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ANALYSIS OF DYNAMIC BEHAVIOUR OF WAVY-PLATES WITH A MEZO-PERIODIC STRUCTURE

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The aim of the contribution is determination of such a form of the mezo-shape function for a mezostructural model, which is suitable to quantitative analysis of dynamic behaviour of a wavy-plate. Governing equations of the averaged theory of wavy-plates, as opposed to those obtained by Michalak et al. (1996), are obtained for different forms of the mezo-shape functions for in-plane and out-of-plane displacements of the plate. The mezo-shape functions of wavy-plates are determined from the solution to the eigenvalue problem of a periodic cell Δ by making use of the finite element method. The comparison of free vibration frequencies obtained from the mezostructural model, the homogenized theory, orthotropic plate model and the finite element method is presented.

Key words: shell, periodic structure, dynamic

1. Introduction

The subject of this paper is determination of mezo-shape functions for a mezostructural model of a periodic shell-like structure, which is referred to as a wavy-plate and dynamic analysis of this structure (Fig. 1). The exact analysis of periodic wavy-plates within the theory of thin elastic shells is too complicated to constitute a basis for investigations of most engineering problems related to those structures. The simplest model of a wavy-plate periodically waved in one direction is the orthotropic plate model e.g. by Troitsky (1976). Investigation of that problem incorporates known asymptotic homogenisation method by Lewiński (1992). However, within the orthotropic plate model and homogenisation theory restricted to the first length scale effect, the effect of periodicity of the cell size on the global response of the wavy-plate becomes negligible. Such approach leads to substructural macrodynamics of wavy-plates, which is based on the modelling approach presented by Woźniak (1997).

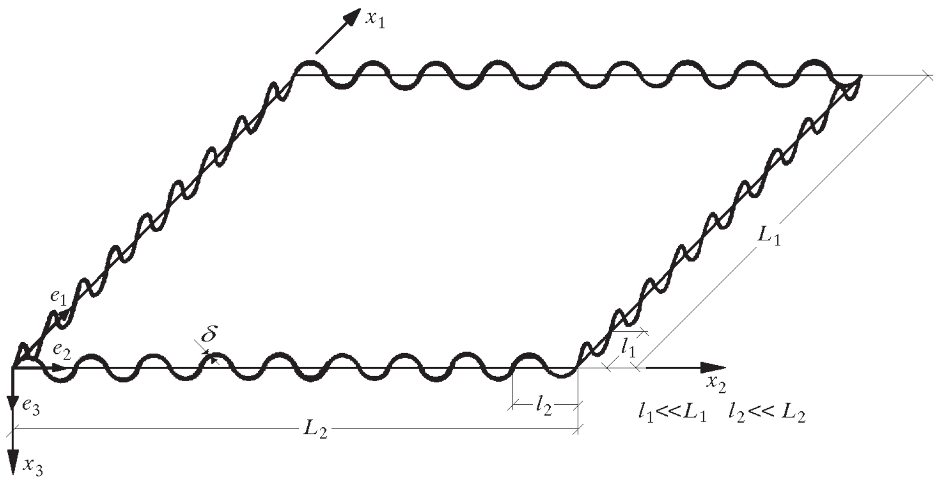

Fig. 1. Scheme of analysed mezo-periodic structure

The model of a wavy-plate is a mezostructural model in which the governing equations of the averaged theory of a wavy-plate depend on the mezostructure length parameter l ($l \equiv \sqrt{(l_1)^2 + (l_2)^2}$, where the wave lengths l_1, l_2 are small enough compared to the minimum characteristic length dimension L of the projection of the wavy-plate on the plane Ox_1x_2. The averaged models of this kind were applied to selected dynamic problems of periodic structures, see e.g. Baron and Woźniak (1995), Cielecka (1995), Jędrysiak (1998), Wierzbicki (1995). The direct description of the wavy-plate is given within the approximated well-known linear theory of thin elastic shells (Green and Zerna, 1954).

