

## ON HEATING PROBLEMS OF FRICTION

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The paper presents a brief survey of literature on heating problems of friction. It examines stationary, quasi-stationary and nonstationary problems.

*Key words:* friction contact problem, heat generation, temperature, thermoelasticity

### 1. Stationary problem

In recent years great attention has been paid to the analysis of contact problems with heat generation. Mechanical energy is transformed into thermal energy whenever friction occurs. This frictional heating is responsible for increase of temperature at the contact interface of contacting couples, which has a considerable influence on the tribological behavior. Therefore, the problem of frictional heat is of great theoretical and practical interest to investigators.

In the formulations given by Jaeger (1942) and Blok (1963) the heating problems of friction consist in finding of the temperature field caused by heat fluxes with given distributions of intensities. It was assumed that the heat flux is in agreement with the distribution of shear stresses and is proportional to the friction forces (Ling, 1973). Next, the determination of shear and normal stresses is reduced to application of Amonton's law (Galini, 1953). The distribution of normal stresses was determined from an appropriate isothermal

problem. In most cases uniform or elliptical distributions obtained from the solutions to Hertz's contact problems (Hertz, 1985) are considered.

The heating problems of friction are examined in stationary, quasi-stationary and nonstationary formulations. If the velocity of slip is small so that the convection caused by the motion does not change the temperature and heat fluxes, as well as the process of heat conduction for given external conditions lasts long enough so that the influence of initial conditions can be ignored, then the thermal contact can be assumed to be stationary. Moreover, it is supposed that the significant part of the heat is transferred into inside of the contacting bodies, so their free surfaces can be treated as adiabatic. This case often occurs, when the Reynolds parameter of the air stream streamlined bodies is smaller and when the Peckle parameter is greater. The assumption connected with the adiabatic free surface of the contacting bodies leads to considerable simplifications in the formulations of the boundary value problems of heat conduction. A review of papers concerning stationary heating problems of friction was given by Korovchinskii (1966, 1968). This author solved also axisymmetrical problems for the stationary condition of friction in the case of torsion and shear of a half-space. He assumed that the half-space was heated by heat fluxes with uniform distribution or with the intensity proportional to Hertzian pressure. The range of variation for the Biot parameters was also determined. It was found, that for maximal values of the Biot parameter of order 0.1 which can take place during a frictional contact forced by the air, the reduction of maximal value of temperature did not exceed 8% in comparison with the maximal value of temperature under the assumption of thermal isolation of the free surface. In a greater part of practical cases the Biot number does not exceed 0.02 and adequate increasing of the maximal temperature is not greater than 2-3% in comparison with the case without giving up the heat. It can be also noticed that the influence of neglecting up the heat on the frictional temperature is greater for the stationary case than for quasi-stationary or nonstationary cases.

## 2. Quasi-stationary problem

A quasi-stationary thermal contact takes place under the condition of sufficiently long duration of friction between bodies in motion. The slip with a constant velocity  $V$  of one body on another, completely thermo-insulated, with heat generation and its transfer through the contact zone becomes one of the classical problems. The shape of the contact zone is a priori unknown. Ho-

wever, as a rule, the contact zone is assumed in the form of a narrow strip or of a bounded and closed region  $\Omega$ . If the characteristic dimension (for example, radius of curvature) of the contacting bodies is considerably greater than the characteristic dimension of the contact zone (for instance, radius) then each contacting body can be treated as a half-space. The quasi-stationary thermal state of the half-space, heated in the region  $\Omega$  on the surface  $z = 0$  by frictional heat fluxes with the intensity  $q(x, y) = fVp(x, y)$ ,  $(x, y) \in \Omega$  (where  $f$  is the friction coefficient,  $p = p(x, y)$  – pressure) and moving with the velocity  $V$  in the positive direction of the axis  $0x$  is governed by the parabolic differential equation of heat conduction

