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MOTION PLANNING FOR WHEELED MOBILE ROBOT USING POTENTIAL FIELD METHOD

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A potential field method in the real-time approach toward avoidance of obstacles for a mobile robot has been developed. A collision-free path and goal-seeking behaviour are calculated using an artificial potential field method. The proposed reactive navigation approach is based on the coordination of elementary responses. To avoid convex obstacles, the navigator generates a "reaching the middle of the collision-free space" and goal-seeking behaviours. A control strategy based on artificial potential fields that generates a trajectory to be followed by a mobile robot that represents a reference for the robot at the same time is proposed. The effectiveness of the proposed method is numerically verified by a series of experiments on the emulator of the wheeled mobile robot Pioneer-2DX.

Key words: mobile robots, obstacle avoidance, artificial potential field

1. Introduction

The navigation problem of passing by obstacles is one of the most important problems in robotics. Complexity of this problem causes that we can not find universal planning methods and realization of movement of autonomous wheeled mobile robots. The planning of motion and its realization was analysed by lots of autors (Arkin, 1998; Berenstain and Koren, 1989; Latombe, 1991). The artificial potential field method was developed by Khatib (1986), Krogh and Thorpe (1986), and was extensively studied in the obstacle avoidance problem for autonomous mobile robots (Berenstain and Koren, 1989; Ge and Cui, 2000; Tilove, 1990). There are two main approaches toward this method. On one hand, the classical method (Khatib, 1986) depends only on the position of the mobile robot. On the other hand, the generalized potential field method (Krogh and Thorpe, 1986) depends also on the velocity of the robot. The underlying idea of the method is to fill the workspace of the robot with an artificial potential field in which the vehicle is attracted to its goal and is repulsed away from obstacles. One of the inherent problems of this method is the existence of local minima which are undesirable equilibrium points of a gradient system. They appear when the sum of the attractive and repulsive forces induced by the potential vanishes in front of the goal.

In this work, the obstacle avoidance problem associated with a mobile robot is considered. This problem will be tackled by means of the method of artificial potential fields. The problem concerned with the reactive navigation of wheeled mobile robots will be analysed in the paper.

2. Generation of motion trajectories in artificial force field

Description of kinematics of a wheeled mobile robot which moves in real environment is shown in Fig. 1. The kinematic equations (Giergiel *et al.*, 2002; Żylski, 1996, 2002) are as follows

$$\begin{bmatrix} \dot{x}_A \\ \dot{y}_A \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} V_{Am}\cos\beta & 0 \\ V_{Am}\sin\beta & 0 \\ 0 & \omega_m \end{bmatrix} \begin{bmatrix} u_v \\ u_\beta \end{bmatrix} \tag{2.1}$$

In this equation V_{Am} and ω_m are maximum value of velocity point A and maximum value of angular velocity of the frame, respectively. The variables u_v, u_β represent the linear and angular velocities of the mobile robot. Those values that result from motion in the artificial forces field should be appropriately normalized so that the inequalities $0 \leqslant u_v \leqslant 1$, $-1 \leqslant u_\beta \leqslant 1$ are true.

The most interesting description of such a mobile robot motion is when it is passing by obstacles. Let us make an assumption that the robot we are analysing has special distance sensors that can give us information about obstacles that the mobile robot is meeting on its way. At this moment, the robot is in a position shown in Fig. 1b. L is an immovable point of the left obstacle, P is an immovable point of the right obstacle and A is a characteristic point of the robot.

