JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
2, **39**, 2001

OPTIMAL DESIGN OF ROTATIONALLY SYMMETRIC SHELLS FOR BUCKLING UNDER THERMAL LOADINGS[1]

Michał Życzkowski
Jacek Krużelecki
Piotr Trzeciak

Institute of Mechanics and Machine Design, Cracow University of Technology
e-mail: m-1@mech.pk.edu.pl

In the present paper the problem of optimal design of a rotationally symmetric shell with immovable supports loaded by a uniform elevated temperature is investigated. We look for the variable thickness and the shape of the middle surface which lead to the maximal increment of the temperature causing buckling of the wall of this shell. The concept of the shell of uniform stability is applied.

Key words: optimal design, shells, thermal loadings, local stability

1. Introductory remarks

Usually, optimal design of structures under stability constraints considers loadings controlled by a system of forces. However, in some practical engineering applications, the loadings which are controlled by displacements, can also occur. This type of problems is, for example, connected with structures having immovable supports and undergoing thermal loading. Then, the compressive forces, which occur due to elevated temperature, depend on the geometry of the structure whereas in the classical optimization problem the forces are independent of the structure.

Instability of shells has very often a local form and buckling does not depend essentially on boundary conditions. This is particularly true in the case of a nonuniform stress distribution and in the case of a "nonuniform" geometry of a shell (variable curvatures, variable thickness). Then, the instability

[1]Full text of the paper presented at the International Congress Thermal Stresses '99 in Cracow

can be determined by the stress state and the curvatures of the shell at individual points, and the buckling is initiated at the weakest point (zone) of the structure, called the dangerous point.

For a shell with a double positive curvature Shirshov (1962) transformed the problem of global stability to a simpler problem of local stability of such a structure. Using the linear theory of shell stability, applying the equations given by Wlassow (1958) and assuming the sinusoidal deflection mode, Shirshov obtained a rather simple formula for the critical loading parameter q, namely

$$q = 2\sqrt{DEh}\,\frac{k_\varphi \cos^2\varphi + k_\theta \sin^2\varphi}{\overline{N}_\theta \cos^2\varphi + 2\overline{S}\cos\varphi\sin\varphi + \overline{N}_\varphi \sin^2\varphi} \tag{1.1}$$

where k_θ and k_φ denote the circumferential and meridional curvatures, respectively, D stands for the shell stiffness, E is the Young modulus, h is the wall thickness of a shell and φ is a certain free parameter with respect to which the loading parameter q should be minimized. In (1.1), the membrane resultant stresses depend on q, namely: $N_\theta = q\overline{N}_\theta$, $N_\varphi = q\overline{N}_\varphi$, $S = q\overline{S}$, where S is the shearing resultant stress due to twisting. For the case $S = 0$ under consideration, the minimization of q with respect to φ leads to two solutions: $\varphi_1 = 0$, $\varphi_2 = \pi/2$, and finally to very simple formulae for the critical membrane resultant stresses, namely

$$N_{\varphi\,cr} = \frac{E}{\sqrt{3(1-\nu^2)}}\,\frac{h^2}{R_\theta} \qquad\qquad N_{\theta\,cr} = \frac{E}{\sqrt{3(1-\nu^2)}}\,\frac{h^2}{R_\varphi} \tag{1.2}$$

where ν is the Poisson ratio, R_θ and R_φ stand for the radius of the circumferential and meridional curvature, respectively. These resultant stresses are assumed positive in compression. The critical value of loading is determined by one of (1.2) whichever leads to the smaller value.

Axelrad (1985), using different governing stability equations obtained also formulae (1.2) describing the critical membrane resultant stresses.

The optimization of shells with respect to their stability presents considerable difficulties connected with very complex stability differential equations, if both the middle surface and variable thickness are unknown. To avoid these difficulties, a simplified local formulation of the stability condition may be applied.

A local condition of shell stability was applied, for example, by Mazurkiewicz and Życzkowski (1966), and by Magnucki (1993) in the optimization of cylindrical shells. A detailed survey of the shell optimization is available in papers by Krużelecki and Życzkowski (1985), and Życzkowski (1992). The monograph by Gajewski and Życzkowski (1988) is devoted to structural optimization under stability constraints.

