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MULTICRITERIA OPTIMIZATION OF SANDWICH CYLINDRICAL PANELS UNDER AXIAL COMPRESSIVE LOADS

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The aim of this paper is to present the bicriteria optimization model of a sandwich cylindrical panel under axial compression. The objective functions are the weight and panel deformability. The deformability is defined as the panel bending rigidity reciprocal, and it represents some qualitative measure of the panel deflections. The design variables are the thicknesses of the layers. The constraints include a stability condition, stress conditions, the validation of theoretical models, and finally, technological and constructional requirements. The problem was solved with the help of Pareto's concept of optimality, with continuous and discrete sets of the design variables. Results of numerical calculations are presented in the form of tables and diagrams. A comparison of the optimal parameters for the unilayer and the sandwich panels is presented.

Key words: multicriteria optimization, sandwich shell, Pareto-optimum

1. Introduction

In recent years thin-walled sandwich-type structures (plates, panels, shells), due to their advantages, have been applied in a wide range of industrial branches. The development of the theoretical basis of sandwich structures is connected with the development of the aircraft and space industry on the one hand, and civil engineering on the other. The development of new materials, significant need for high technologies, and low-weight laminated structures are very important motivations for intensive research in that field. The progress in the development of new materials as well as in the technology of their manufacturing is very important for distribution of these structures. These premises show that the further development of sandwich structures will continue to be in demand. The fundamental distinguishing advantage of multilayer structures is the beneficial relation of load-carrying capacity to their weight. The application of more precise techniques for assembling structures eliminates the influence of so-called geometrical imperfections to some extent. The condition of stability is the main condition to be regarded in the design of thin-walled structures.

The advantages of sandwich structures can be better exploited and their faults can be minimized if the basic geometric and physical parameters are calculated with the help of multidisciplinary optimization. The multicriteria optimization procedures allow a designer to model a structure including the real behaviour of the structure. The multicriteria optimization is nearer to the technical reality than the traditional scalar optimization both in the sense of structure modelling and data interpretation. In engineering practice the multicriteria optimization based on the Pareto-optimality concept is applied broadly. The best optimal solutions are selected from the set of compromise Pareto-optimal solutions by means of additional preference functions (Osyczka, 1992).

This paper is a continuation of the research concerning the optimal design of sandwich structures. In the work by Ostwald (1990) the scalar optimization of sandwich shells with the weight as the optimization criterion is presented, whereas the works by Ostwald (1993, 1996) deal with the bicriteria optimization of sandwich shells under combined loads. Further, Ostwald (1997) proposes the vector optimization of sandwich plates with a foam plastic core, and Kasperska and Ostwald (1998) discusses the sandwich plates with a trapezoidal core. The scalar optimization of sandwich panels is presented in the work by Ostwald and Sekulski (1989). The later works by Kasperska and Ostwald (2000a,b) present the sandwich panels under combined loads with elements of an expert system.

2. The bicriteria optimization model of a cylindrical panel

This paper presents the bicriteria optimization model of a sandwich cylindrical panel under axial compression. An open cylindrical shell is composed of two thin carrying layers (faces) made of a material with high strength properties having different thicknesses h_1 and h_2 and different elastic properties. Between the faces the core of the thickness h_3, made of a material with relatively low strength properties, is stiffly placed. The basic dimensions of the model and the load are presented in Fig. 1.

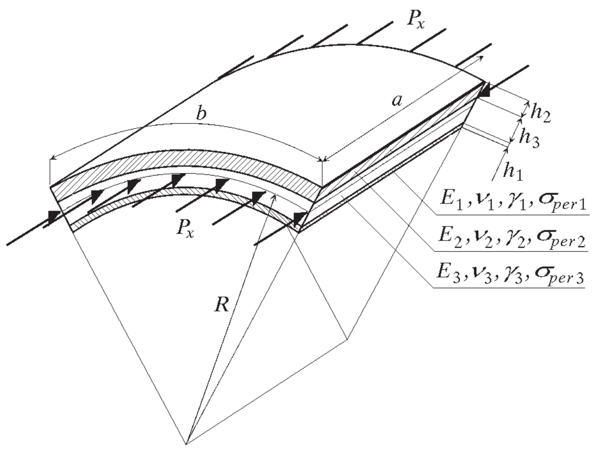

Fig. 1. Model of the sandwich cylindrical panel

The following assumptions were made in the panel model. The panel is thin-walled. The materials of the faces have elastic isotropic properties. It was assumed that the deformations of the panel are linear and elastic. The core is incompressible in the direction normal to the middle surface of the shell. The core has a relatively small rigidity in comparison with the faces (it is so-called light core). The displacement of any point of the panel is described based on the Kirchhoff-Love hypothesis of the broken line. The panel edges are simply supported and they have a membrane joining the individual layers together.

The bicriteria optimization problem, which uses the concept of Pareto's optimum, is formulated as follows

$$\boldsymbol{Q}(\boldsymbol{x}) = [Q_1(\boldsymbol{x}), Q_2(\boldsymbol{x})] = (1 - W)Q_1(\boldsymbol{x}) + WQ_2(\boldsymbol{x}) \rightarrow \text{minimum}$$

where

$\boldsymbol{x} = [h_1, h_2, h_3]$	–	vector of design variables
$Q_1(\boldsymbol{x}), Q_2(\boldsymbol{x})$	–	optimization criteria
W	–	weighting coefficient, $0 \leqslant W \leqslant 1.0$.

