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INFLUENCE OF THE LUBRICATING AGENT ON THE PROPERTIES OF CONTACT JOINTS

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The paper presents nonlinear mathematical models describing the properties of dry and lubricated contact joints loaded in the normal direction within the range 0.5-2.5 MPa. The surfaces of joints were coated with an assembly paste and with a hydraulic oil. The dependences of the energy dissipation coefficient Ψ as a function of the load for applied lubricating agents were presented. The structure of the models was determined on the basis of the experimentally determined spectral-response characteristics of the relative displacements of the nominal contact of the surface and of the contact load in the normal direction. The forms of the functional factors of models were estimated using the methods of the linear regression analysis.

Key words: contact joint, nonlinear models, energy dissipation

1. Introduction

Contact joints weakly loaded in the normal direction frequently occur in various technical applications e.g., they are commonly found in connections of movable elements of technological equipments. The physical phenomena describing the motion in the zone of such connections are characterised by significant nonlinearity. The application of different types of lubrication for the contact joints with various surface roughness may considerably change the character of their deformability and damping. Thereby, there is a need to develop mathematical models which describe more precisely the properties of lubricated contact joints than those used until now.

2. Estimation of nonlinear mathematical models of lubricated contact joints

2.1. The test stand

The experimental tests were carried out on a specially designed and constructed stand, which enables the realization of static and dynamic loadings in the normal directions. The block diagram of the stand is given in Fig. 1 (Skrodzewicz, 1999; Skrodzewicz and Gutowski, 2000). The samples for investigations were made of steel 45. The surfaces forming the investigated contact joints were machined by the method of cylindrical grinding, and their nominal area of contact amounted to $A = 50\,\mathrm{cm}^2$. Hardness of the samples was within the range 15-50 HRC. The surfaces of the contact joint were machined in the manner, which allows one to achieve a markedly different roughness. Roughness of the more precisely machined surface was characterised by a value of the indicator $R_q = 0.86\,\mu\mathrm{m}$. The roughness indicator of the second surface was $R_q = 2.37\,\mu\mathrm{m}$. The contact joints were placed in such a way that the mutual location of the traces of machining of the examined surfaces was perpendicular to each other. Before the actual experiment, the dry contact joints had been stabilised by loading them with 2000 cycles with the average value of the loading $\sigma_0 = 2.5\,\mathrm{MPa}$ and the amplitude $\sigma_m = 2.25\,\mathrm{MPa}$.

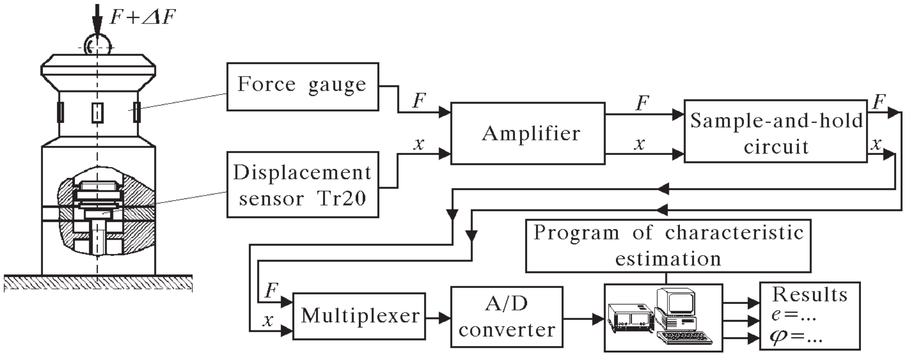

Fig. 1. Block diagram of the system for recording and processing signals

The experiment was carried out for the dry joint, and subsequently for the joint coated with NUTO 46 hydraulic oil. After careful washing of the joint with birol it was coated with the assembly paste OKS200 containing MoS_2. All measurements were carried out in temperature $22 \pm 1°\mathrm{C}$. Before each measurement, the contact joint had been stabilised by loading it with

1000 cycles with the average value of the loading $\sigma_0 = 2.5$ and the amplitude $\sigma_0 = 2.25$ MPa, which resulted in squeezing out of the excess of the lubricating agent.