The number of papers devoted to the optimal design of shells under stability constraints is fairly large. However, most of them are confined to parametric optimization or to optimal design of stiffening elements of cylindrical shell. Very general computer programs PANDA and PANDA2, going this line, are due to Bushnell (1983, 1987). On the other hand, the difficulties connected with variational optimization with unknown both the middle surface and variable thickness are substantial and, to avoid them, we employ the local stability condition.

Making use of the hypothesis of the locality of buckling to the problem of optimal design, Życzkowski and Krużelecki (1975) proposed a concept of the *shell of uniform stability* which can be stated here as follows: if the condition of local stability is satisfied in the form of equality not only at the dangerous point but at any point of the shell, such a structure is called "the shell of uniform stability". This concept was applied to optimization of cylindrical shells by Życzkowski and Krużelecki (1975), Krużelecki and Życzkowski (1984), Krużelecki (1988). The optimization based on the concept of uniform stability was numerically verified by Krużelecki and Trzeciak (2000), who used the BOSOR 4 Code, and relatively high accuracy of the solution was found.

From the point of view of the stability the improvement of performance of an initially cylindrical shell can be obtained by at least in three different ways. The first one is connected with optimization of the wall thickness of a cylindrical shell. Let us mention the papers by Hyman and Lucas (1971) (parametrical optimization) and by Gajewski (1991) (multimodal optimization). The second way deals with changing of the shape of an initially cylindrical shell – the optimal shape of the middle surface is looked for. Only a few papers strictly devoted to the problem under consideration are quoted here, namely by Błachut (1987a,b) and Krużelecki (1997), Krużelecki and Trzeciak (1998), which are devoted mainly to parametrical optimization.

The optimization of both the thickness and middle surface of a shell is the most general optimization problem. Such problems were investigated by Życzkowski and Krużelecki (1975) (thin-walled tube under pure bending), Krużelecki and Życzkowski (1984) (bending and torsion), Krużelecki (1988) (bending and axial compression). They applied the concept of the shell of uniform stability. The same concept was used by Rysz and Życzkowski (1989) for the optimization of a cylindrical shell under creep conditions. Also, the same concept was applied to the optimization of shells with a double curvature by Krużelecki and Trzeciak (2000) (elastic shell under hydrostatic pressure) and

Krużelecki and Trzeciak (1999) (inelastic shell under hydrostatic pressure).

Most of the above-mentioned papers were confined to optimization of cylindrical shells, not necessarily circular ones. The present work is devoted to variational optimization of both shape functions of a shell with a double curvature loaded by a uniform elevated temperature. The hypothesis of the locality of buckling is utilised and the optimal structure is sought in the class of the shells of uniform stability. Stiffening by ribs or frames will not be considered, but the results obtained here may be regarded as an introductory step towards the variational optimization of stiffened shells.

2. Assumptions

- The shell is elastic, isotropic, axisymmetrical, subject to an elevated temperature without additional surface loadings.
- The shell of the length $2l_0$ is simply supported by axially immovable supports at both ends.
- Loss of the stability is described by a local condition of the Shirshov type. Following his idea, we restrict our considerations to doubly convex shells. Additional strength condition will not be introduced.
- Prebuckling bending state is neglected – to satisfy this assumption we introduce an additional constraint on the meridional slope.
- Neither ribs nor any kind of reinforcement will be taken into consideration.

3. Formulation of the optimization problem

As the design objective we assume the maximization of the temperature

$$\varDelta T_{cr} \to \max \tag{3.1}$$

Both the thickness $h = h(x)$ and variable shape described by the radius $R = R(x)$ serve as the design variables.

The right-hand part of the shell $0 \leqslant x \leqslant l$, where x is the independent variable measured along the axis of the shell, will be the only considered; the left-hand side $-l \leqslant x \leqslant 0$ will be assumed symmetric.

Such an optimization problem is stated under two equality constraints. It is assumed that the optimal shell has the same volume of material (weight) as the cylindrical reference shell with the wall thickness h_0 and the radius R_0

$$2\pi l_0 R_0 h_0 = 2\pi \int\limits_0^{l_0} Rh\sqrt{1+R'^2}\, dx \tag{3.2}$$

and the internal capacity of the both containers is also the same

$$2\pi l_0 R_0^2 = 2\pi \int\limits_0^{l_0} R^2\, dx \tag{3.3}$$

where $(\cdot)' = d/dx$, R is the distance between the shell axis and the middle surface, and h is the wall thickness of the optimal structure. Additionally, the minimal value of the coordinate R is constrained by the lower bound

$$R(l_0) = R_{\min} \geqslant R_{adm} \tag{3.4}$$

the slope of the meridian is limited by the upper bound

$$|R'| \leqslant R'_{adm} \tag{3.5}$$

and our investigation is restricted to a doubly convex shell

$$R'' \leqslant 0 \tag{3.6}$$

where R_{adm}, R'_{adm} are certain assumed values.