The aim of this paper is, first, to obtain the averaged model of a wavy-plate for different forms of local oscillations of in-plane and out-of-plane displacements. Second, to determine such a form of the mezo-shape function which is suitable to quantitative analysis of dynamic behaviour of the wavy-plate. The correctness of the assumed mezo-shape functions is to be verified by comparing free vibration frequencies obtained from the mezostructural model with those found from the homogenized theory, orthotropic plate model and finite element method.

Throughout the paper, the indices $i, j, k, ...$ run over $1, 2, 3$, being related to the orthogonal Cartesian coordinates x_1, x_2, x_3 with the base vectors $\boldsymbol{e}_i$.

The indices $\alpha, \beta, \gamma, \ldots$ run over $1,2$ and are related to the midsurface shell parameters θ^1, θ^2.

2. Modelling approach and choice of mezo-shape functions

The modelling approach to the mezostructural theory of wavy-plates was presented by Michalak eta al. (1996). For the sake of self-consistency, we recall here its key concepts.

The model is based on the well-known, for thin elastic shells, linear approximated theory including its strain-displacement equations, stress-strain relations and equations of motion in the weak form. By $\boldsymbol{u} = u^i(\boldsymbol{x},t)\boldsymbol{e}_i$ we denote the displacement vector field of the wavy-plate midsurface, by $\boldsymbol{p} = p^i(\boldsymbol{x},t)\boldsymbol{e}_i$ the external forces, and by ρ the mass density averaged over a shell thickness with respect to the midsurface. Let the midsurface of the undeformed plate be given by the parametric representation $x^i = R^i(\theta^1,\theta^2)$, where θ^1, θ^2 are the surface parameters. In the sequel, the above parameterisation is defined by $x^1 = \theta^1$, $x^2 = \theta^2$ and $x^3 = z(\theta^1,\theta^2) = z(\theta^1 + l_1, \theta^2) = z(\theta^1, \theta^2 + l_2)$. By $\varDelta \equiv (o, l_1) \times (0, l_2)$, we denote the basic cell of the periodic plate structure on the Ox_1x_2 plane. For an arbitrary integrable function $f(\boldsymbol{z})$ defined on $\varPi$ we denote its averaged value by

$$\langle f \rangle(\boldsymbol{x}) = \frac{1}{l_1 l_2} \int\limits_{\varDelta(x)} f(\boldsymbol{z})\, dz_1 dz_2 \tag{2.1}$$

For a $\varDelta$-periodic function f formula (2.1) yields a constant averaged value.

Kinematics hypothesis

Contrary to Michalak eta al. (1996), we assume in this approach, that the local in-plane and out-of-plane displacement oscillations have different forms. We restrict considerations to the motion, in which the macrodisplacements $U_i(\boldsymbol{x},t) = \langle u_i \rangle(\boldsymbol{x},t)$ describing the averaged motion of the wavy-plate and their derivatives are slow-varying functions of $\boldsymbol{x}$, i.e. can be treated as constant in calculations of the averages $\langle \cdot \rangle(\boldsymbol{x})$. The displacement field $u_i(\boldsymbol{x},t)$ of the wavy-plate we approximate by

$$\begin{aligned} u_\alpha(\boldsymbol{x},t) &= U_\alpha(\boldsymbol{x},t) + h(\boldsymbol{x})V_\alpha(\boldsymbol{x},t) \\ u_3(\boldsymbol{x},t) &= U_3(\boldsymbol{x},t) + g(\boldsymbol{x})V_3(\boldsymbol{x},t) \end{aligned} \qquad \boldsymbol{x} = (x^1,x^2) \in \varPi \qquad t \geqslant 0 \tag{2.2}$$