2.2. Formulation of the mathematical model

The formulation and identification of the mathematical model was performed for two independent variables σ_0 and σ_m establishing the values of the preliminary load for the variable σ_0 at 5 levels with the values: 0.5 MPa, 1 MPa, 1.5 MPa, 2 MPa and 2.5 MPa, respectively. Next, each level of the variable σ_0 was divided into 6 parts, which resulted in determining 5 values of the input signal amplitudes simultaneously determining the values of the range for the variable σ_m in such a way that $\sigma_{m1} = \Delta\sigma_m \approx 0.165\sigma_0$ achieved the value $\sigma_{m1} \approx 0.83\sigma_0$ at the fifth level. As a result of those divisions a 25-point even experimental plan was obtained. A variable component of the input signal of the frequency of 1 Hz in the form similar to the harmonic signal was applied.

The structure of the mathematical model coupling the relative displacement of the contact joint with its load in the form of the system of nonlinear equations was selected

$$\begin{aligned}
&\sigma(t) = \sigma_0 + \sigma_m \sin(\omega t) \\
&x(\sigma_0, \sigma_m, t) = a_1(\sigma_0, \sigma_m) \sin[\omega t + \varphi(\sigma_0)] + a_2(\sigma_0, \sigma_m)\{\cos 2[\omega t + \varphi(\sigma_0)] - 1\} - \\
&-a_3(\sigma_0, \sigma_m) \sin 3[\omega t + \varphi(\sigma_0)] - a_4(\sigma_0, \sigma_m)\{\cos 4[\omega t + \varphi(\sigma_0)] - 1\}
\end{aligned} \tag{2.1}$$

where $\sigma_0 > \sigma_m$.

As a result of the performed analyses, the best fitting model from among simple mathematical ones was the model in the form

$$a_1(\sigma_m, \sigma_0) = \alpha_{11} + \alpha_{12}\frac{\sigma_m}{\sigma_0^l} + \alpha_{13}\Big(\frac{\sigma_m}{\sigma_0}\Big)^2 \tag{2.2}$$

For the remaining components, the best fitted model proved to be the multiplicative one in the form

$$a_k(\sigma_m, \sigma_0) = \alpha_k\Big(\frac{\sigma_m}{\sigma_0}\Big)^{k\frac{\pi}{2}} \tag{2.3}$$

for $2 \leqslant k \leqslant 4$.

The model of the component representing damping for one independent variable σ_0 was established in the form

$$\varphi(\sigma_0) = \varphi_0 + \gamma \log \sigma_0 \tag{2.4}$$

2.3. Experimental results

As a result of the estimation for the grinding, dry and lubricated surfaces, the following values of the components of the model given by equation (2.1) were obtained:

— dry surface

$$
\begin{aligned}
a_1(\sigma_m,\sigma_0) &= 0.021 + 0.205\frac{\sigma_m}{\sigma_0^{0.25}} + 0.25\Big(\frac{\sigma_m}{\sigma_0}\Big)^2\\
a_2(\sigma_m,\sigma_0) &= 0.135\Big(\frac{\sigma_m}{\sigma_0}\Big)^{\pi}\\
a_3(\sigma_m,\sigma_0) &= 0.06\Big(\frac{\sigma_m}{\sigma_0}\Big)^{\frac{3\pi}{2}}\\
a_4(\sigma_m,\sigma_0) &= 0.018\Big(\frac{\sigma_m}{\sigma_0}\Big)^{2\pi}\\
\varphi(\sigma_0) &= -0.004 + 0.011\log\sigma_0 \ \mathrm{rad}
\end{aligned}
$$

— surface coated with assembly paste OKS 200

$$
\begin{aligned}
a_1(\sigma_m,\sigma_0) &= 0.0134 + 0.35\frac{\sigma_m}{\sigma_0^{0.375}} + 0.255\Big(\frac{\sigma_m}{\sigma_0}\Big)^2\\
a_2(\sigma_m,\sigma_0) &= 0.14\Big(\frac{\sigma_m}{\sigma_0}\Big)^{\pi}\\
a_3(\sigma_m,\sigma_0) &= 0.05\Big(\frac{\sigma_m}{\sigma_0}\Big)^{\frac{3\pi}{2}}\\
a_4(\sigma_m,\sigma_0) &= 0.012\Big(\frac{\sigma_m}{\sigma_0}\Big)^{2\pi}\\
\varphi(\sigma_0) &= -0.022 + 0.047\log\sigma_0 \ \mathrm{rad}
\end{aligned}
$$