$$\nabla^2 T = \frac{V}{k} \frac{\partial T}{\partial x} \quad (2.1)$$

with the conditions

$$k \frac{\partial T}{\partial z} \Big|_{z=0} = \begin{cases} -q(x, y) & \text{for } (x, y) \in \Omega \\ hT & \text{for } (x, y) \notin \Omega \end{cases} \quad (2.2)$$

and

$$T \rightarrow 0 \quad \text{for } \sqrt{x^2 + y^2 + z^2} \rightarrow \infty \quad (2.3)$$

where

- $\nabla^2$  – Laplace operator in the cartesian coordinate system  $(x, y, z)$
- $T$  – temperature
- $h$  – coefficient of the heat exchange
- $K$  – coefficient of the thermal conductivity.

For a thermally insulated free surface  $h = 0$ , the solution to the quasi-stationary problem of the heat conduction defined by (2.1)-(2.3) for the contact zone in the form of  $\Omega = \{(x, y), |x| \leq a, |y| < \infty\}$  or in the form of the rectangle  $\Omega = \{(x, y), |x| \leq a, |y| \leq \infty\}$  are presented by Jaeger (1942), and in the form of a circle of the radius  $a$  are given by Rykalin (1941). In the case of convective heat conduction ( $h \neq 0$ ) with free surfaces of the frictional bodies, the approximate solutions for the linear distribution of heat fluxes were obtained by Kolyano and Didyk (1977) by using the method of averaged characteristic and by Yevtushenko and Ukhanska (1992) by using the method of compensation. Analogous problems of thermoelasticity were solved in papers by Heekman and Burton (1980), Barber (1984), Podstrigach and Kolyano (1972). The solutions to quasi-stationary problems of thermoelasticity for moving linear thermal sources were presented by using Fourier's integral transforms by Bryant (1988) and Yevtushenko and Ukhanska (1994). The papers by Matysiak and Ukhanska (1997a,b) were devoted to solutions to the

heat conduction problems of a periodic convective composite half-space with a moving heat source (periodically layered body and a body composed of a matrix and unidirectional periodically distributed fibers with identical rectangular cross-sections). The problems of heat exchange during fast slip between bodies in contact belong to a group among the practical cases of frictional heat generation. In the paper by Mow and Cheng (1967) the authors ascertained that „a fast motion conforms to the state, when the velocity of motion  $V$  of a thermal source is considerably greater than the ratio of the coefficient of thermal diffusivity and a characteristic dimension of heated region  $\Omega$ ”, so it can be written as

$$Pe = \frac{Va}{k} \gg 1$$

where  $Pe$  is the Peckle parameter. It was obtained by Blok (1940), that for the condition  $Pe > 10$  the gradients of thermal fluxes in the direction  $Ox$  of motion and in the perpendicular direction  $Oy$  are considerably smaller than in the direction of heating  $Oz$ , thus the equation of heat conduction (2.1) can be approximated by

$$\frac{\partial^2 T}{\partial z^2} = \frac{V}{k} \frac{\partial T}{\partial x} \quad (2.4)$$

The solution to equation (2.4) with boundary conditions (2.2), (2.3) for a linear distribution of the heat source was given by Ling and Jang (1971) and the corresponding problems of thermoelasticity were solved by Ling and Mow (1965), Barber and Comninou (1989). The theory of fast moving during frictional heating was significantly developed with great attention paid to technical applications. It is worth to mention here the papers by Matysiak et al. (1998), Yevtushenko and Yevtushenko (1999), in which the temperature field in metals caused by cold rolling was investigated. Gesim and Winer (1984), Yevtushenko et al. (1996) solved the problems connected with fast rotating cylinders, Knothe and Liebelt (1995), Yevtushenko and Semerak (1999) examined the temperature generated during friction of wheels on rails.