The concept of Pareto's optimality does not give a single solution, but a set of Pareto-optimal solutions, also called the nondominated or compromise solutions. The graphic models of the scalar optimization (with the single objective function) and multiobjective optimization are presented in the work by Ostwald (1996). The best optimal solution must be chosen from the set of Pareto-optimal solutions with the help of additional criteria. These criteria are formulated in the form of so-called preference functions (Osyczka, 1992; Ostwald, 1993).

The multicriteria optimal design of structures requires the formulation of the optimization criterion set, design variables and the set of constraints. The most commonly used optimization criterion is the weight of the structure. This criterion has also some economic meaning. According to the symbols presented in Figure 1, the first criterion has the following form

$$Q_1(\boldsymbol{x}) = ab(h_1\gamma_1 + h_2\gamma_2 + h_3\gamma_3)\ [\mathrm{kg}] \ \rightarrow \text{minimim}$$

where γ_i is the material density of the ith layer.

The second criterion is the demand of the maximum structural stiffness in the form of the minimal deformability of the shell. This paper adopted a simplified model of the procedure, because the relations describing the panel deflections are not sufficiently reliable. Therefore, the condition of the minimal structural deformability was formulated by Ostwald (1993). For the cylindrical panel this criterion is defined as follows

$$Q_2(\boldsymbol{x}) = \frac{1}{D(\boldsymbol{x})}\ [1/\mathrm{MNm}] \ \rightarrow \text{minimum}$$

where $D(\boldsymbol{x})$ – bending shell rigidity (Ostwald, 1993)

$$D(\boldsymbol{x}) = \frac{E_n h_1 h_2 (h_1 + 2h_3 + h_2)^2}{4(h_1 + h_2)(1 - \nu_n^2)}$$

$E_n = E_1 = E_2$ – Young's modulus of the faces
$\nu_n = \nu_1 = \nu_2$ – Poisson's constants of the faces.

Similarly, the optimization criteria for cylindrical shells were established by Ostwald (1993,1997).

The thicknesses h_i $(i = 1, 2, 3)$ of the panel layers are taken as the design variables. The set of constraints is defined as follows:

1. Permissible critical load must be greater than the axial compressive force P_x

$$P_{crit\ per} = \frac{P_{crit}^{lin}}{\alpha n} \geqslant P_x$$

The upper critical load P_{crit}^{lin} [MN/m] is calculated as the smallest positive root of the algebraic equation (Sekulski, 1984)

$$A_1\widetilde{P}_x^4 + A_2\widetilde{P}_x^3 + A_3\widetilde{P}_x^2 + A_4\widetilde{P}_x + A_5 = 0$$

$$[\widetilde{P}_x] = 1 \qquad\qquad P_{crit}^{lin} = \frac{B(h_1+h_2+h_3)^2}{ab}\widetilde{P}_x$$

$$B = \sum_{i=1}^{3} B_i = \sum_{i=1}^{3} \frac{E_i h_i}{1-\nu_i^2}$$

where A_i, $i = 1,...,5$ are factors that depend on the physical and geometrical parameters of the panel (Sekulski, 1984).

The factor $\alpha = 1.4$ (Bushnell, 1987) takes into account the influence of initial deflections (geometrical imperfections) on the value of the critical force, and $n = 1.25$ is a safety factor.

2. Normal stresses in each layer of the panel cannot exceed the permissible stresses

$$\sigma_{xi} = \frac{B_i}{B}\frac{P_x}{h_i} \leqslant \sigma_{per\ i}$$

3. Validation of the critical load equations leads to the assumption (Sekulski, 1984)

$$\frac{R}{h_1+h_2+h_3} \geqslant 30$$

4. Considering the technological and constructional requirements, the following conditions are assumed (so-called geometric constraints)

$$0.1 \text{ mm} \leqslant h_1, h_2 \leqslant 3.0 \text{ mm}$$
$$1 \text{ mm} \leqslant h_3 \leqslant 50 \text{ mm}$$

The set of constraints for the sandwich panel encompasses 10 conditions simultaneously.

3. Numerical calculations and conclusions

The bicriteria optimization model of the sandwich cylindrical panel presented above is solved with the help of the PANELA program, which has been written in Delphi 4.0 language for Windows 95/98/NT. The program includes some elements of an expert system. It enables one to choose an optimization procedure and way of normalization of objective functions, as well as a proper preference function and control parameters (Kasperska and Ostwald, 2000a,b).

The PANELA program contains four optimization procedures. The first procedure generates a set of the Pareto-optimal solutions based on a systematic search method, with discrete design variables, which can be in accordance with the standards (MESP procedure). These solutions have practical signification. The second procedure is based on the Hooke-Jeeves method with an application of a penalty function (HJ procedure). The solutions from this procedure are exact with the theoretical results (these solutions show which of the constraints are active). The PANELA program contains also two other discrete optimization procedures, as simulated annealing (SA procedure) and genetic algorithm (GA procedure). The best optimization solution is generated from a set of preferred solutions. This set is a basic guide for the designer and may be helpful in making the right decisions and selecting the best optimal solution.