where $U_i(\cdot,t)$, $V_i(\cdot,t)$ – slow-varying functions (basic unknowns). The functions $h(\cdot)V_\alpha(\cdot,t)$ and $g(\cdot)V_3(\cdot,t)$ describe local displacement oscillations, caused by the mezostructure of the periodic plate. The functions $h(\cdot)$ and $g(\cdot)$ are referred to as the mezo-shape functions and the choice of these functions is obtained as an approximate solution to the eigenvalue problem of a periodic cell Δ together with periodic boundary conditions. These functions are continuous functions defined on R^2, having continuous derivatives of the first and second order. Moreover, values $h(\boldsymbol{x})$ and $g(\boldsymbol{x})$ satisfy the conditions $h(\boldsymbol{x}) \in O(l^2)$, $h_{,\alpha}(\boldsymbol{x}) \in O(l)$, $h_{|\alpha\beta}(\boldsymbol{x}) \in O(l)$, $\langle \rho h \rangle = 0$, $g(\boldsymbol{x}) \in O(l^2)$, $g_{,\alpha}(\boldsymbol{x}) \in O(l)$, $g_{|\alpha\beta}(\boldsymbol{x}) \in O(l)$, $\langle \rho g \rangle = 0$. The choice of these functions will be determined by analysis of free vibrations of the periodic cell Δ with the use of the finite element method.

The form of the mezo-shape functions is obtained as eigenvibration forms of the periodic cell Δ. From numerical calculations of the free vibrations with the use of the finite element method for the periodic cell Δ described by the function $z = f\sin(2\pi x/l)$ (where we have assumed $f/l = 0.1$ and $\delta/l = 0.1$) together with the periodic boundary conditions, the forms of the eigenvibrations are determined. These forms are determined on the nodal displacements of the finite elements and can be approximated by analytical functions: for in-plane vibrations $h = l^2\sin(2\pi x/l)$, for out-of-plane vibrations $g = l^2\sin(4\pi x/l)$.

3. Averaged description: mezo-structural theory (MST)

The macromodelling procedure proposed by Woźniak (1997) and the aforementioned kinematics hypotheses lead from the direct description of the wavy-plate to a system of equations within respect to the macrodisplacements U_i and correctors V_i, constituting the governing equations of the averaged theory of wavy-plates.

The equations of motion presented below in the coordinate form are

$$
\begin{aligned}
&M^{i\alpha\beta}{}_{,\alpha\beta} - M^{i\alpha}{}_{,\alpha} - N^{i\alpha}{}_{,\alpha} + N^i + \langle \widetilde{\rho} \rangle \ddot{U}^i = \widetilde{p}^i \\
&K^\gamma + L^\gamma + \langle \widetilde{\rho} hh \rangle \ddot{V}^\gamma = \langle \widetilde{p}^\gamma h \rangle \\
&K^3 + L^3 + \langle \widetilde{\rho} gg \rangle \ddot{V}^3 = \langle \widetilde{p}^3 g \rangle
\end{aligned}
\tag{3.1}
$$

The constitutive equations have the form

$$
\begin{aligned}
&N^{i\alpha} = D^{i\alpha|j\beta}U_{j,\beta} + H^{i\alpha|\mu}V_{\mu} + H^{i\alpha|3}V_3\\
&N^{i} = D^{i|j\beta}U_{j,\beta} + C^{i|\mu}V_{\mu} + C^{i|3}V_3\\
&K^{\alpha} = H^{\alpha|j\beta}U_{j,\beta} + H^{\alpha|\mu}V_{\mu} + H^{\alpha|3}V_3\\
&K^{3} = H^{3|j\beta}U_{j,\beta} + H^{3|\mu}V_{\mu} + H^{3|3}V_3\\
&M^{i\alpha\beta} = B^{i\alpha\beta|j\gamma\delta}U_{j,\gamma\delta} - B^{i\alpha\beta|j\gamma}U_{j,\gamma} + B^{i\alpha\beta|\mu}V_{\mu} + B^{i\alpha\beta|3}V_3\\
&M^{i|\alpha} = -B^{i\alpha|j\gamma\delta}U_{j,\gamma\delta} + B^{i\alpha|j\gamma}U_{j,\gamma} - B^{i\alpha|\mu}V_{\mu} - B^{i\alpha|3}V_3\\
&L^{\alpha} = B^{\alpha|j\gamma\delta}U_{j,\gamma\delta} - B^{\alpha|j\tau}U_{j,\tau} + B^{\alpha|\mu}V_{\mu} + B^{\alpha|3}V_3\\
&L^{3} = B^{3|j\gamma\delta}U_{j,\gamma\delta} - B^{3|j\tau}U_{j,\tau} + B^{3|\mu}V_{\mu} + B^{3|3}V_3
\end{aligned}
\tag{3.2}
$$