### 3. Nonstationary problem

A nonstationary thermal contact is conditioned by a nonstationary distribution of the contact pressure or by a nonstationary velocity of the slip as well as by the fact that the development of the frictional process (heating process) is considered from some initial time. The frictional processes during braking are nonstationary and of short duration. Krachenskii and Chinchinadze (see

for references Chinchinadze (1967), Chinchinadze et al. (1979)) formulated a criterion for evaluation of the friction and wear of the contacting couples, in which the principal role plays the temperature of friction. It enabled determination of the frictional thermal strength of materials and capacity of frictional couples in accordance with the specified values of frictional coefficient and wear intensity.

The experimental methods of investigating materials in terms of the frictional thermal strength during braking can be used to find the mean temperature of the frictional surface, temperature flash, volumetric temperature, distribution of heat fluxes, effective depth of overheating (see Chinchinadze et al., 1979). In particular, the following models for calculation of temperature regimes in brakes are presented in the monograph by Chinchinadze et al. (1979):

- maximum temperature in the contact zone is given in the form of sum of the temperature flash and the mean temperature on the nominal friction surface (or contour of it) caused by a uniform heat flux on this surface
- for calculation of the temperature flash, motion of a thin rod on the surface of a smooth half-space is considered
- the average temperature is determined from the solution to a one-dimensional nonstationary thermal problem of friction disregarding thermal exchange in the braking process.

The other method for solving the problems of friction heating during braking was demonstrated by Rowson (1978). The author analysed an axisymmetrical nonstationary problem of heat conduction for a half-space heated in a circle by the heat flux with intensity

$$q(r, t) = fV(t)p$$

where

$$V(t) = V_0 \left(1 - \frac{t}{t_s}\right) \quad 0 \leq t \leq t_s$$

and  $V(t)$  is the velocity of braking,  $t$  is time,  $t_s$  is the time to the full stop. This approach was applied in papers by Yevtushenko and Ivanyk in order to find the temperature in the center of the heating region. Moreover, in those papers the temperature field and quasi-static stresses were given in the cases of Hertzian distribution of the contact pressure, see Yevtushenko and Ivanyk (1995a,b,c, 1997a), quasi-Hertzian (in which roughness of the friction surface is taken into account), see Yevtushenko et al. (1996), and on the assumption

of exponential dependence on time, see Yevtushenko and Ivanyk (1996, 1999), Yevtushenko et al. (1997, 1999).

It can be underlined that the determination of frictional temperature in a contact region with fixed dimensions was obtained experimentally or by empirical formulae. The influence of thermal deformations on the friction temperature is considered in papers by Yevtushenko and Chapovska (1995, 1997).

The nonstationary heating problems of friction were applied to investigations of thermal processes on separated protrusions of micro-irregularities of surfaces for a nominal contact zone. It is known that the heat exchange during friction takes place on small pieces of the contact zone, which change their shapes during the friction process. The character of these changes is determined by the temperature, velocity, load and the region of the real contact. If the protrusions of the micro-irregularities are sufficiently distant one from another (if the distance between the two adjacent irregularities is of order greater than the dimensions of the irregularities), then they can be considered separately by using the solutions to the axisymmetrical nonstationary problem of heat transfer for a half-space with a circular source of small radius acting on the boundary, see Ting and Winer (1989). The problem can be investigated with regard to convective cooling of free surfaces, see Yevtushenko and Ivanyk (1997b), or without cooling, see Yevtushenko and Ukhanska (1996), Yevtushenko and Kulchytsky-Zhyhailo (1997). Experimental registrations of typical durations of temperature processes during heating and cooling of the contact micro-irregularities reveal their short duration, which is of order 0.1-1 ms. For this reason the calculation model for determination of the temperature of separated micro-irregularities should include the solution to the auxiliary problem of pure convective cooling of the body excluding the heat source. In this problem, the distribution of temperature obtained for the nonstationary problem of heat conduction for the half-space with the circular heat source is given as the initial distribution of temperature. In the cases when the thermal interactions between the protrusions cannot be omitted, the analysis of heating processes is rather complicated. For calculation of transient processes the dimensions and distributions of micro-potrusions for an arbitrary time must be known. These data can be described approximately by profile measurements, see Bouden and Tabor (1964), Rudzit (1975). In papers by Wisniewski (1986), Greenwood and Williamson (1966), for calculation of nonstationary temperature of friction surfaces the model based on the random processes was presented, in which the contact region was divided on parts characterized by small areas.

