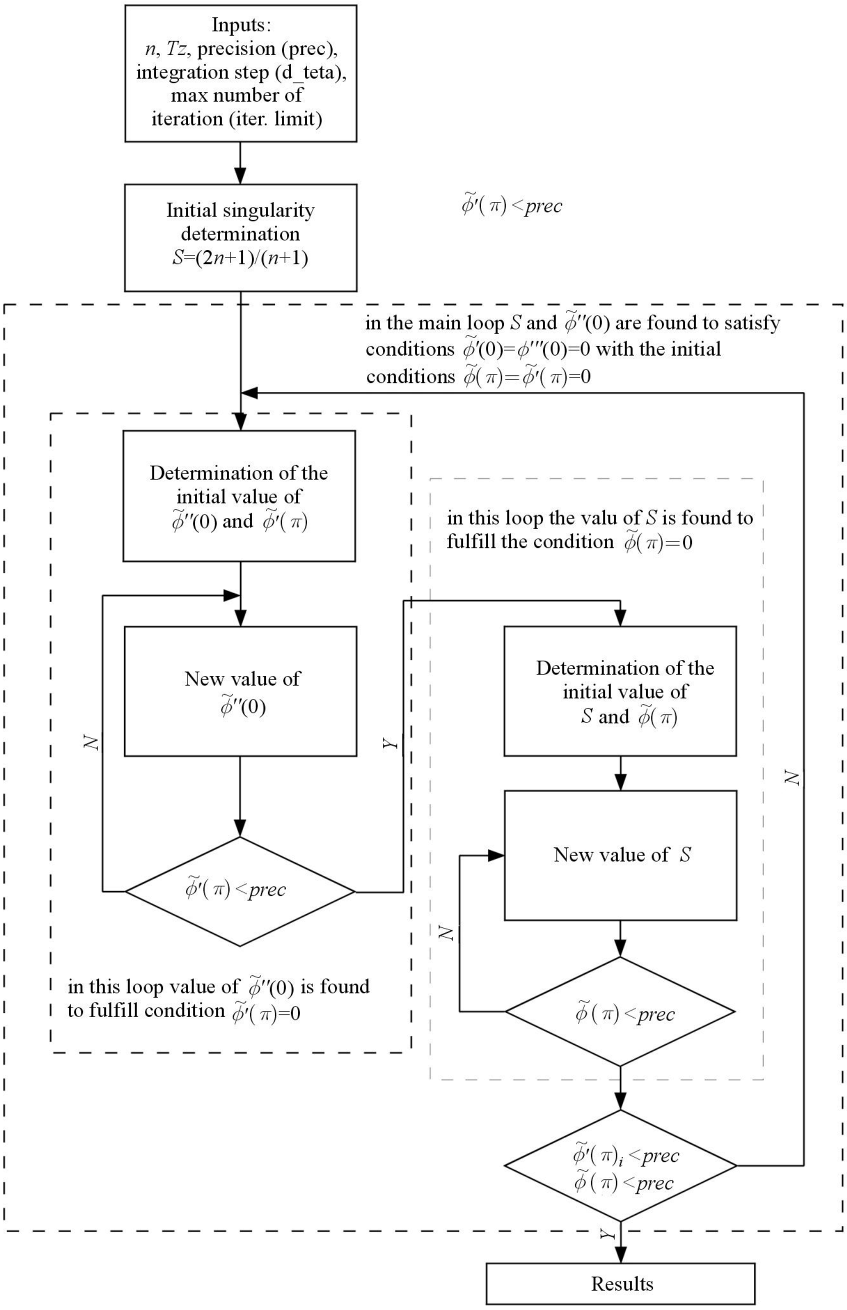

Fig. 1. The algorithm for numerical solution to Eq. (3.2)

was found for the strain hardening exponent $n \leqslant 20$ in the case of the plane stress, and for $n \leqslant 30$ in the plane strain. The convergence depends on T_z and the worst situation was observed for $T_z \in [0.23-0.27]$. Exemplary results for the strain hardening exponent $n = 20$ and the plane stress are presented in Fig. 2.