The following data were assumed in the numerical calculations:

- the carrying layers are made of aluminium alloy PA6, $E_{1,2} = 7.06 \cdot 10^4$ MPa, $\nu_{1,2} = 0.3$, $\gamma_{1,2} = 2780\,\mathrm{kg/m^3}$, $\sigma_{per} = 0.75\ R_e = 195\,\mathrm{MPa}$,
- the core is made of a foam plastic, $E_3 = 53\,\mathrm{MPa}$, $\nu_{1,2} = 0$, $\gamma_{1,2} = 210\,\mathrm{kg/m^3}$
- panel middle surface radius $R = 2\,\mathrm{m}$
- panel length and width $a, b = 0.8\,\mathrm{m}$, $1\,\mathrm{m}$, $1.25\,\mathrm{m}$
- axial compressive force $P_x = 0.1$, 0.2, 0.3, 0.4, 0.5 and 1.0 MN/m.

The sizes of the panel were selected for the calculations in such a way, that the area of the panel $ab = 1\,\mathrm{m}^2$. In the following panels the sizes are:

(a) $a = 0.8\,\mathrm{m}$ $b = 1.25\,\mathrm{m}$ $a/b = 0.64$

(b) $a = 1\,\mathrm{m}$ $b = 1\,\mathrm{m}$ $a/b = 1.00$

(c) $a = 1.25\,\mathrm{m}$ $b = 0.8\,\mathrm{m}$ $a/b = 1.56$

The panel configurations, taken into consideration in this work, are presented in Figure 2.

According to the technological and constructional requirements, the values of the design variables h_1 and h_2 varied from 0.1 to 3.0 mm, with the step equal to 0.1 mm. These parameters ought to be in accordance with the standards. The third design variable, thickness of the core h_3, varied from 1 to 50 mm,

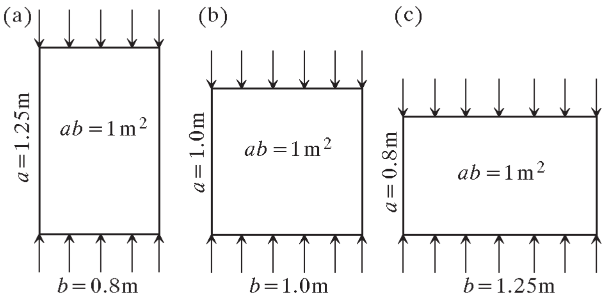

Fig. 2. Basic sizes of analysed panels

with the step equal to 1 mm. These assumptions mean, that the space of decision variables contains 45000 vectors $\boldsymbol{x}$. Some of these vectors do not satisfy at least one of the constraints, so the set of feasible solutions contains 37571 points. In Figure 3 an image of the set of feasible solutions into the criteria space R^2 is shown.

In Figure 4 a set of the Pareto-optimal solutions is presented – it is a part of the criteria space from Fig. 3. These solutions were obtained using MESP procedure. This figure shows the ideal solution, which is determined by the minimization of every criterion $\boldsymbol{Q}(\boldsymbol{x})$ separately.

The results of numerical calculations for the panel with $a/b = 1.00$ and the compressive force $P_x = 0.1\,\mathrm{MN/m}$ are presented in Table 1 (MESP, SA, GA – discrete models, HJ – a continuous model). The presented results are the same as in Figure 4.

Table 1. Optimal thicknesses of the sandwich panel according to the different optimization procedures for $P_x = 0.1\,\mathrm{MN/m}$, $a = b = 1\,\mathrm{m}$

No.	W	Method	Optimal thickness [mm]			Q_1	Q_2	Active	Preferred
			h_1	h_2	h_3	[kg]	[1/MNm]	constr.	solutions
1	0	MESP	0.3	0.3	9.0	3.5580	993.5267		
		SA	0.3	0.3	9.0	3.5580	993.5267		
		GA	0.3	0.3	9.0	3.5580	993.5267		
		HJ	0.2705	0.2722	9.0818	3.4157	1086.1028	Stability	
2	0.1	MESP	0.3	0.3	13.0	4.3980	485.7828		
		SA	0.3	0.3	12.0	4.1880	567.9828		
		GA	0.3	0.3	13.0	4.3980	485.7828		
		HJ	0.2594	0.2594	12.9557	4.1627	569.1466		

3	0.2	MESP	0.3	0.3	17.0	5.2380	287.1132		
		SA	0.3	0.3	17.0	5.2380	287.1132		
		GA	0.3	0.3	17.0	5.2380	287.1132		
		HJ	0.3177	0.3177	15.8674	5.0983	309.8042		
4	0.3	MESP	0.4	0.4	18.0	16.0040	190.3580		
		SA	0.4	0.4	18.0	6.0040	190.3580		
		GA	0.4	0.4	18.0	6.0040	190.3580		
		HJ	0.3635	0.3635	18.1563	5.8337	206.7878		
5	0.4	MESP	0.4	0.4	21.0	6.6340	140.7276		MM, P2
		SA	0.3	0.4	26.0	7.4060	108.2910		MM, P2
		GA	0.4	0.4	21.0	6.6340	140.7276		MM, P2
		HJ	0.4059	0.4059	20.2768	6.5150	148.4604		MM, P2
6	0.5	MESP	0.5	0.5	22.0	7.4000	101.8431		P1
		SA	0.5	0.5	22.0	7.4000	101.8431		P1
		GA	0.5	0.5	22.0	7.4000	101.8431		P1
		HJ	0.4492	0.4492	22.4399	7.2101	109.5323		P1
7	0.6	MESP	0.5	0.5	26.0	8.2400	73.4184		
		SA	0.5	0.5	26.0	8.2400	73.4184		
		GA	0.5	0.5	26.0	8.2400	73.4184		
		HJ	0.4972	0.4972	24.8339	7.9793	80.8116		
8	0.7	MESP	0.6	0.6	28.0	9.2160	52.5271		
		SA	0.6	0.6	28.0	9.2160	52.5271		
		GA	0.6	0.6	28.0	9.2160	52.5271		
		HJ	0.5552	0.5552	27.7342	8.9112	58.0176		
9	0.8	MESP	0.6	0.6	33.0	10.2660	38.0572		
		SA	0.7	0.7	32.0	10.6120	34.4408		
		GA	0.6	0.6	33.0	10.2660	38.0572		
		HJ	0.6353	0.6353	31.7348	10.1966	38.7255		
10	0.9	MESP	0.8	0.8	40.0	12.8480	19.3578		
		SA	0.8	0.8	40.0	12.8480	19.3578		
		GA	0.8	0.8	40.0	12.8480	19.3578		
		HJ	0.7780	0.7780	38.8671	12.4882	21.0795		
11	1.0	MESP	3.0	3.0	50.0	27.1800	3.0591		
		SA	2.9	3.0	50.0	26.9020	3.1177		
		GA	3.0	3.0	50.0	27.1800	3.0591		
		HJ	2.9999	2.9999	49.9999	27.1797	3.0592	h_1, h_2, h_3	