We did not manage, however, to obtain an accurate mathematical model with the structure of functions (2.2) and (2.3) for the surface of the contact joint, which was coated with the hydraulic oil. The reason for this failure was a non-monotonic character of the functions coupling σ_m with σ_0, which was distinctly demonstrated by the numerical values of the coefficients $a_1(\sigma_m)$ of the models of the contact joints coated with the oil. A trial of application of the formulated two-variable model would lead to further considerable compilation of the forms of equations (2.2) and (2.3). Thus, the mathematical models for one independent variable σ_m were estimated with a sufficient accuracy assuming σ_0 as a parameter. For:

— the surface coated with hydraulic oil NUTO 46 for $\sigma_0 = 0.5\,\mathrm{MPa}$

$$a_1(\sigma_m) = -0.05 + 1.42\sigma_m - 1.17\sigma_m^2 \qquad a_2(\sigma_m) = 1.0\sigma_m^{\pi}$$
$$a_3(\sigma_m) = 0.68\sigma_m^{\frac{3\pi}{2}} \qquad a_4(\sigma_m) = 0.9\sigma_m^{2\pi}$$

— the surface coated with hydraulic oil NUTO 46 for $\sigma_0 = 1\,\mathrm{MPa}$

$$a_1(\sigma_m) = 0.08 - 0.04\sigma_m + 0.81\sigma_m^2 \qquad a_2(\sigma_m) = 0.145\sigma_m^{\pi}$$
$$a_3(\sigma_m) = 0.022\sigma_m^{\frac{3\pi}{2}} \qquad a_4(\sigma_m) = 0.0005\sigma_m^{2\pi}$$

— the surface coated with hydraulic oil NUTO 46 for $\sigma_0 = 1.5\,\mathrm{MPa}$

$$a_1(\sigma_m) = 0.025 + 0.36\sigma_m + 0.16\sigma_m^2 \qquad a_2(\sigma_m) = 0.043\sigma_m^{\pi}$$
$$a_3(\sigma_m) = 0.0045\sigma_m^{\frac{3\pi}{2}} \qquad a_4(\sigma_m) = 0.0004\sigma_m^{2\pi}$$

— the surface coated with hydraulic oil NUTO 46 for $\sigma_0 = 2\,\mathrm{MPa}$

$$a_1(\sigma_m) = 0.02 + 0.3\sigma_m + 0.125\sigma_m^2 \qquad a_2(\sigma_m) = 0.018\sigma_m^{\pi}$$
$$a_3(\sigma_m) = 0.0012\sigma_m^{\frac{3\pi}{2}} \qquad a_4(\sigma_m) = 0.00005\sigma_m^{2\pi}$$

— the surface coated with hydraulic oil NUTO 46 for $\sigma_0 = 2.5\,\mathrm{MPa}$

$$a_1(\sigma_m) = 0.04 + 0.265\sigma_m + 0.09\sigma_m^2 \qquad a_2(\sigma_m) = 0.01\sigma_m^{\pi}$$
$$a_3(\sigma_m) = 0.0006\sigma_m^{\frac{3\pi}{2}} \qquad a_4(\sigma_m) = 0.00001\sigma_m^{2\pi}$$

and

$$\varphi(\sigma_0) = -0.012 + 0.034\log\sigma_0 \text{ rad} \qquad \text{for} \qquad 0.5 \leqslant \sigma_0 \leqslant 2.5 \text{ MPa}$$

Graphical images of the nonlinear mathematical models illustrating the deformability of the contact joints and the phase planes of the motion in these joints are presented in Fig. 2. A distinct difference can be seen between the characteristic of the contact joint coated with the oil and the remaining contact joints. For this reason, the experiment with the contact joint coated with the oil was repeated, and the same result was obtained.

The contact joint coated with the oil caused a pronounced weakening of the dynamics of motion as a function of σ_0, and demonstrated definitely the greatest nonlinearity.