A uniform elevated temperature ΔT generates axial compressive force due to the immovable supports.

For rotationally symmetrical shells the radii of curvature amount to

$$R_\varphi = -\frac{\sqrt{(1+R'^2)^3}}{R''} \qquad\qquad R_\theta = R\sqrt{1+R'^2} \tag{3.7}$$

and hence, for the shell loaded by the compressive axial force N_x the membrane meridional and circumferential resultants take the form

$$N_\varphi = N_x\frac{\sqrt{1+R'^2}}{2\pi R} \qquad\qquad N_\theta = N_x\frac{R''}{2\pi\sqrt{1+R'^2}} \tag{3.8}$$

The meridional strain ε_φ can be evaluated from Hooke's law

$$\varepsilon_\varphi = \frac{1}{E}(\sigma_\varphi - \nu\sigma_\theta) + \alpha\Delta T \tag{3.9}$$

in which the stresses σ are expressed by the resultants N with changed signs. After introducing Eq (3.8) the meridional strain assumes the form

$$\varepsilon_\varphi = -\frac{N_x}{2\pi Eh}\Big(\frac{\sqrt{1+R'^2}}{R} - \frac{\nu R''}{\sqrt{1+R'^2}}\Big) + \alpha\varDelta T \tag{3.10}$$

where α is the thermal expansion coefficient. The total elongation of the shell in the axial direction is assumed to be zero. Since, we have

$$\int\limits_0^{l_0} \varepsilon_\varphi \, dx = 0 \tag{3.11}$$

Substituting Eq (3.10) into Eq (3.11) we can evaluate the axial force

$$N_x = \frac{2\pi E\alpha l_0 \varDelta T}{\int\limits_0^{l_0} \frac{1}{h}\Big(\frac{\sqrt{1+R'^2}}{R} - \frac{\nu R''}{\sqrt{1+R'^2}}\Big)\, dx} \tag{3.12}$$

For elevated temperatures the membrane resultant N_φ is positive (compressive), whereas N_θ is negative (tensile), see Eq (3.8). Hence, the critical loading is determined by N_φ. Introducing safety the factor against buckling j, $N_\varphi = N_{\varphi\, cr}/j$ and $\varDelta T = \varDelta T_{cr}/j$, and utilizing Eq (1.2), (3.8) and (3.12) we have

$$\frac{\varDelta T_{cr}}{j}\sqrt{3(1-\nu^2)}\, \alpha l_0(1+R'^2) = h^2 \int\limits_0^{l_0} \frac{1}{h}\Big(\frac{\sqrt{1+R'^2}}{R} - \frac{\nu R''}{\sqrt{1+R'^2}}\Big)\, dx \tag{3.13}$$

It results from Eq (3.13) that the variable thickness h of the shell of uniform stability is described by

$$h = h_1\sqrt{1+R'^2} \tag{3.14}$$

where h_1 is the wall thickness for $x = 0$.

We formulate the problem of optimization as a classical problem of calculus of variations employing the Lagrangian multiplier method. To maximize the critical elevated temperature $\varDelta T_{cr}$ the integral in Eq (3.13) should be maximal under the conditions of constant integrals in Eqs (3.2) and (3.3).

The Lagrangian function can be written as

$$L = \frac{h_1}{R} - \nu h_1 \frac{R''}{1+R'^2} + \varLambda_1 h_1 R(1+R'^2) + \varLambda_2 R^2 \tag{3.15}$$

where $\varLambda_1$ and $\varLambda_2$ are the Lagrangian multipliers. This function is linear with respect to R''. In such a case the Euler-Lagrange equation, usually of the fourth order, is reduced to a second-order equation (Appendix, Eq (A.3)). Now, we are going to prove that the Poisson ratio ν will not appear in this equation. Indeed, the function F_1 in (A.1) is here of the form