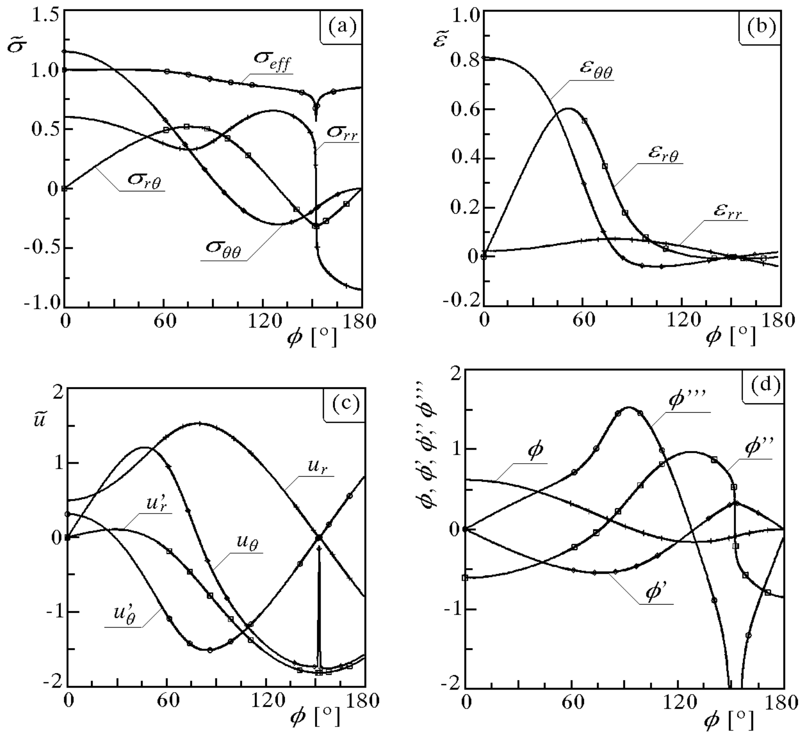

Fig. 2. Exemplary results for $n = 3$, $T_z = 0$; (a) stress functions $\widetilde{\sigma}(\theta)$, (b) strain functions $\widetilde{\varepsilon}(\theta)$, (c) displacement functions $\widetilde{u}(\theta)$ and their first derivatives, (d) function $\phi(\theta)$ and its derivatives

4. Calculations of d_n and I_n functions

In order to compute the crack opening displacement (Neimitz, 1998), one must know the value of $d_n(\alpha, \varepsilon, n, T_z)$

$$\delta_T = d_n(\alpha, \varepsilon, n, T_z)\frac{J}{\sigma_0} \tag{4.1}$$

Using Eq. (2.1) and the method proposed by Shih (1981), (see Fig. 3), one can compute d_n for any material from the following formula

$$d_n = \frac{2}{I_n}\widetilde{u}_2(\pi,n,T_z)\Big(\frac{\alpha\sigma_0}{E}[\widetilde{u}_1(\pi,n,T_z)+\widetilde{u}_2(\pi,n,T_z)]\Big)^{\frac{1}{n}} \tag{4.2}$$

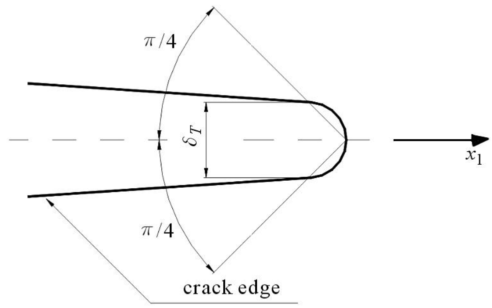

Fig. 3. Definition of CTOD

The value of the function $I_n(n,T_z)$ follows form the path-independency of the J-integral. The J-integral is path-independent when

$$I(T_z,n) = \int\limits_{-\pi}^{\pi}\Big\{\frac{n}{n+1}\widetilde{\sigma}_e^{n+1}\cos\theta - \frac{3}{2}\Big(\sin\theta[\widetilde{\sigma}_r(\widetilde{u}_\theta-\widetilde{u}_r')-\widetilde{\sigma}_{r\theta}(\widetilde{u}_r+\widetilde{u}_\theta')]\Big)] + \tag{4.3}$$
$$+\frac{3}{2}\cos\theta[n(s-2)+1](\widetilde{\sigma}_r\widetilde{u}_r+\widetilde{\sigma}_{r\theta}\widetilde{u}_\theta)\Big\}\;d\theta$$

5. Comparison of results

The values obtained with the help of the program hrr_par.exe for $I_n(n)$ and $(\pi/I_n)^{1/(n+1)}$ which depend on the R-O power exponent n, may be compared with the Hutchinson results obtained for $T_z = 0.5$ or 0 (Hutchinson, 1968). Differences are presented in Table 1. The differences are smaller than 0.3% both for the plane stress and plane strain.

The values of the singularity exponent $s(n,T_z)$ are close to Guo's results (Guo, 1993a,b). The differences are less than 0.15% almost for all cases (Table 2). Only for $n = 8$ and $T_z = 0.45$ this difference is about 1.35%. In Fig. 4, the function d_n (obtained using hrr_par.exe) is presented. Results are close to those presented by Guo (1995).