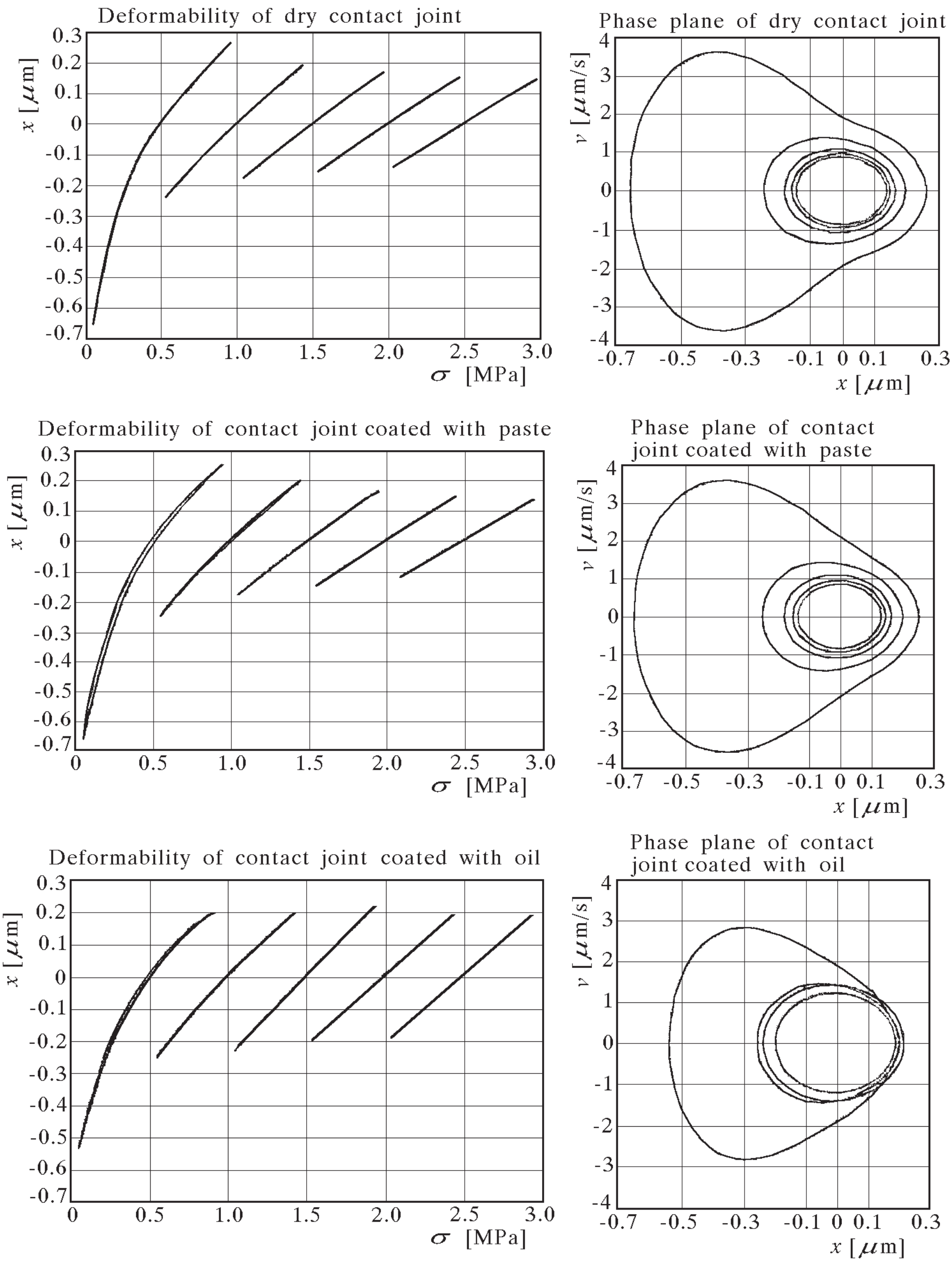

Fig. 2. Deformability characteristics and phase plane of motion in the tested contact joint for $\sigma_m = 0.425\,\mathrm{MPa}$ and $0.5 < \sigma_0 < 2.5\,\mathrm{MPa}$