2.1. Avoidance of convex obstacles

Point A determines the position of the robot, β angle gives the attitude, point A the velocity vector that has direction of the axis x_1. Location of points

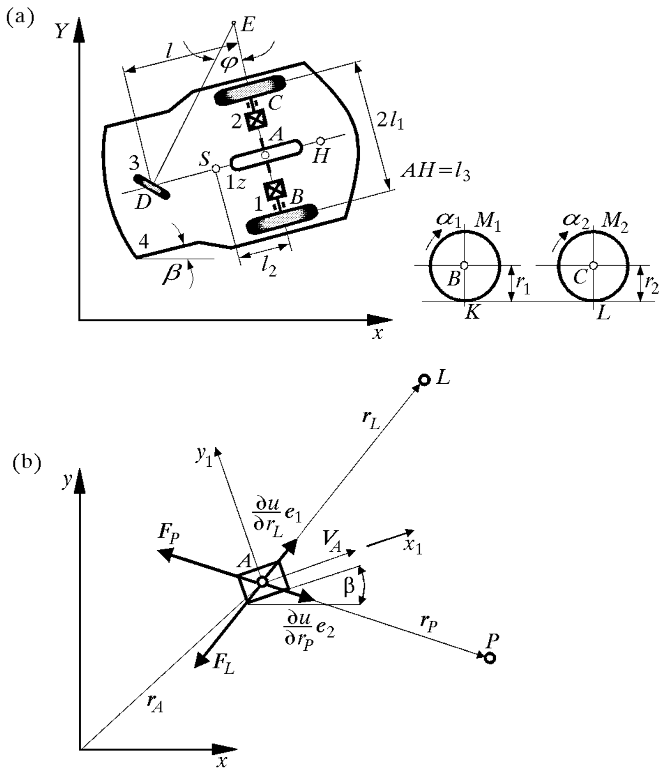

Fig. 1. (a) A scheme of the model; (b) forces in the potential field

L, P in the co-ordinate plane x_1y_1, and location of point A in the co-ordinate plane xy, determine equivalent vectors radii shown in Fig. 1b. Point A has to move along a trajectory so that the wheeled mobile robot could safely move between the obstacles. Assume that the motion is in the xy configuration in the artificial force field, where the forces come from the obstacles L, P shown in Fig. 1b. Those forces are defined as (Żylski, 2002)

$$\boldsymbol{F}_L = -\frac{k^2}{r_L^2}\boldsymbol{e}_1 \qquad\qquad \boldsymbol{F}_P = -\frac{k^2}{r_P^2}\boldsymbol{e}_2 \tag{2.2}$$

The forces in equation (2.2) are called repulsive forces, k is a selected coefficients, r_L, r_P denote distances between point A and obstacles, $\boldsymbol{e}_1$, $\boldsymbol{e}_2$ are the left and right directional unit vectors. The equivalent vector radii are set in the form

$$\boldsymbol{r}_L = r_L\boldsymbol{e}_1 \qquad\qquad \boldsymbol{r}_P = r_P\boldsymbol{e}_2 \tag{2.3}$$

From equation (2.3) we can determine the unit vectors $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, and putting them to dependence (2.2) we get

$$\boldsymbol{F}_L = -\frac{k^2}{r_L^3}\boldsymbol{r}_L \qquad \boldsymbol{F}_P = -\frac{k^2}{r_P^3}\boldsymbol{r}_P \tag{2.4}$$

Because we assume that forces (2.4) come from the force field, then the differential of the force function is

$$dU_R = \boldsymbol{F}_L d\boldsymbol{r}_L + \boldsymbol{F}_P d\boldsymbol{r}_P = -k^2\Big(\frac{dr_L}{r_L^2} + \frac{dr_P}{r_P^2}\Big) \tag{2.5}$$

After integration we get a function of the force field

$$U_R = k^2\Big(\frac{1}{r_L} + \frac{1}{r_P}\Big) + C \tag{2.6}$$

where C is a constans. The gradient of the force function determined in point A is set as

$$\operatorname{grad} U_R = \frac{\partial U_R}{\partial r_L}\boldsymbol{e}_1 + \frac{\partial U_R}{\partial r_P}\boldsymbol{e}_2 = \frac{k^2}{r_L^2}\boldsymbol{e}_1 + \frac{k^2}{r_P^2}\boldsymbol{e}_2 \tag{2.7}$$