Papers by Bogdanovich and Belov (1992), Bogdanovich et al. (1993) gave

some new experimental results which were connected with the dynamics of temperature fields on material surfaces with regular topography. There are three different heating regimes of contact regions (stationary, transient and dynamic), describing the character of temperature evolutions. Present investigations are being carried out on the grounds of the formulations of mathematical models for calculation of temperature fields corresponding to the regimes. The generation and heat transfer during friction are dependent on the geometry and characteristics of the contacting bodies, their mechanical and thermophysical properties, regime of working of the kinematic frictional pair. The generated frictional heat in the contact region in the form of heat fluxes is distributed to the contacting bodies. For the analysis of heat fluxes in the neighborhood of the contact zone, the coefficient of heat flux  $\gamma$  is applied. For example, if the body 1 undergoes the heat flux  $q_1 = \gamma q$ , then the second body is exposed to  $q_2 = (1 - \gamma)q$ , so that  $q = q_1 + q_2$ . The first determination of the coefficients of distribution of heat fluxes was given by Blok (1937), who considered the slip of single protrusions in the form of circles, squares or lateral surfaces of a cylinder on the surface of a half-space. The dimension of the contact regions is sufficiently small in comparison with the dimension of the contacting bodies, thus the bodies can be considered as half-spaces. The intensity of heat generation is assumed to be independent of time, and it is distributed on the width of the contact zone according to the law of contact pressure. If the body 1 is immovable and the body 2 is uniformly slipping with a small velocity ( $P_e \leq 0.32$ ) the value of  $\gamma$  is defined by Blok (1940) in the form

$$\gamma = \frac{K_1}{K_1 + K_2} \quad (3.1)$$

where  $K_i$ ,  $i = 1, 2$  is the coefficient of thermal conductivity of the  $i$ th body. In the case of the lateral surface of the cylinder and a great velocity ( $P_e \geq 10$ ), Blok (1940) gives the result

$$\gamma = \frac{K_1 \sqrt{\pi}}{K_1 \sqrt{\pi} + K_2 \sqrt{\frac{16}{P_e}}} \quad (3.2)$$

Comparing the averaged temperature on the contact surface, Jaeger (1942) found that

$$\gamma = \frac{1.75 K_1}{1.75 K_1 + K_2 \sqrt{P_e}} \quad (3.3)$$

From relation (3.3) it follows that an increase in the velocity leads to a decrease in the distribution coefficient of the heat flux  $\gamma$  and to diminishing of  $q_1$ .

It should be emphasised that the wide application of new antifrictional materials to mechanical systems makes the investigations of problems of friction focus on friction temperature of contacting bodies with taking into consideration thermophysical characteristics dependent on temperature (for example: metal-ceramics, graphite). The methods of solving boundary value problems of heat conduction can be reduced by linearization of partial differential equations by using Kirkhoff's transform (Kolyano, 1992). There are well-investigated nonstationary problems of heat conduction, when the coefficients of therm conductivity and specific heat are dependent on temperature whereas the other coefficients are constants.

Some contact problems connected with the geometrical microstructure of real surfaces of contacting solids (roughness, waviness) were considered by Dundurs et al. (1973), Panek and Dundurs (1979), Pauk (1994), Yevtushenko and Pauk (1994), Grilitskij and Pauk (1995), Pauk and Woźniak (1997), Pauk and Yevtushenko (1997).

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### O zagadnieniach generacji ciepła podczas tarcia

#### Streszczenie

W pracy przedstawiono krótki przegląd zagadnień dotyczących generacji ciepła podczas tarcia. Omawiane problemy podzielono na stacjonarne, quasistacjonarne i niestacjonarne.

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