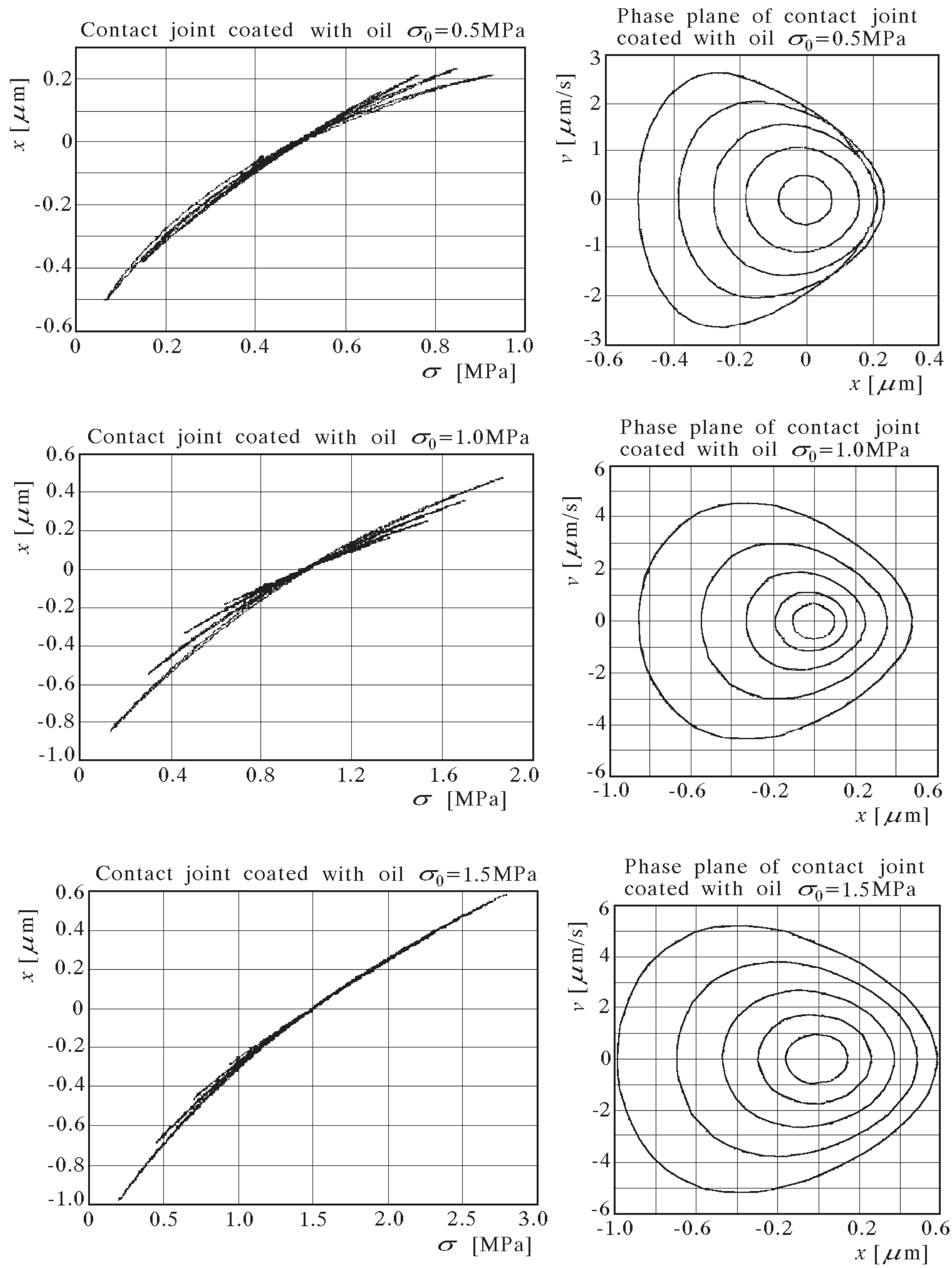

Fig. 3. Deformability characteristics and phase plane of motion in the tested contact joint coated with oil

2.4. Determination of the energy dissipation coefficient

The estimation of the energy losses in the nonlinear contact joint requires the elaboration of the method of its calculation. The main assumption in this process is its similarity to the linear models (Skrodzewicz, 1999). In the case of nonlinear contact joints, the most appropriate measure of the energy dissipation seems to be a dimensionless energy dissipation coefficient Ψ, which expresses the ratio of the dissipated and potential energy. The definition is proposed in the integral form.