Table 1 presents the solutions for the parameter $W \in [0, 1]$, obtained by means of MESP procedure (systematic search method), SA (simulated annealing) and GA (genetic algorithm), and based on the discrete set of the design variables, which is of practical importance. The lower line presents the results of the Hooke-Jeeves procedure (HJ) with an interior penalty function, based on the continuous set of the design variables, which enables the identification

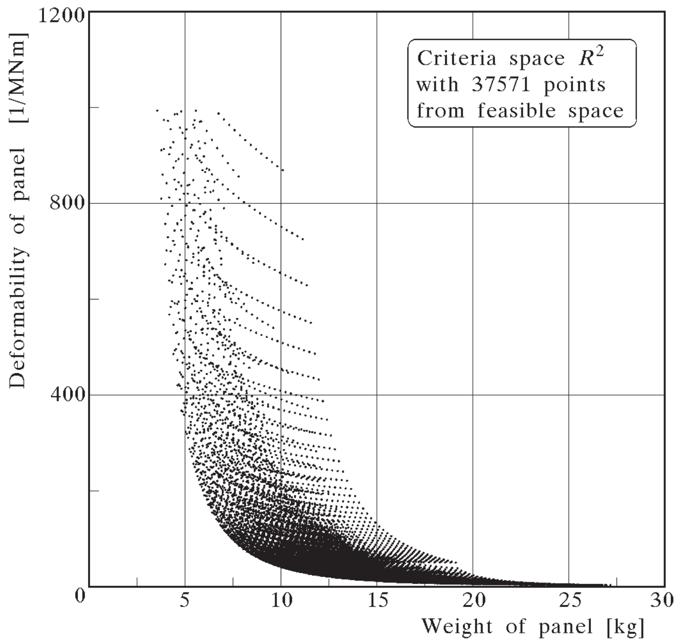

Fig. 3. The criteria space for the sandwich panel $a = b = 1\,\mathrm{m}$, $P_x = 0.1\,\mathrm{MN/m}$

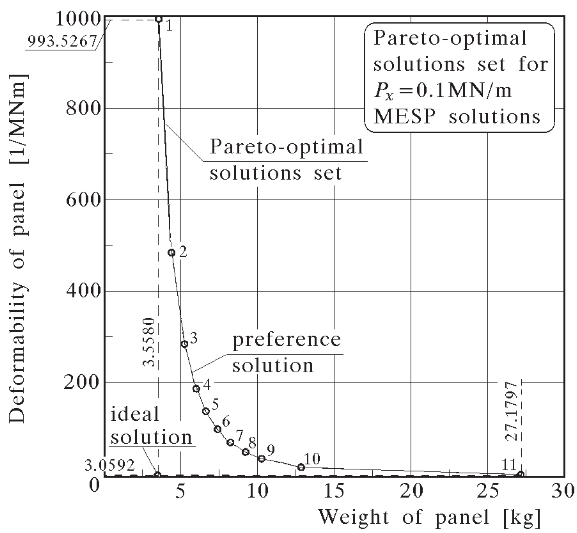

Fig. 4. The set of the Pareto-optimal solutions in the objective space for $a = b = 1\,\mathrm{m}$

of the active constraints. The average calculation time for one W parameter is about 6.5 s for MESP, 11 s for SA, 3.5 s for GA and 4.5 s for HJ (processor Pentium-MMX 166MHz). The last column presents the preferred solutions. The abbreviation MM means that the preferred optimal solution was obtained by means of the min-max method, P1 – by means of the global criterion method with the norm $p = 1$ and P2 – with the norm $p = 2$.

The following tables present the results of numerical calculations for panels with different ratios a/b. At first, the scalar optimizations with the weight as the objective function are presented (Tables 2, 3 and 4 refer respectively to the a/b ratios).