$$F_1 = -\frac{\nu h_1}{1+R'^2} \tag{3.16}$$

where all the derivatives of this function shown in (A.3) vanish

$$\frac{\partial F_1}{\partial R} = \frac{\partial^2 F_1}{\partial x \partial R'} = \frac{\partial^2 F_1}{\partial R \partial R'} = \frac{\partial^2 F_1}{\partial x \partial R} = \frac{\partial^2 F_1}{\partial R^2} = \frac{\partial^2 F_1}{\partial x^2} = 0$$

and F_1 is absent in the final equation. A simple explanation of this fact looks as follows: the integral containing ν may readily be evaluated

$$\int\limits_0^{l_0} \frac{\nu h_1 R''}{1+R'^2}\, dx = \int\limits_0^{R'_l} \frac{\nu h_1 d\psi}{1+\psi^2} = \nu h_1 \arctan R'_l \tag{3.17}$$

where the new variable of integration ψ is equal to R', which implies that $R''dx = d\psi$, and R'_l denotes the slope at the simply supported end of the shell. Hence, this integral does not depend on the integration path, that is on the function $R = R(x)$, and obviously, it cannot appear in the Euler-Lagrange equation. This is an important statement since we proved that the optimal shape does not depend directly on ν. It depends only via Lagrangian multipliers. The profit of the optimization depends on ν.

The Euler-Lagrange equation (A.3) takes finally the form

$$r'' = -\frac{l_0^2}{R_0^2}\frac{1}{2\lambda_1 r^3}\Big\{1 - \Big[\lambda_1\Big(1 - \frac{R_0^2}{l_0^2}r'^2\Big) + 2\lambda_2 r\Big]r^2\Big\} \tag{3.18}$$

where the dimensionless variables and dimensionless Lagrangian multipliers are introduced as follows

$$\begin{array}{ll} \xi = \dfrac{x}{l_0} & r = \dfrac{R}{R_0} \\ \lambda_1 = \varLambda_1 h_1 R_0 & \lambda_2 = \varLambda_2 R_0^2 \end{array} \tag{3.19}$$

and the primes denote now the differentiation with respect to ξ. The boundary condition for Eq (3.18), ensuring "smooth" shape, can be written as follows: $r'(0) = 0$. Eqs (3.2), (3.3) and (3.13) can be rewritten in the dimensionless form

$$\begin{aligned}
&\frac{h_1}{h_0}\int\limits_0^1 r\Big(1+\frac{R_0^2}{l_0^2}r'^2\Big)\,d\xi = 1\\
&\int\limits_0^1 r^2\,d\xi = 1 \qquad\qquad (3.20)\\
&\frac{\Delta T_{cr}}{j}\sqrt{3(1-\nu^2)}\,\alpha = \frac{h_0}{R_0}\frac{h_1}{h_0}\int\limits_0^1\Big\{\frac{1}{r}+\nu\frac{1-\Big[\lambda_1\Big(1-\frac{R_0^2}{l_0^2}r'^2\Big)+2\lambda_2 r\Big]r^2}{2\lambda_1 r^3\Big(1+\frac{R_0^2}{l_0^2}r'^2\Big)}\Big\}\,d\xi
\end{aligned}$$

4. Numerical results

Calculations were performed for various parameters describing the length of the shell, namely for $l_0/R_0 = 1$, 4/3, 2, 4, under the inequality constraints (Eqs (3.4), (3.5) and (3.6)): $r(1) \geqslant 0$, $-r'(1) \leqslant l_0/R_0$ (the slope is smaller than $45°$), $r'' \leqslant 0$, and for the Poisson ratio $\nu = 0$, 0.5.

Differential Eq (3.18) is integrated numerically using the Runge-Kutta method starting from the point $\xi = 0$, $r(0) = r_0$ and satisfying the boundary condition. The starting values of r_0, h_1/h_0 and the Lagrangian multipliers λ_1 and λ_2 are the unknowns. For the assumed value of r_0, the Lagrangian multipliers λ_1 and λ_2 are chosen to satisfy constraint $(3.20)_2$. For such parameters the dimensionless thickness h_1/h_0 is evaluated from Eq $(3.20)_1$. Such a procedure is repeated to obtain the maximal critical elevated temperature ΔT_{cr} defined by Eq $(3.20)_3$. It occurs that for short shells $(l_0/R_0 = 1,\ 4/3)$ and $\nu = 0$ the optimization leads to a cylindrical shell with constant thickness whereas for $\nu = 0.5$ both the short and long shells are not cylindrical structures. It also turns out that for $l_0/R_0 = 2$ the inequality constraint enssuring the convexity of the optimal shell $r''(1) = 0$ as well as the inequality constraint limiting the slope of the meridian $(-r'(1) = 2)$ are active. For the shell with $l_0/R_0 = 4$ only the convexity constraint is the active one.