If the experiment applies a periodic excitation signal

$$F(t) = \sum_{k=0}^{n} F_k \sin(k\omega t + \phi_k) \tag{2.5}$$

then the output signal, in which the nonlinear object is examined, can be obtained also in a periodic form

$$x(t) = \sum_{k=0}^{n} x_k \sin(k\omega t + \delta_k) \tag{2.6}$$

The energies E_h and E_p are represented by the appropriate areas of the plot of the relative displacements of the surfaces which constitute the contact in the function of the load, see Fig. 4.

Applying definite integrals, one can calculate the area of the curved trapezium in the parametric form as follows

$$S = \int_{t_1}^{t_2} y(t)\, dt \tag{2.7}$$

where

$$y(t) = F(t)\frac{dx(t)}{dt}$$

Applying this formula, the definition of the Ψ coefficient (for nonlinear case) can be transformed into four identical integral equations described by equation (2.8). Separating the integration limits, the following is obtained

$$s_1 = \int_{0}^{\frac{T}{4}} y(t)\, dt \qquad s_2 = \int_{\frac{T}{2}}^{\frac{T}{4}} y(t)\, dt \qquad s_3 = \int_{\frac{T}{2}}^{\frac{3T}{4}} y(t)\, dt \qquad s_4 = \int_{T}^{\frac{3T}{4}} y(t)\, dt \tag{2.8}$$

for $T = 2\pi/\omega$.

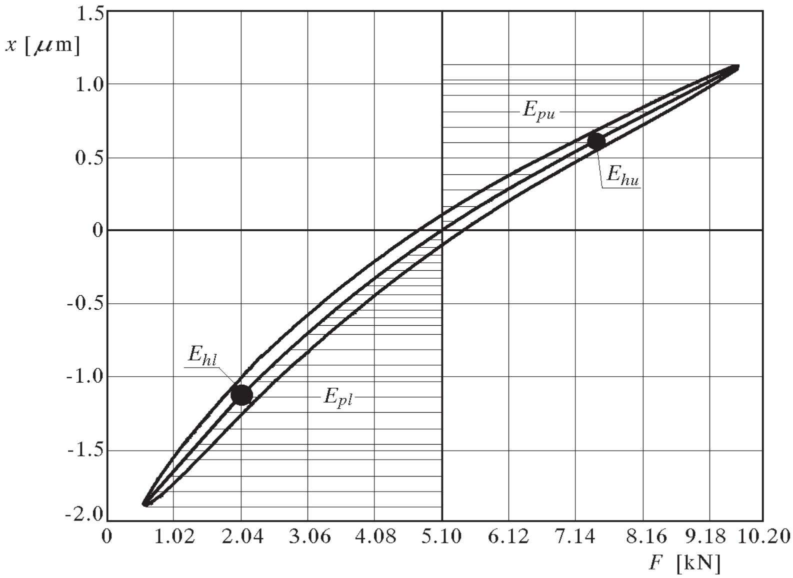

Fig. 4. Graphical interpretation of the integral definition of the energy dissipation coefficient

Assuming notation

$$
\begin{aligned}
E_{pu} &= \frac{1}{2}(s1 + s2) \qquad & E_{pl} &= \frac{1}{2}(s3 + s4) \\
E_{hu} &= s1 - s2 \qquad & E_{hl} &= s3 - s4
\end{aligned}
\tag{2.9}
$$

the potential energy for the nonlinear system can be presented in the form

$$
E_p = \frac{1}{2}(E_{pu} + E_{pl}) \tag{2.10}
$$

when the energy of losses in this system is

$$
E_h = E_{hu} + E_{hl} \tag{2.11}
$$

Hence, the energy dissipation coefficient Ψ can be described by the relationship

$$
\Psi = \frac{E_h}{E_p} \tag{2.12}
$$

It should be noticed that the presented definition of the energy dissipation coefficient in the linear case is reduced to the relationship consistent with the Kelvin-Voigt linear model. Making use of relationship (2.12), the dependence $\Psi = f(\sigma_0)$ was estimated as:

— for the contact joint coated with the assembly paste

$$\Psi(\sigma_0) = 0.137 - 0.293 \log \sigma_0$$

— for the contact joint coated with the hydraulic oil

$$\Psi(\sigma_0) = 0.075 - 0.214 \log \sigma_0$$

— for the dry contact joint

$$\Psi(\sigma_0) = 0.025 - 0.062 \log \sigma_0$$

Based on the experimental results, it can be practically assumed that the values of the energy dissipation coefficients Ψ for the presented nonlinear models are very close to the values which could be calculated on the basis of relationship (2.13) obtained from the Kelvin-Voigt linear model (for small values of the angle φ)

$$\Psi(\sigma_0) = 2\pi \tan[\varphi(\sigma_0)] \approx 2\pi\varphi(\sigma_0) \tag{2.13}$$

The characteristics illustrating the dependence of the energy dissipation coefficient Ψ on σ_0 are shown in Fig. 5.