Table 2. Scalar optimization: $a = 0.8$ m and $b = 1.25$ m ($a/b = 0,64$), $W = 0$ (weight as the objective function)

P_x [MN/m]	Optimal thicknesses [mm]			Q_1 [kg]	Q_2 [1/MNm]	$P_{crit\ per}$ [MN/m]	$\sigma_{1,2}$ [MPa]	Active constraints
	h_1	h_2	h_3					
0.1	0.3	0.3	9.0	3.5580	993.5267	0.1026	164.98	S
0.2	0.6	0.6	11.0	5.3680	354.2780	0.2037	180.58	S, σ_1, σ_2
0.3	0.6	1.0	14.0	7.3880	156.9213	0.3110	186.39	S, σ_1, σ_2
0.4	1.0	1.1	16.0	9.1980	84.6476	0.4007	189.49	S, σ_1, σ_2
0.5	1.3	1.3	19.0	11.2180	48.1206	0.5058	191.35	S, σ_1, σ_2
1.0	2.6	2.6	33.0	21.3860	7.8234	1.047	191.48	S, σ_1, σ_2

Table 3. Scalar optimization: $a = 1.0$ m and $b = 1.0$ m ($a/b = 1.00$), $W = 0$ (weight as the objective function)

P_x [MN/m]	Optimal thicknesses [mm]			Q_1 [kg]	Q_2 [1/MNm]	$P_{crit\ per}$ [MN/m]	$\sigma_{1,2}$ [MPa]	Active constraints
	h_1	h_2	h_3					
0.1	0.3	0.3	9.0	3.5580	993.5267	0.1082	164.98	S
0.2	0.5	0.6	10.0	5.1580	424.6227	0.2015	180.70	S, σ_1, σ_2
0.3	0.6	1.0	12.0	6.9680	209.7903	0.3146	186.54	S, σ_1, σ_2
0.4	0.9	1.2	13.0	8.5680	126.9637	0.4156	189.67	S, σ_1, σ_2
0.5	0.9	1.7	15.0	10.3780	82.4410	0.5222	191.55	S, σ_1, σ_2
1.0	2.6	2.6	23.0	19.2860	15.1291	1.0051	191.73	S, σ_1, σ_2

Table 4. Scalar optimization: $a = 1.25\,\mathrm{m}$ and $b = 0.8\,\mathrm{m}$ ($a/b = 1.56$), $W = 0$ (weight as the objective function)

P_x [MN/m]	Optimal thicknesses [mm]			Q_1 [kg]	Q_2 [1/MNm]	$P_{crit\ per}$ [MN/m]	$\sigma_{1,2}$ [MPa]	Active constraints
	h_1	h_2	h_3					
0.1	0.3	0.3	7.0	3.1380	1612.5000	0.1043	165.35	S
0.2	0.3	0.3	7.0	3.1380	1612.5000	0.1043	165.35	S
0.3	0.6	1.0	9.0	6.3380	357.8931	0.3127	186.78	S, σ_1, σ_2
0.4	0.9	1.2	10.0	7.9380	205.2616	0.4347	189.86	S, σ_1, σ_2
0.5	1.2	1.4	10.0	9.3280	156.2226	0.5068	191.80	S, σ_1, σ_2
1.0	2.6	2.6	23.0	17.1860	41.2919	1.0056	191.98	S, σ_1, σ_2

The tables below present the results of numerical calculations for the multicriteria optimization for panels with different ratios a/b (Tables 5, 6 and 7 respectively to a/b ratios). These tables show the preferred solutions based on the global criterion in most cases.

Table 5. Multicriteria optimization: $a = 0.8\,\mathrm{m}$ and $b = 1.25\,\mathrm{m}$ ($a/b = 0.64$), (multicriteria preferred solutions)

P_x [MN/m]	W	Optimal thicknesses [mm]			Q_1 [kg]	Q_2 [1/MNm]	$P_{crit\ per}$ [MN/m]	$\sigma_{1,2}$ [MPa]	Active constraints
		h_1	h_2	h_3					
0.1	0.40	0.4	0.4	21.0	6.6340	140.7276	0.3085	122.80	
0.2	0.50	0.6	0.6	29.0	9.4260	49.0379	0.5458	163.96	
0.3	0.50	0.8	0.8	33.0	11.3780	28.2061	0.7043	184.89	σ_1, σ_2
0.4	0.50	1.0	1.1	36.0	13.3980	17.9261	0.8253	188.27	σ_1, σ_2
0.5	0.50	1.3	1.3	38.0	15.2080	12.8392	0.9110	190.41	σ_1, σ_2
1.0	0.50	2.5	2.6	46.0	23.8380	4.2906	1.2839	194.88	σ_1, σ_2

Table 6. Multicriteria optimization: $a = 1.0\,\mathrm{m}$ and $b = 1.0\,\mathrm{m}$ ($a/b = 1.0$) (multicriteria preferred solutions)

P_x [MN/m]	W	Optimal thicknesses [mm]			Q_1 [kg]	Q_2 [1/MNm]	$P_{crit\ per}$ [MN/m]	$\sigma_{1,2}$ [MPa]	Active constraints
		h_1	h_2	h_3					
0.1	0.40	0.4	0.4	21.0	6.6340	140.7276	0.3384	122.80	
0.2	0.50	0.6	0.6	27.0	9.0060	56.4024	0.5856	164.14	
0.3	0.50	0.8	0.8	30.0	10.7480	33.9648	0.8082	185.44	σ_1, σ_2
0.4	0.50	1.0	1.0	32.0	12.5580	22.5279	0.9593	188.51	σ_1, σ_2
0.5	0.50	1.3	1.3	33.0	14.1580	16.8552	1.0691	190.65	σ_1, σ_2
1.0	0.40	2.6	2.6	32.0	21.1760	8.2821	1.2775	191.50	σ_1, σ_2