where we have denoted

$$
\begin{aligned}
&D^{i\alpha|j\beta} \equiv D\Big\langle H^{\delta\alpha\gamma\beta}G^{i}{}_{\delta}G^{j}{}_{\gamma}\sqrt{a}\Big\rangle\\
&H^{i\alpha|\mu} = H^{\mu|i\alpha} \equiv D\Big\langle H^{\delta\alpha\gamma\beta}G^{i}{}_{\delta}G^{\mu}{}_{\gamma}h_{,\beta}\sqrt{a}\Big\rangle\\
&H^{i\alpha|3} = H^{3|i\alpha} \equiv D\Big\langle H^{\delta\alpha\gamma\beta}G^{i}{}_{\delta}G^{3}{}_{\gamma}g_{,\beta}\sqrt{a}\Big\rangle\\
&D^{i|j\beta} \equiv D\Big\langle H^{\alpha\delta\gamma\beta}\left\{ {\lambda \atop \alpha\delta} \right\}G^{i}{}_{\lambda}G^{j}{}_{\gamma}\sqrt{a}\Big\rangle\\
&C^{i|\mu} \equiv D\Big\langle H^{\alpha\beta\gamma\delta}\left\{ {\lambda \atop \alpha\beta} \right\}G^{i}{}_{\lambda}G^{\mu}{}_{\gamma}h_{,\delta}\sqrt{a}\Big\rangle\\
&C^{i|3} \equiv D\Big\langle H^{\alpha\beta\gamma\delta}\left\{ {\lambda \atop \alpha\beta} \right\}G^{i}{}_{\lambda}G^{3}{}_{\gamma}g_{,\delta}\sqrt{a}\Big\rangle\\
&H^{\alpha|\mu} \equiv D\Big\langle H^{\tau\beta\gamma\delta}G^{\alpha}{}_{\tau}G^{\mu}{}_{\gamma}h_{,\delta}h_{,\beta}\sqrt{a}\Big\rangle\\
&H^{\alpha|3} = H_{3|\alpha} \equiv D\Big\langle H^{\tau\beta\gamma\delta}G^{\alpha}{}_{\tau}G^{3}{}_{\gamma}g_{,\delta}h_{,\beta}\sqrt{a}\Big\rangle\\
&B^{i\alpha\beta|j\gamma\delta} \equiv B\Big\langle H^{\alpha\beta\gamma\delta}n^{i}n^{j}\sqrt{a}\Big\rangle\\
&B^{i\alpha\beta|\mu} = B^{\mu|i\alpha\beta} \equiv B\Big\langle H^{\alpha\beta\gamma\delta}h_{|\gamma\delta}n^{i}n^{\mu}\sqrt{a}\Big\rangle
\end{aligned}
$$

$$B^{i\alpha\beta|3} = B^{3|i\alpha\beta} \equiv B\Big\langle H^{\alpha\beta\gamma\delta} g_{|\gamma\delta} n^i n^3 \sqrt{a}\Big\rangle$$

$$B^{i\alpha\beta|j\gamma} = B^{j\gamma|i\alpha\beta} \equiv B\Big\langle H^{\alpha\beta\mu\delta} \left\{ {\gamma \atop \mu\delta} \right\} n^i n^j \sqrt{a}\Big\rangle$$

$$B^{i\alpha|j\gamma} \equiv B\Big\langle H^{\mu\delta\tau\nu} \left\{ {\alpha \atop \mu\delta} \right\} \left\{ {\gamma \atop \tau\nu} \right\} n^i n^j \sqrt{a}\Big\rangle$$