In Fig. 1 the radius in terms of the longitudinal coordinate for the shells of uniform stability are presented for choosen l_0/R_0 and $\nu = 0.5$. The appropriate variable thicknesses are plotted in Fig. 2.

In Fig. 3 the final shapes of the optimal shells are presented for $l_0/R_0 = 1$ and $l_0/R_0 = 4$. It occurs that the profit of the optimization, measured by the temperature ratio of the critical temperature increment ΔT_{cr} for the optimal

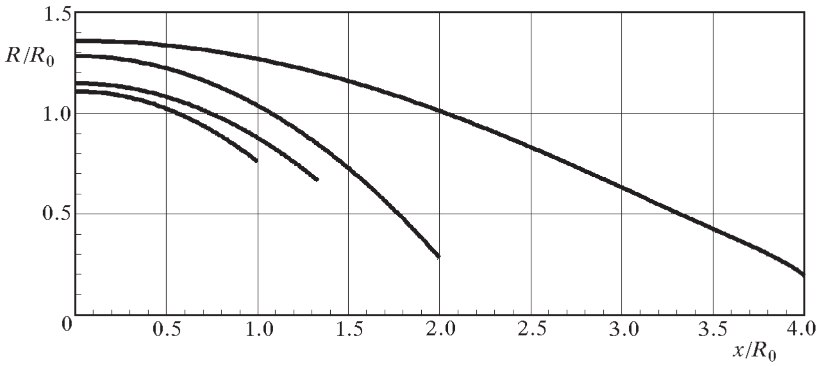

Fig. 1. Radius in terms of longitudinal coordinate, $\nu = 0.5$

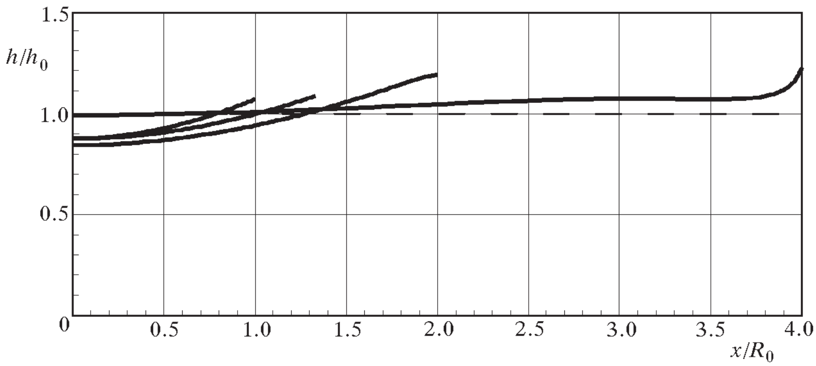

Fig. 2. Thickness of therms of longitudinal coordinate, $\nu = 0.5$

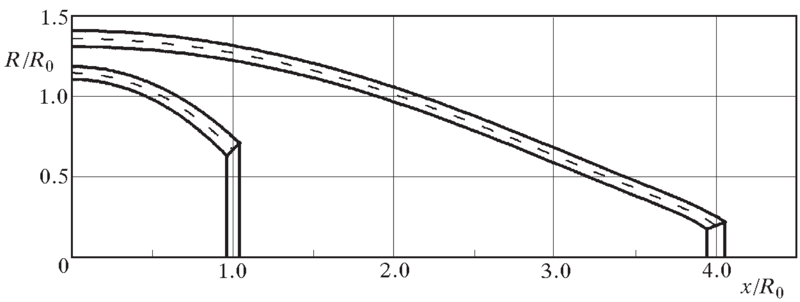

Fig. 3. Final shapes of optimal shells, $\nu = 0.5$

shell to such an increment ΔT_{cr}^{c} for the cylindrical reference shell, clearly depends on the length of the shell, namely for long shells the profit is higher than for short ones (Fig. 4) and also depends on ν. On the other hand, it does not depend on the thickness parameter h_0/R_0.