Table 7. Multicriteria optimization: $a = 1.25\,\mathrm{m}$ and $b = 0.8\,\mathrm{m}$ $(a/b = 1.56)$, (multicriteria preferred solutions)

P_x [MN/m]	W	Optimal thicknesses [mm]			Q_1 [kg]	Q_2 [1/MNm]	$P_{crit\ per}$ [MN/m]	$\sigma_{1,2}$ [MPa]	Active constraints
		h_1	h_2	h_3					
0.1	0.40	0.4	0.4	18.0	6.0040	190.3580	0.2933	123.11	
0.2	0.50	0.5	0.6	24.0	8.0980	78.4161	0.5374	179.15	σ_1, σ_2
0.3	0.50	0.8	0.8	25.0	9.6980	48.4102	0.7594	185.52	σ_1, σ_2
0.4	0.50	1.0	1.1	27.0	11.5080	31.2750	1.0072	188.82	σ_1, σ_2
0.5	0.50	1.3	1.3	27.0	12.8980	24.7600	1.1710	190.95	σ_1, σ_2
1.0	0.50	2.6	2.6	27.0	20.1260	11.3164	1.7357	191.63	σ_1, σ_2

In Tables 2-7 the values of the permissible critical axial compressive loads $P_{crit\ per}$ and the normal stresses in the faces $\sigma_{1,2}$ are shown. In the last column the active constraints are presented. The abbreviation S means that the stability condition is active, the abbreviations σ_1 or σ_2 mean, that the conditions connected with the normal stresses in lower (1) or upper (2) faces are active. In the scalar optimization (Tables 2-4) the stability and stress conditions are active in most cases. In the multicriteria optimization (Tables 5-7) the stress conditions are active.

The solutions of the scalar optimization $(W = 0$, weight as the criterion) and the preferred solutions for different loads P_x for the panel with $a/b = 0.64$, $a/b = 1.00$ and $a/b = 1.56$ are presented in Fig. 5.

From the comparison of the values with the index Pb/Q_1 (load to weight) it follows that this index achieves higher values for $a/b = 0.64$ both in the scalar and vector optimization. A panel with the ratio $a/b = 1.56$ is characterized by lower weight and increased deformability in relation to the panel with the ratio $a/b = 0.64$ and 1.00.

Figures 6 and 7 present the optimal values of the design variables h_1, h_2 and h_3 for the scalar and multicriteria optimization depending on the load P_x for the panel with different ratios a/b. The change in the face thickness, both in the scalar $(W = 0)$ and vector optimization, is approximately a linear function of the load, and this function does not depend on the ratio a/b (Fig. 6). The change in the core thickness depends on the ratio a/b. The change in the core thickness h_3 is a non-linear function of the load, and this relationship has different character in the scalar and vector optimization (see Fig. 7).

The condition of structure stability is of decisive importance to the task of the scalar optimization for $W = 0$ (weight as the optimization criterion). The geometrical constraints, which determine the permissible thickness of the panel layers, are of crucial importance for $W = 1$ (deformability as the optimization

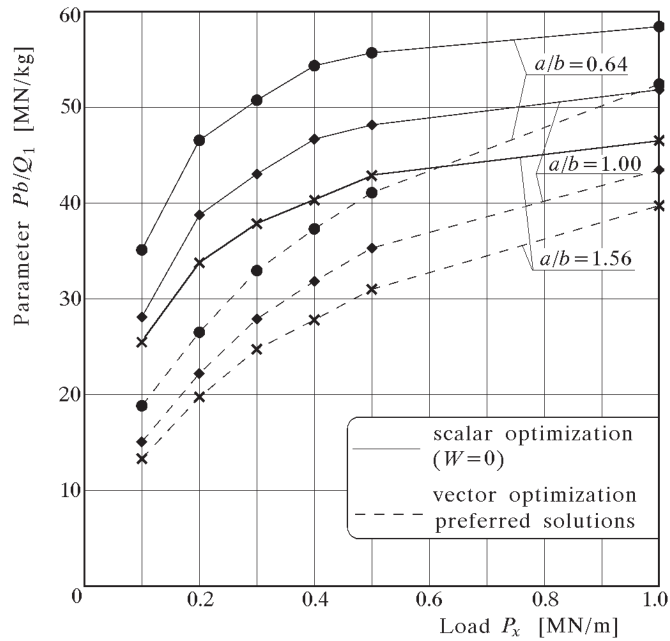

Fig. 5. Comparison of the scalar and preferred solutions for the panel with different ratios a/b