$$B^{i\alpha|\mu} = B^{\mu|i\alpha} \equiv B\Big\langle H^{\beta\tau\gamma\delta} \left\{ {\alpha \atop \beta\tau} \right\} n^i n^\mu h_{|\gamma\delta} \sqrt{a}\Big\rangle$$

$$B^{i\alpha|3} = b^{3|i\alpha} \equiv B\Big\langle H^{\beta\tau\gamma\delta} \left\{ {\alpha \atop \beta\tau} \right\} n^i n^3 g_{|\gamma\delta} \sqrt{a}\Big\rangle$$

$$B^{\alpha|\mu} \equiv B\Big\langle H^{\gamma\beta\tau\delta} h_{|\gamma\beta} h_{|\tau\delta} n^\alpha n^\mu \sqrt{a}\Big\rangle$$

$$B^{\alpha|3} = B^{3|\alpha} \equiv B\Big\langle H^{\gamma\beta\tau\delta} h_{|\gamma\beta} g_{|\tau\delta} n^\alpha n^3 \sqrt{a}\Big\rangle$$

$$B^{3|3} \equiv B\Big\langle H^{\alpha\beta\gamma\delta} g_{|\alpha\beta} g_{|\gamma\delta} n^3 n^3 \sqrt{a}\Big\rangle$$

The above equations (3.1) and (3.2) represent a system of 9 differential equations for the 3 macro- displacements U_i and 3 internal variables V_i.

4. Applications

To compare the mezo-structural theory (MST), homogenized theory (HT), orthotropic plate model and finite element method we shall investigate a simple problem of cylindrical bending of a rectangular wavy-plate (Fig. 2). In this case the basic unknowns U_i and V_i depend only on the arguments x_2 and t.

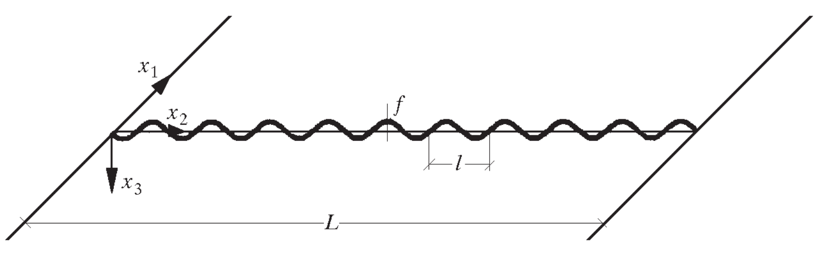

Fig. 2. Simply supported wavy-plate

4.1. Mezo-structural theory

In this case, neglecting an external loading, the system of equations of motion will take the form

$$\begin{aligned}
&M^{122}{}_{,22} - M^{12}{}_{,2} - N^{12}{}_{,2} + N^1 + \langle\widetilde{\rho}\rangle\ddot{U}^1 = 0\\
&M^{222}{}_{,22} - M^{22}{}_{,2} - N^{22}{}_{,2} + N^2 + \langle\widetilde{\rho}\rangle\ddot{U}^2 = 0\\
&M^{322}{}_{,22} - M^{32}{}_{,2} - N^{32}{}_{,2} + N^3 + \langle\widetilde{\rho}\rangle\ddot{U}^3 = 0\\
&K^1 + L^1 + \langle\widetilde{\rho}hh\rangle\ddot{V}^1 = 0\\
&K^2 + L^2 + \langle\widetilde{\rho}hh\rangle\ddot{V}^2 = 0\\
&K^3 + L^3 + \langle\widetilde{\rho}gg\rangle\ddot{V}^3 = 0
\end{aligned} \tag{4.1}$$

After substituting the right-hand sides of Eqs (3.2) into Eqs (4.1), we obtain a system of equations for $U_i = U_i(\boldsymbol{x}, t)$ and $V_i = V_i(\boldsymbol{x}, t)$.