Assuming that the gradient components are directed normally to the surface in the direction of increment of an appropriate function. Obviously, component forces of the force field have directions of the equivalent gradient components with sensess corresponding to minimal values of the function. Because of the fact that point A moves in the assumed force field, its velocity vector has the direction of the resultant force and so has the gradient of the forces in point A of the field. The vector product of the force field gradient and the velocity vector in point A will be

$$\operatorname{grad} U_R \times \boldsymbol{V}_A = \boldsymbol{0} \tag{2.8}$$

Figure 1b shows components of the force field gradient function and velocity vector in point A. The vector of instantaneous angular velocity of the frame is proportional to vector (2.8), thus

$$\dot{\boldsymbol{\beta}} = \varepsilon(\operatorname{grad} U_R \times \boldsymbol{V}_A) = \boldsymbol{\omega}_4 \qquad \varepsilon > 0 \tag{2.9}$$

In xy configuration we can write

$$\operatorname{grad} U_R \times \boldsymbol{V}_A = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \dfrac{\partial U_R}{\partial x_A} & \dfrac{\partial U_R}{\partial y_A} & 0 \\ \dot{x}_A & \dot{y}_A & 0 \end{vmatrix} \tag{2.10}$$

Then, the angular velocity vector of the frame is described as

$$\dot{\boldsymbol{\beta}} = \varepsilon\Big(\frac{\partial U_R}{\partial x_A}\dot{y}_A - \frac{\partial U_R}{\partial y_A}\dot{x}_A\Big)\boldsymbol{k} = \boldsymbol{\omega}_4 \tag{2.11}$$

where

$$\begin{aligned} \frac{\partial U_R}{\partial x_A} &= \frac{\partial U_R}{\partial r_L}\cos(\alpha_L+\beta) + \frac{\partial U_R}{\partial r_P}\cos(\alpha_P-\beta) \\ \frac{\partial U_R}{\partial y_A} &= \frac{\partial U_R}{\partial r_L}\sin(\alpha_L+\beta) - \frac{\partial U_R}{\partial r_P}\sin(\alpha_P-\beta) \end{aligned} \tag{2.12}$$

From the position of the vector in point A in this nonholonomic configuration, the following relations can be written

$$\dot{x}_A = V_A\cos\beta \qquad\qquad \dot{y}_A = V_A\sin\beta \tag{2.13}$$

Substituting (2.12), (2.13) into (2.11), we get

$$\dot{\boldsymbol{\beta}} = \varepsilon k^2\Big(-\frac{1}{r_L^2}\sin\alpha_L + \frac{1}{r_P^2}\sin\alpha_P\Big)\boldsymbol{k} = \boldsymbol{u}_\beta \tag{2.14}$$

Equation (2.8) is true, when at a given moment, the angular instantaneous velocity equals zero. Otherwise if (2.8) is false, then at a given moment the angular velocity of the frame occurs and the mobile robot moves approaching the position where (2.8) will hold. According to formula (2.11) it ensues that when the value of the angular velocity of the frame equals zero the proportion is true, from which one can determine the components of the velocity vector of point A in the xy axis, which are proportional to appropriate components of the gradient, which is

$$\begin{aligned} \dot{x}_A &= \varepsilon_1\frac{\partial U_R}{\partial x_A} = \varepsilon_1\Big[\frac{k^2}{r_L^2}\cos(\alpha_L+\beta) + \frac{k^2}{r_P^2}\cos(\alpha_P-\beta)\Big] \\ \dot{y}_A &= \varepsilon_1\frac{\partial U_R}{\partial y_A} = \varepsilon_1\Big[\frac{k^2}{r_L^2}\sin(\alpha_L+\beta) - \frac{k^2}{r_P^2}\sin(\alpha_P-\beta)\Big] \qquad \varepsilon_1 > 0 \end{aligned} \tag{2.15}$$

The velocity vector in point A is determined as

$$\boldsymbol{V}_A = \dot{x}_A\boldsymbol{i} + \dot{y}_A\boldsymbol{j} = \boldsymbol{u}_v \tag{2.16}$$

If point A is moving with a velocity found from (2.16) with components determined from (2.15) and the angular velocity vector of the frame is determined from (2.14), then the mobile robot moving in the force field is safely passing by the obstacles.