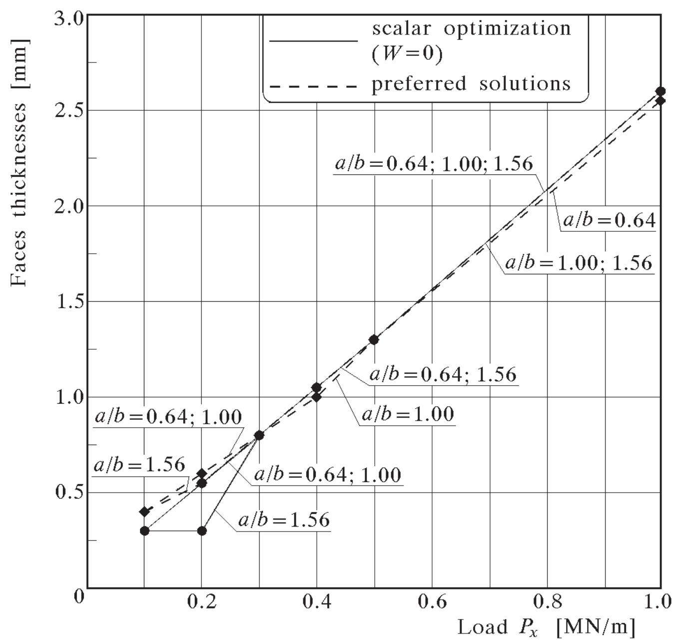

Fig. 6. Change in the face thicknesses for panels with different ratios a/b

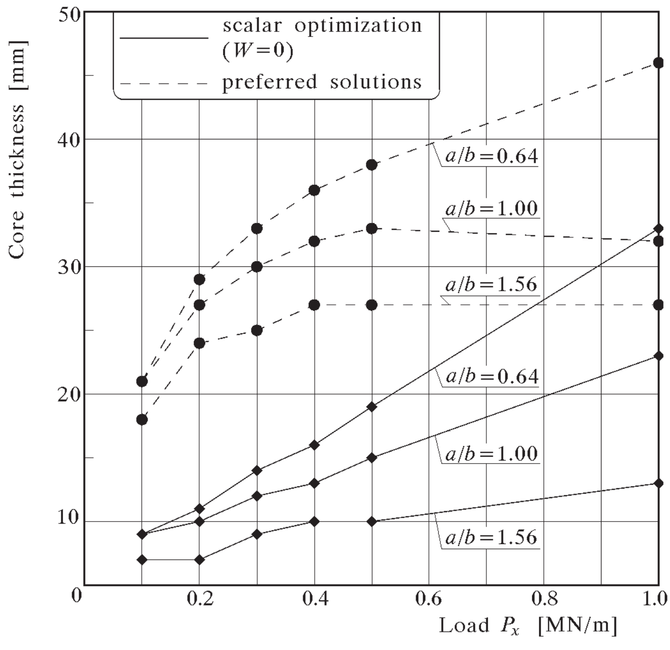

Fig. 7. Change in the core thicknesses for panels with different ratios a/b

criterion). For $W = 0.7$-0.9 the constraint on the thickness h_3 of the core is activated under higher loads. Under loads $P_x \geqslant 0.3\,\mathrm{MN/m}$ the strength conditions are active too. In most cases, the optimal parameters of the panel in the vector optimization are received for $W = 0.5$. It corresponds to equal weights of both optimization criteria.

Sandwich structures have many advantages, and hence they are an attractive option for designers in many practical applications. Among other things, the relation between the weight and load is better than in other structures. In Table 8 the optimization results of unilayer panels, made of the same material as the faces of the sandwich panels, under axial compression are presented. In the two last columns a comparison between the optimal weight and the optimal deformability is shown. The relationships Q_1^U/Q_1^S and Q_2^U/Q_2^S show the superiority of sandwich structures. The results presented here demonstrate that the use of sandwich panels, in comparison with the use of corresponding unilayer panels, results in considerable weight savings in the scalar and multicriteria optimization with preferred solutions. In the case of the scalar optimization with the deformability as the criterion the unilayer panels are lighter than the sandwich panels – these results are connected with the activity of the geometric constraints.

Table 8. Optimal parameters of the unilayer and sandwich cylindrical panel under axial compression (bicriteria optimization)

P [MN]	W	Unilayer panel h [mm]	Q_1^U [kg]	Q_2^U [1/MNm]	Sandwich panel h_1 [mm]	h_2 [mm]	h_3 [mm]	Q_1^S [kg]	Q_2^S [1/MNm]	$\frac{Q_1^U}{Q_1^S}$	$\frac{Q_2^U}{Q_2^S}$
0.1	0	2.8	7.784	19180.8	0.3	0.3	9.0	3.556	993.527	2.189	19.306
	0.5	3.2	8.896	6293.71	0.5	0.5	22.0	7.400	101.843	1.202	61.798
	1.0	6.0	16.668	954.779	3.0	3.0	50.0	27.180	3.059	0.613	312.121
0.2	0	4.1	11.398	3613.55	0.5	0.6	10.0	5.158	424.623	2.210	8.510
	0.5	4.6	12.788	2118.77	0.6	0.6	27.0	9.006	56.402	1.420	27.566
	1.0	6.0	16.668	954.779	3.0	3.0	50.0	27.180	3.059	0.613	312.121
0.3	0	5.0	13.900	1693.20	0.6	1.0	12.0	6.968	209.790	1.995	8.071
	0.5	5.4	15.012	1307.71	0.8	0.8	30.0	10.748	33.968	1.397	38.498
	1.0	6.0	16.668	954.779	3.0	3.0	50.0	27.180	3.059	0.613	312.121
0.4	0	5.7	15.846	1113.95	0.9	1.2	13.0	8.568	126.964	1.849	8.774
	0.5	5.8	16.124	1056.99	1.0	1.1	32.0	12.558	22.528	1.284	46.919
	1.0	6.0	16.668	954.779	3.0	3.0	50.0	27.180	3.059	0.613	312.121

Figure 8 shows the optimal thicknesses of the faces for the unilayer and sandwich panels subject to different axial compressions P_x. Figure 9 shows the weight of the unilayer and sandwich panels. Parameter $W = 0$ refers to the scalar optimization with the weight as the objective function, and $W = 0.5$ refers to the multicriteria preferred solutions.