Let the wavy-plate midsurface be defined by $z = f\sin(2\pi x_2/l)$ and the mezo-shape functions (obtained in Section 2) by $h = l^2\sin(2\pi x_2/l)$, $g = l^2\sin(2\pi x_2/l)$. Let us restrict the considerations to analysis of free vibrations of an unbounded wavy-plate. In this case, we shall look for solutions to Eqs (4.1) in the form

$$\begin{aligned}
&U_1 = 0 && V_1 = 0\\
&U_2 = A_2\sin(kx_2)\cos(\omega_2 t) && V_2 = C_2\cos(kx_2)\cos(\omega t)\\
&U_3 = A_3\sin(kx_2)\cos(\omega t) && V_3 = C_3\cos(kx_2)\cos(\omega t)
\end{aligned} \tag{4.2}$$

where $k := \pi/L$ is the wavenumber, L – vibration wavelength $(L \ll l)$. Substituting the right-hand sides of Eqs (4.2) into Eqs (4.1), we obtain non-trivial solutions if only

$$\begin{vmatrix}
\omega^2\langle\widetilde{\rho}\rangle - C_{33} & C_{35} & C_{36}\\
C_{53} & \omega^2\langle\widetilde{\rho}hh\rangle - C_{55} & C_{56}\\
C_{63} & C_{65} & \omega^2\langle\widetilde{\rho}gg\rangle - C_{66}
\end{vmatrix} = 0 \tag{4.3}$$

where we have denoted

$$\begin{aligned}
C_{33} \equiv{}& B\Big\langle H^{2222}(n^3)^2\sqrt{a}\Big\rangle k^4 +\\
&+\Big[B\Big\langle H^{2222}\Big(\Big\{ {2 \atop 22} \Big\} n^3\Big)^2\sqrt{a}\Big\rangle + D\Big\langle H^{2222}(G^3{}_2)^2\sqrt{a}\Big\rangle\Big]k^2
\end{aligned}$$

$$C_{35} = C_{53} \equiv \Big[B\Big\langle H^{2222} \left\{ {}^{\,2}_{22} \right\} n^2 n^3 h_{,22}\sqrt{a}\Big\rangle - D\Big\langle H^{2222} G^2{}_2 G^3{}_2 h_{,2}\sqrt{a}\Big\rangle\Big]k$$

$$C_{36} = C_{63} \equiv \Big[B\Big\langle H^{2222} \left\{ {}^{\,2}_{22} \right\} (n^3)^2 g_{,22}\sqrt{a}\Big\rangle - D\Big\langle H^{2222} (G^3{}_2)^2 g_{,2}\sqrt{a}\Big\rangle\Big]k$$

$$C_{55} \equiv D\Big\langle H^{2222}(G^2{}_2 h_{,2})^2\sqrt{a}\Big\rangle + B\Big\langle H^{2222}(n^2 h_{,22})^2\sqrt{a}\Big\rangle k \tag{4.4}$$

$$C_{56} = C_{65} \equiv -B\Big\langle H^{2222} n^2 n^3 h_{,22} g_{,22}\sqrt{a}\Big\rangle - D\Big\langle H^{2222} G^2{}_2 G^3{}_2 h_{,2} g_{,2}\sqrt{a}\Big\rangle$$

$$C_{66} \equiv D\Big\langle H^{2222}(G^3{}_2 g_{,2})^2\sqrt{a}\Big\rangle + B\Big\langle H^{2222}(n^3 g_{,22})^2\sqrt{a}\Big\rangle$$

Let the amplitude of the shell midsurface is equal $f = l/10$. In this case, formulae (4.4), for the constant thickness δ and with the notation $\lambda := \delta/l$, $\gamma := \alpha l$, yield

$$C_{33} = \frac{E}{\delta(1-\nu^2)}(0.05641174\lambda^4\gamma^4 + 0.031026495\lambda^4\gamma^2 + 0.13539982\lambda^2\gamma^2)$$

$$C_{35} = \frac{E}{\delta(1-\nu^2)}(0.36019325\lambda^2\gamma + 1.35399818\gamma)$$

$$C_{36} = \frac{E}{\delta(1-\nu^2)}(4.58612251\lambda^2\gamma - 0.74263257\gamma)$$

$$C_{55} = \frac{E\delta^3}{1-\nu^2}\Big(4.21014436 + 13.53998280\frac{1}{\lambda^2}\Big) \tag{4.5}$$

$$C_{56} = \frac{E\delta^3}{1-\nu^2}\Big(53.60521952 - 7.42632580\frac{1}{\lambda^2}\Big)$$

$$C_{66} = \frac{E\delta^3}{1-\nu^2}\Big(682.522787 + 10.08234978\frac{1}{\lambda^2}\Big)$$