2.2. Goal-seeking behaviour

Most of theproposed potential functions are based on the following idea: the mobile robot should be attracted toward its goal configuration, while being repulsed by the obstacles. In this section, we illustrate the attractive potential.

Analysing the problem of goal-seeking behaviour, we can assume that point B is the goal, as shown in Fig. 2. The motion takes place in the plane xy. Point B effects the robot with an attractive force $\boldsymbol{F}$, which can be determined as

$$\boldsymbol{F} = -k_r r_1 \boldsymbol{e}_1 \tag{2.17}$$

where $r_1 = d$ is the distance between points A and B, k_r is an appropriate coefficient, $\boldsymbol{e}_1 = \boldsymbol{r}_1/d$ is the unit vector of the A-B axis.

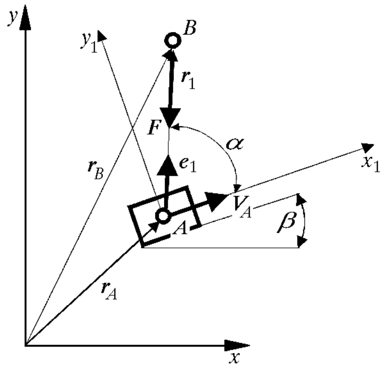

Fig. 2. Attractive potential

Equation (2.17) can be written as

$$\boldsymbol{F} = -k_r r_1 \frac{\boldsymbol{r}_1}{d} \tag{2.18}$$

Assume now the above defined force (2.18) comes from a force field, then

$$dU_B = \boldsymbol{F}\ d\boldsymbol{r}_1 \tag{2.19}$$

Using equation (2.18), we can transform relation (2.19) to the form

$$dU_B = -k_r r_1\ dr_1 \tag{2.20}$$

Integrating equation (2.20), assuming the integration constant equal to zero, we get the field force function

$$U_B = -\frac{1}{2} k_r r_1^2 = -\frac{1}{2} k_r [(x_B - x_A)^2 + (y_B - y_A)^2] \tag{2.21}$$

Assuming that the elements of the velocity vector in point A on appropriate axes are proportional to the corresponding elements of the gradient field force, we can receive

$$\begin{aligned} \dot{x}_A &\cong \varepsilon \frac{\partial U_B}{\partial x_A} = \varepsilon k_r (x_B - x_A) \\ \dot{y}_A &\cong \varepsilon \frac{\partial U_B}{\partial y_A} = \varepsilon k_r (y_B - y_A) \end{aligned} \tag{2.22}$$

where ε and k_r are coefficients adjusted through experiments to generate the best trajectory.

To complete the planning method, we set the rotational part to be $\dot{\beta} = \omega$. It is convenient to use the following formula

$$\dot{\beta} \cong k_\beta \alpha = \arctan \frac{y_B - y_A}{x_B - x_A} - \beta \tag{2.23}$$

The proportionality coefficient k_β is chosen in such a way so it does not deviate too much from the A-B direction. It is implicit in equation (23) that the forward direction characterised by β will be aligned. Using (2.22) and (2.23) in such an approach, the resulting command will be

$$u_v = \sqrt{\dot{x}_A^2 + \dot{y}_A^2} \qquad\qquad u_\beta = \dot{\beta} \tag{2.24}$$

which should be appropriately normalised so that the inequalities $0 \leqslant u_v \leqslant 1$, $-1 \leqslant u_\beta \leqslant 1$ are true.

2.3. Fusion of behaviour scenarios

In order to make the mobile robot be attracted toward its goal configuration, while being repulsed from the obstacles, the potential U is constructed as the sum of two additional elementary potential functions

$$U(x) = U_R(x) + U_B(x) \tag{2.25}$$

where U_R is the repulsive potential associated with given obstacles and U_B is the attractive potential associated with the goal configuration x_B, y_B. In both cases, appropriate control coefficients for u_V, u_β should be adjusted experimentally to get the best generation of the trajectory.