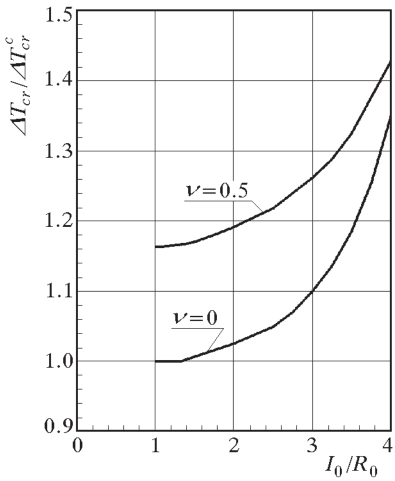

Fig. 4. Profit of optimization vs. length of shell

5. Discussion of the obtained results

The results presented in this paper are based on the concept of the shell of uniform stability. These results can be considered satisfactory but they do not have to constitute the absolute optimum. Such a global optimum may be obtained using the full shell stability equations.

It should be stressed that in a similar case of the optimization, namely when the loading is controlled by the axial force, the optimization leads to a cylindrical shell of constant thickness and no profit is obtained. In the problem under consideration such a profit reaches several dozen per cent.

The optimal solutions based on the concept of the shell of uniform stability are limited in the case of very long shells by the additional condition of global buckling of the shells treated as columns. For long shells both types of the stability conditions should be satisfied and the optimal structure can be considered as a shell of equal stability. In this case, buckling mode interaction may occur; a more detailed analysis of this interaction is difficult, but the simplest practical way to take it into consideration is to raise accordingly the safety factor j.

A. On functionals depending linearly on the second derivative

Usually, the Euler-Lagrange equation for functionals depending on the second derivative of the unknown function is of the fourth order. However, if this dependence is linear, then the reduction to a second-order equation takes place.

Consider a functional depending linearly on the second derivative:

$$J = \int\limits_a^b [y'' F_1(x,y,y') + F_2(x,y,y')]\, dx \tag{A.1}$$

The second term obviously leads to a second-order equation but for the sake of uniform notation we retain it in the analysis. The Euler-Lagrange equation for (A.1) takes first the form

$$y''\frac{\partial F_1}{\partial y} + \frac{\partial F_2}{\partial y} - \frac{d}{dx}\Big(y''\frac{\partial F_1}{\partial y'} + \frac{\partial F_2}{\partial y'}\Big) + \frac{d^2 F_1}{dx^2} = 0 \tag{A.2}$$

The vanishing of y^{IV} in Eq (A.2) is seen immediately: it might appear just in the last term, but F_1 does not depend on y'' and hence y^{IV} is absent. The third derivative y''' appears in (A.2) twice: in the third term we obtain $-y'''\partial F_1/\partial y'$ and in the last term using the chain rule of differentiation we find $y'''\partial F_1/\partial y'$. Hence, these expressions cancel each other. Finally, we obtain the following second-order equation

$$\begin{gathered}\Big(2\frac{\partial F_1}{\partial y} + \frac{\partial^2 F_1}{\partial x \partial y'} + \frac{\partial^2 F_1}{\partial y \partial y'}y' - \frac{\partial^2 F_2}{\partial y'^2}\Big)y'' + \Big(2\frac{\partial^2 F_1}{\partial x \partial y} + \frac{\partial^2 F_1}{\partial y^2}y' - \frac{\partial^2 F_2}{\partial y \partial y'}\Big)y' + \\ + \frac{\partial^2 F_1}{\partial x^2} + \frac{\partial F_2}{\partial y} - \frac{\partial^2 F_2}{\partial x \partial y'} = 0\end{gathered} \tag{A.3}$$

This equation is linear with respect to y'' since neither F_1 nor F_2 depend on this derivative.

Acknowledgment

This paper was partly supported by the State Committee for Scientific Research (KBN) under Grant No. PB 7-T07A-031-16.