From Eqs (4.3) we conclude that for the above form of vibrations we have three free vibration frequencies: the lower vibration frequency ω_1 and two higher one ω_2, ω_3 (which can be called the mezo-resonance frequencies) caused by the mezo-periodic structure of the wavy-plate.

4.2. Homogenized theory (HT)

The homogenized model of dynamics of the wavy-plate can be derived from Eqs (4.1)-(4.5) by the asymptotic approximation in which the mezostructure of the wavy-plate is scaled down $l \to 0$. Keeping in mind that $\delta/l = \text{const}$, we shall neglect the mezoinertial terms $\langle\widetilde{\rho}hh\rangle \to 0$, $\langle\widetilde{\rho}gg\rangle \to 0$, and we can eliminate the correctors V_i in Eqs (4.1). Now, formula (4.3) leads to

$$\langle\widetilde{\rho}\rangle\omega^2 = C_{33} - \frac{C_{66}(C_{35})^2 + C_{55}(C_{36})^2 + 2C_{35}C_{36}C_{56}}{C_{55}C_{66} - (C_{56})^2} \tag{4.6}$$

where ω is the lower free vibration frequency.

4.3. Orthotropic plate model

Let us restrict the considerations to analysis of transverse vibrations of orthotropic plates. In this case, the equation of motion has the form (Troitsky, 1976)

$$B_{22}U_{3,2222} + \frac{\partial^2}{\partial t^2}(\widetilde{\rho}U^3 - J_1 U_{3,22}) = 0 \tag{4.7}$$

where

$$B_{22} = B\frac{1}{1 + \left(\pi\frac{f}{l}\right)^2} \qquad\qquad J_1 = \frac{1}{l}\int\limits_s (z^2 + x^2)\rho\, ds \tag{4.8}$$

We shall look for a solution to Eqs (4.7) in the form $U_3 = A_3\sin(kx_2)\cos\omega t)$. For the free vibration frequency, we obtain the following expression

$$\omega^2 = \frac{B_{22}k^4}{\widetilde{\rho} + J_1 k^2} \tag{4.9}$$

4.4. Finite element method

Now we shall look for a solution to free vibrations of a simply supported wavy-plate with the use of the finite element method. The span of the wavy-plate is equal $L = 10l = 10.0\,\text{m}$, where l is the mezostructure length parameter (length of the periodic cell). From the solution obtained by making use of the finite element method we have many free vibration frequencies for the corresponding form of the eigenvibrations. In Table 1 free vibration frequencies correspnding to the eigenforms approximated by the function $U_3 = A\sin(kx_2)$, where $k = \pi/10l$, see Eqs (4.2), are presented.

We have analysed free vibration frequencies for the above-mentioned models of the wavy-plates. In Table 1, the free vibration frequencies versus δ/l ratio found from the mezostructural theory, homogenized theory, orthotropic plate model and finite element method, are shown, where the amplitude of the wave is assumed as $f = l/10$. Table 2 presents the free vibration frequencies versus f/l ratio, where the thickness of wavy-plate is assumed as *delta* $= l/10$. The free vibration frequencies in Table 1 and Table 2 are determined for $l = 1.0\,\mathrm{m}$, $E = 210\,\mathrm{GPa}$ and $\rho = 785\,\mathrm{kg/m^2}$.