The presented results of numerical calculations lead to the following conclusions:

- The stability constraint has the decisive meaning in the scalar optimization with the weight as the objective function.
- The geometric constraints have significant influence on the optimal thicknesses of layers.
- The condition $h_3 = 50\,\mathrm{mm}$ is active for $W = 0.7$-0.9, for superior P_x.
- For the axial compression $P_x \geqslant 0.3\,\mathrm{MN/m}$ the strength constraints connected with the normal stresses are active.
- The optimal parameters of the sandwich panels were received for $W = 0.5$ in most cases. It means that both criteria have the same importance.
- The relationship Pb/Q_1 is the greatest for $a/b = 0.64$ (for the scalar and multicriteria optimization). The panels with $a/b = 1.56$ weigh less and

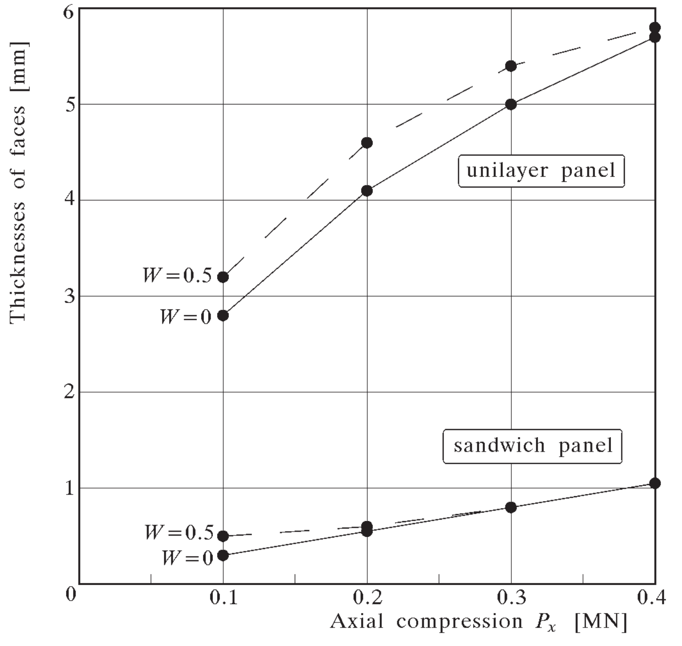

Fig. 8. Thicknesses of the faces for unilayer and sandwich panels

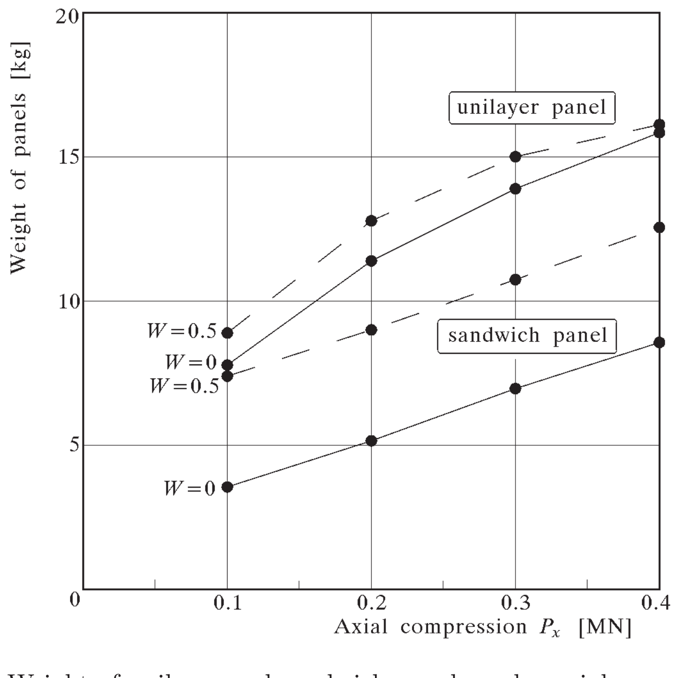

Fig. 9. Weight of unilayer and sandwich panels under axial compression

have greater deformability than the panels with $a/b = 0.64$ and 1.00.

- The thickness of the faces h_1 and h_2, for the scalar and multicriteria optimization, is approximately a linear function of the compressive load and does not depend on the ratio a/b.
- The thickness of the core h_3 depends on the ratio a/b and is a nonlinear function of the compressive load. This relationship is different for the scalar and multicriteria optimization.

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Wielokryterialna optymalizacja trójwarstwowej paneli walcowej przy osiowym ściskaniu

Streszczenie

W pracy sformułowano model dwukryterialnej optymalizacji cienkościennej trójwarstwowej paneli walcowej obciążonej osiowymi siłami ściskającymi. Kryteriami optymalizacyjnymi są masa paneli i jej podatność na odkształcenia. Podatność paneli, zdefiniowana jako odwrotność sztywności na zginanie, jest jakościową miarą odkształcalności konstrukcji. Zmiennymi decyzyjnymi są grubości warstw paneli. Warunkami ograniczającymi są warunek stateczności, warunki wytrzymałościowe, warunek ważności stosowanych modeli teoretycznych oraz warunki technologiczno-konstrukcyjne. Zadanie rozwiązano w oparciu o koncepcję optimum Pareto, uwzględniając dyskretne i ciągłe zbiory zmiennych decyzyjnych. Wyniki obliczeń numerycznych przedstawione są w postaci tabel i wykresów. Porównano optymalne parametry paneli jedno- i trójwarstwowych.

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