3. Numerical simulation

The used method of generation of the trajectory of motion, has been verified by simulation on the mobile wheeled robot Pioneer 2DX emulator in a scene with rectangular obstacles. An input saturation was included, with bounds on V_A and $\dot{\beta}$, respectively and with $v_{Am} = 0.4\,\mathrm{m/s}$, $\omega_m = 0.3\,\mathrm{rad/s}$.

The described mobile robot is equipped with eight ultrasonic sensor rings. The sensors are divided into two groups. Group A is composed of four neighbouring sensors which measure the distance to the obstacle. Simulations were performed using the Matlab/Simulink package with the sampling interval $T = 0.01\,\mathrm{s}$.

Some of the chosen simulations of the robot behaviour, among others the reaching of the middle of the collision-free space, are shown in Fig. 3.

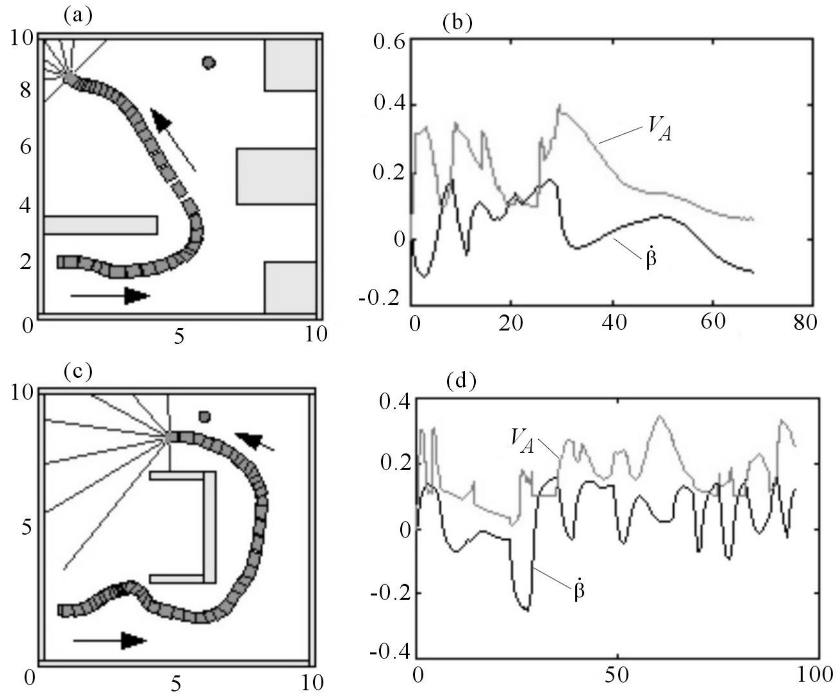

Fig. 3. Realisation of analysed motion

In both cases, the nonholonomic motion obtained for $x_A(0) = 1$, $y_A(0) = 2$, $\beta(0) = 0$ was successful. In Fig. 3b,d the associated parameters of motion V_A, $\dot{\beta}$ are shown.

The examples of the resulting navigation behaviour of the goal-seeking is shown in Fig. 4.