Table 1. Values of $I_n(n)$ function, $(\pi/I_n)^{1/(n+1)}$

n	$I_n(n)$	$I_n(n)_{HRR}$	Difference	$(\pi/I_n)^{\frac{1}{n+1}}$	$(\pi/I_n)^{\frac{1}{n+1}}\vert_{HRR}$	Difference
Plane stress $(T_z = 0)$						
3	3.85	3.86	0.26%	0.950	0.949	0.15%
5	3.41	3.41	0.00%	0.986	0.987	0.06%
9	3.03	3.03	0.00%	1.004	1.004	0.04%
13	2.87	2.87	0.00%	1.006	1.006	0.05%
Plane strain $(T_z = 0.5)$						
3	5.51	5.51	0.00%	0.869	0.869	0.00%
5	5.02	5.01	0.20%	0.925	0.925	0.02%
9	4.60	4.60	0.00%	0.963	0.963	0.04%
13	4.40	4.40	0.00%	0.976	0.976	0.02%

Table 2. Values of singularity exponent $s(n, T_z)$

n	T_z				
	0	0.3	0.4	0.45	0.5
Our results					
3	−0.2500000	−0.2380839	−0.2380157	−0.2426839	−0.2500000
3.6364	−0.2156929	−0.2031756	−0.2034906	−0.2084886	−0.2156854
8	−0.1111266	−0.0997175	−0.1019225	−0.1059254	−0.1111112
10	−0.0909101	−0.0803697	−0.0828057	−0.0863690	−0.0909083
Guo results					
3	−0.250000	−0.237825	−0.237730	−0.242500	−0.250000
3.6364	−0.215686	−0.203186	−0.203186	−0.208336	−0.215686
8	−0.111111	−0.099617	−0.101835	−0.104511	−0.111111
10	−0.090909	−0.080280	−0.082909	−0.086409	−0.090909
Differences between our results and Guo ones (Shih, 1981)					
3	0.00%	0.11%	0.12%	0.08%	0.00%
3.6364	0.00%	0.01%	0.15%	0.07%	0.00%
8	0.01%	0.10%	0.09%	1.35%	0.00%
10	0.00%	0.11%	0.12%	0.05%	0.00%

The results presented in this section concern an infinite plate with the crack loaded at infinity. The results for a finite body can be found in Gałkiewicz and Graba (2004).

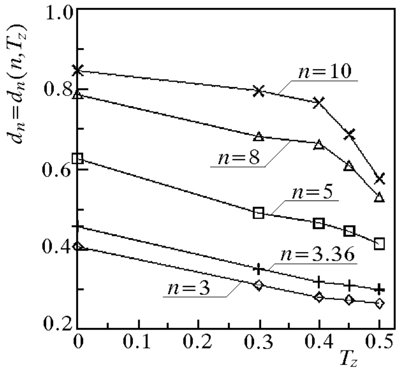

Fig. 4. Results of normalized crack tip opening displacement for different n vs. T_z

6. Conclusions

Using the program hrr_par.exe, one can obtain results for a wide range of materials which are characterised by n, σ_0, ε_0, E in an easy and fast way. The obtained results are accurate when compared to those published in the literature.

The program hrr_par.exe – the source version (for Delphi 6) and the compiled source are available on:

http://www.tu.kielce.pl/~pgfm/HRR.htm
http://www.tu.kielce.pl/~mgraba → Fracture
http://www.tu.kielce.pl/~mgraba/Fracture_03.htm

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Algorytm wyznaczania funkcji $\widetilde{\sigma}_{ij}(n,\theta)$, $\widetilde{\varepsilon}_{ij}(n,\theta)$, $\widetilde{u}_i(n,\theta)$, $d_n(n)$, $I_n(n)$ w rozwiązaniu HRR i jego trójwymiarowym uogólnieniu

Streszczenie

W artykule zaprezentowano algorytm pozwalający na określenie funkcji $\widetilde{\sigma}_{ij}(n,\theta)$, $\widetilde{\varepsilon}_{ij}(n,\theta)$, $\widetilde{u}_i(n,\theta)$, $d_n(n)$, $I_n(n)$ niezbędnych do opisu pola naprężeń, odkształceń i przemieszczeń w materiałach nieliniowych według prawa Ramberga-Osgooda. Algorytm pozwala uzyskać również wartości funkcji $\widetilde{\sigma}_{ij}(n,\theta,T_z)$, $\widetilde{\varepsilon}_{ij}(n,\theta,T_z)$, $\widetilde{u}_i(n,\theta,T_z)$ dla próbek trójwymiarowych przy wykorzystaniu parametru T_z, który zależy od grubości próbki i może być wyznaczony numerycznie.

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