References

1. Axelrad E.I., 1985, On Local Buckling of Thin Shells, *Int. J. Non-Linear Mech.*, **24**, 4, 249-259
2. Błachut J., 1987a, On Optimal Barrel-Shaped Shells Under Buckling Constraints, *AIAA J.*, **25**, 186-188
3. Błachut J., 1987b, Combined Axial and Pressure Buckling of Shells Having Optimal Positive Gaussian Curvature, *Comp. and Struct.*, **26**, 3, 513-519
4. Bushnell D., 1983, PANDA-Interactive Program for Minimum Weight Design of Stiffened Cylindrical Panels and Shells, *Comput. and Struct.*, **16**, 1-4, 167-185
5. Bushnell D., 1987, PANDA2-Program for Minimum Weight Design of Stiffened, Composite, Locally Buckled Panels, *Comput. and Struct.*, **25**, 4, 469-605
6. Gajewski A., Życzkowski M., 1988, *Optimal Structural Design Under Stability Constraints*, Dordrecht-Boston-London: Kluwer Academic Publishers
7. Gajewski A., 1991, Multimodal Optimization of Uniformly Compressed Cylindrical Shells, *Proc. Engineering Optimization in Design Processes*, 275-282, Karlsruhe
8. Hyman B.I., Lucas A.W., 1971, An Optimum Design for the Instability of Cylindrical Shells under Lateral Pressure, *AIAA J.*, **9**, 4, 738-740
9. Krużelecki J., 1988, Optimal Design of a Cylindrical Shell Under Overall Bending with Axial Force, *Bull. Polish Acad. Sci., Tech. Sci.*, **36**, 3-4, 141-150
10. Krużelecki J., 1997, On Optimal Barrel-Shaped Shells Subjected to Combined Axial and Radial Compression, *Proc. Structural and Multidisciplinary Optimization, WCSMO-2*, 467-472, Zakopane
11. Krużelecki J., Trzeciak P., 1998, Optimal Design of Axially Symmetrical Shells under Compression Using the Concept of a Shell of Uniform Stability, *Proc. Zbiorniki Cienkościenne*, 39-42 (in Polish with English summary), Karłów
12. Krużelecki J., Trzeciak P., 1999, Optimal Design of Rotationally Symmetric Shells for Inelastic Buckling under Hydrostatic Pressure, *Proc. WCSMO-3*, (CD-ROM only), ISMO/UBCAD/UB/AIAA, Buffalo
13. Krużelecki J., Trzeciak P., 2000, Optimal Design of Axially Symmetrical Shells under Hydrostatic Pressure with Respect to their Stability, *Struct. Multidisc. Optim*, **19**, 2, 148-154
14. Krużelecki J., Życzkowski M., 1984, Optimal Design of an Elastic Cylindrical Shell under Overall Bending with Torsion, *Solid Mech. Archives*, **9**, 3, 269-306
15. Krużelecki J., Życzkowski M., 1985, Optimal Design of Shells – a Survey, *Solid Mech. Archives*, **10**, 101-170

16. Magnucki K., 1993, *Some Problems of Optimization of Rod Structures and Shells with Elastic Stability Condition Taken into Account*, Poznań: Rozprawy No. 292, Polit. Poznańska (in Polish with English summary)

17. Mazurkiewicz S., Życzkowski M., 1966, Optimum Design of Cross-Section of Thin-Walled Bar under Torsion and Bending, *Bull. Acad. Pol., Ser. Sci. Techn.*, **14**, 4, 273-282

18. Rysz M., Życzkowski M., 1989, Optimal Design of a Cylindrical Shell under Overall Bending with Axial Force with Respect to Creep Stability, *Structural Optimization*, **1**, 1, 29-36

19. Shirshov V.P., 1962, Local Buckling of Shells, *Proc. II Vses. Konf. Teorii Plastin i Obol.*, 314-317 (in Russian)

20. Wlassow W.S., 1958, *Allgemeine Schalentheorie und ihre Anwendung in der Technik*, Berlin, Akademie-Verlag, (translated from Russian, Moskva-Leningrad 1949)

21. Życzkowski M., 1992, Recent Advances in Optimal Structural Design of Shells, *Eur. J. Mech., A/Solids*, **11**, Special issue, 5-24

22. Życzkowski M., Krużelecki J., 1975, Optimal Design of Shells with Respect to their Stability, *Proc. IUTAM Symposium on Optimization in Structural Design*, 229-247, Warsaw

Optymalne kształtowanie powłok obrotowo symetrycznych z uwagi na stateczność pod działaniem obciążeń termicznych

Streszczenie

W pracy rozważano zagadnienie optymalnego kształtowania obrotowo symetrycznej powłoki na nieprzesuwnych podporach obciążonej równomiernym polem temperatur. Poszukiwano takiej zmiennej grubości ścianki i kształtu powierzchni środkowej, które prowadzą do maksymalnej temperatury powodującej wyboczenie ścianki powłoki. Wykorzystano koncepcję powłoki równomiernej stateczności.

Manuscript received November 20, 2000; accepted for print December 18, 2000