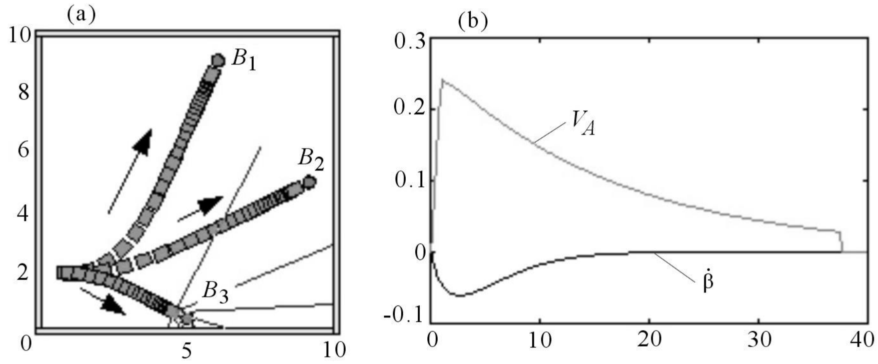

Fig. 4. Goal seeking behaviour

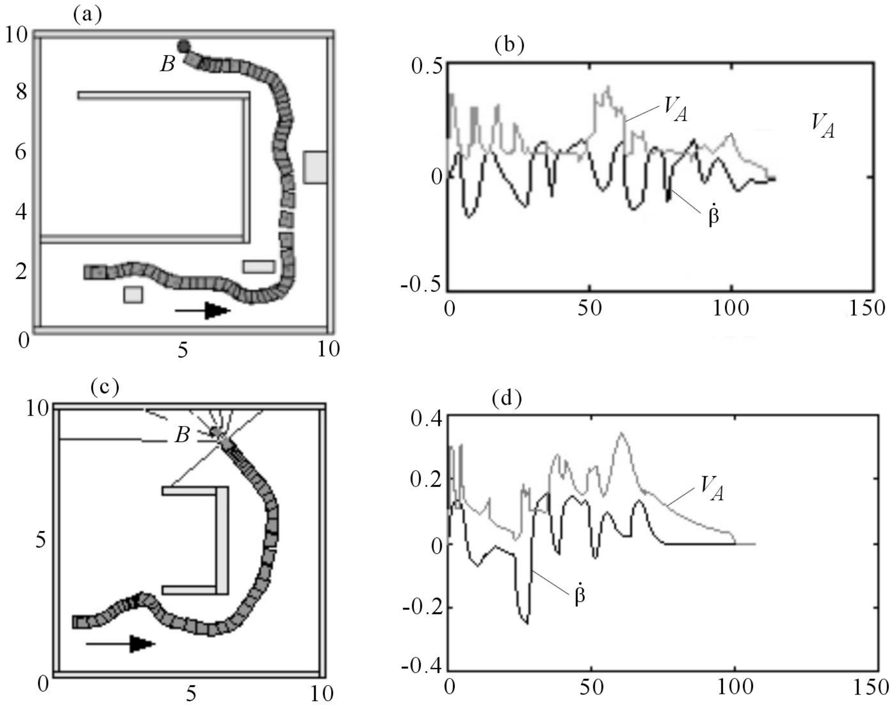

Fig. 5. Fusion of the behaviours

In these cases, the action according to (2.24) guides the robot to its destinations B_i, which are depicted in Fig. 4a. The parameters of motion generated by such an action, Eq. (2.24), for point $G(5, 0.5)$ are depicted in Fig. 4b.

In order to make the mobile robot be attracted toward its goal configuration, while being repulsed from the obstacles, we conducted other simulations. An example of the resulting potential field combiner is shown in Fig. 5. The mobile robot received mission to reach the given goal position B from a given start position with the reaching of the middle of the collision-free space. The fusion of the "reaching the middle of the collision free space" and "goal-seeking" behaviours scenarios showed its efficiency in going from the source to the goal without collision.

We have applied the proposed method to several other situations and the performance was always satisfactory.

4. Summary

We have proposed an approach for planning the motion of a wheeled mobile robot moving between obstacles. The method of the artificial force field can be a basis for realisation of reactive navigation of a wheeled mobile robot. The extracted solutions can be used in the control of elementary behaviour scenarios of the robot reveal, in an unknown environment and in real time. However, experiments that some failures may occur when a concave obstacle emerges between the robot and the goal. In order to solve of such a blocking situation an additional behaviours should be proposed.

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Planowanie ruchu mobilnego robota kołowego z zastosowaniem sztucznego pola potencjalnego

Streszczenie

W pracy rozważa sie problem generowania bezkolizyjnej trajektorii ruchu mobilnego robota w czasie rzyczywistym z zastosowaniem sztucznego pola potencjalnego. Analizuje się elementarne zachowanie mobilnego robota, takie jak: osiągnij środek wolnej przestrzeni oraz idź do celu. Wygenerowana trajektoria ruchu umożliwiająca omijanie przeszkód wypukłych uwzględnia elementarne zachowania robota. Stanowi ona trajektorię zadaną, która realizuje układ sterowania. Efektywność zaproponowanego rozwiązania została numerycznie zweryfikowana na emulatorze mobilnego robota kołowego Pioneer-2DX.

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