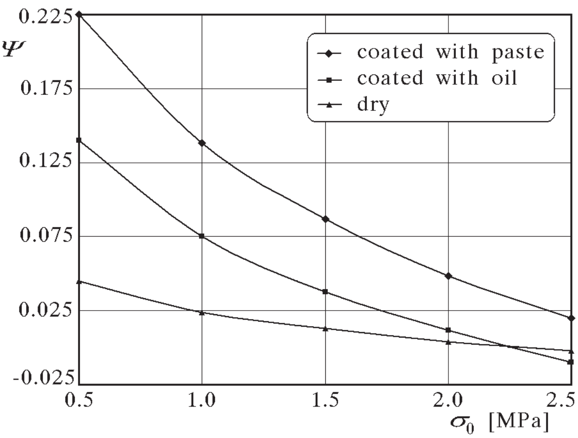

Fig. 5. Characteristics of the energy dissipation coefficient Ψ of the tested contact joints

3. Summary

The phenomena occurring in the contact joint take place at the boundary of micro and nanometers, and they are characterised by strong nonlinearity.

The contact joint coated with the oil was found to be the most difficult for a mathematical description among the examined joints. This joint causes a pronounced drop in the dynamics of motion as a function of σ_0, and decisively demonstrates the greatest nonlinearity. The complexity of the phenomena occurring is such a joint illustrates Fig. 3. The changes of deformability for $\sigma_0 = 0.5\,\mathrm{MPa}$ reassemble a slightly unfolded fan with the deformability decreasing in the function of σ_m. On the contrary, for $\sigma_0 = 1\,\mathrm{MPa}$ the deformability as a function of σ_m increased more strongly. This fact explains the opposite signs of the components $a_1(\sigma_m)$ for $\sigma_0 = 0.5\,\mathrm{MPa}$ and $a_1(\sigma_m)$ for $\sigma_0 = 1\,\mathrm{MPa}$. The changes of deformability as a function of σ_m were significantly smaller for the value of the parameter $\sigma_0 = 1.5\,\mathrm{MPa}$. These phenomena may by utilized for the control, in a limited range, of the deformability of the contact joint as a function of the static load σ_0 as well as the amplitude of the dynamic load σ_m.

4. Conclusion

Finding an answer to the question: what is the influence of relation of the type of surface roughness with the physical properties of the lubricating liquid (viscosity?) on the properties of the contact joints (especially the deformability) seems to be important. When do the pastes possess the similar properties as lubricating liquids on other types of roughness in the contact area? Does any of the roughness indicators determines the best the type of such a surface? Or, maybe, one should look for another, better indicator? The answer to these questions would allow the designers of the technological equipment to control (in a limited range) the deformability of the contact joints occurring in many constructions in the natural way, and would enable optimization of these constructions with regard to e.g. their vibrostability. The thus performed optimization would not require significant changes to the equipment structure, but only an appropriate selection of the constant surface, the type of its roughness and the lubricating agent. For this reason, the recognition and description of the phenomena-taking place in the lubricated joints seems to be extremely interesting.

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Wpływ czynnika smarującego na właściwości połączeń stykowych

Streszczenie

W pracy przedstawiono nieliniowe modele matematyczne opisujące właściwości połączeń stykowych suchych i smarowanych obciążonych w kierunku normalnym w zakresie 0.5-2.5 MPa. Powierzchnie połączeń pokryto pastą montażową i olejem hydraulicznym. Podano zależności współczynnika rozproszenia energii Ψ w funkcji obciążenia dla zastosowanych czynników smarujących. Strukturę modeli określono na podstawie eksperymentalnie wyznaczonych charakterystyk widmowych sygnałów przemieszczeń względnych nominalnych powierzchni styku i obciążenia styku w kierunku normalnym. Postaci czynników funkcyjnych modelu wyestymowano stosując metody analizy regresji liniowej.

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