Table 1. Free vibration frequencies versus ratio δ/l, $f/l = 1/10 =$ const

ω [1/s]	δ/l	1/10	1/25	1/50	1/100
MST	ω_1	12.800	5.263	2.650	1.349
	ω_2	$21.221 \cdot 10^3$	$16.420 \cdot 10^3$	$15.467 \cdot 10^3$	$15.214 \cdot 10^3$
	ω_3	$34.954 \cdot 10^3$	$33.049 \cdot 10^3$	$32.837 \cdot 10^3$	$32.787 \cdot 10^3$
HT	ω	12.803	5.265	2.654	1.339
Orth.	ω	14.050	5.620	2.810	1.405
FEM	ω	13.925	5.570	2.785	1.392
MST/FEM	ω_1/ω	91.9%	94.4%	95.0%	96.9%

Table 2. Free vibration frequencies versus ratio f/l, $\delta/l = 1/100 =$ const

ω [1/s]	f/l	1/20	1/10	1/7
MST	ω_1	1.444	1.349	1.806
	ω_2	$9.977 \cdot 10^3$	$15.214 \cdot 10^3$	$16.283 \cdot 10^3$
	ω_3	$33.685 \cdot 10^3$	$32.787 \cdot 10^3$	$31.814 \cdot 10^3$
HT	ω	1.441	1.339	1.813
Orth.	ω	1.502	1.405	1.294
FEM	ω	1.486	1.392	1.302
MST/FEM	ω_1/ω	97.2%	96.9%	138.7%

5. Conclusions

In this paper we have applied the modelling approach, which leads to the length-scale model, and is different from that known from the homogenized theory, orthotropic plate theory and finite element method, because it takes into account the effect of the mezo-structure size on dynamic behaviour of the plate. The mezo-structural model is based on the assumption that the displacements of a periodic wavy-plate can be described by slowly varying macro-displacements, on which the oscillations are superimposed as a sum of products of mezo-shape functions and internal variables. The internal variables are assumed to be slowly varying functions, and they play the role of unknown amplitudes for these oscillations. The mezo-shape functions have been derived as eigenvibration forms of a periodic cell Δ. In this paper the choice of these functions has been determined by analysis of free vibrations of the periodic cell Δ with the use of the finite element method. In Section 4, different models of the wavy-plate have been discussed. On the basis of the results, we can formulate the following conclusions:

- Free vibration frequencies can be successfully applied for determination of the form of the mezo-shape functions.
- Analysing the results presented in Table 1, we can observe that the assumed approximate form of the mezo-shape functions $h = l^2 \sin(2\pi x/l)$, $g = l^2 \sin(4\pi x/l)$ well describes dynamic behaviour of the wavy-plates for different ratios δ/l and wavy amplitude $f \leqslant l/10$.
- Analysing the results in Table 2, for the plates with wave amplitudes $f > l/10$, we conclude differences between the values of free vibration frequencies for the mezo-structural theory and finite element method, because the assumed form of the mezo-shape functions describes the free vibration frequencies for the periodic cell Δ only in an approximate way. For the wavy-plate with amplitudes $f > l/10$ the form of the eigenvibrations of the periodic cell Δ should be described by more accurate functions.
- Only the mezo-structural model gives us lower and higher free vibration frequencies for the assumed vibration form of the wavy-plate.

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Analiza dynamicznych zachowań płyty pofałdowanej

Streszczenie

Celem pracy było znalezienie takich postaci funkcji mezo-kształtu, które dawałyby poprawne wyniki w analizie ilościowej zachowań dynamicznych płyty pofałdowanej. Uśrednione równania opisujące płytę pofałdowaną w przeciwieństwie do przedstawionych w Michalak i inni (1996) są otrzymane dla różnych postaci oscylacji przemieszczeń w płaszczyźnie płyty i w kierunku prostopadłym do płaszczyzny środkowej. Funkcje mezo-kształtu zostały określone w wyniku rozwiązania, przy pomocy metody elementów skończonych, problemu drgań własnych komórki periodyczności Δ. Porównano częstości drgań własnych otrzymane z modelu mezostrukturalnego, teorii homogenizacji, modelu płyty ortotropowej i z metody elementów skończonych.

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