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DYNAMIC STABILITY OF ELECTRORHEOLOGICAL FLUID-FILLED LAMINATE

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In the present paper the technique of the dynamic stability analysis proposed for conventional laminated structures is extended to cover the activated electrorheological fluid-filled beam under the time-dependent axial loading. The thin symmetrically laminated structure consists of the uniform beam and the multi-cell chains containing the electrorheological fluid in an extensional configuration. Changing the electric field on the cell electrodes modifies basic mechanical properties such as Young's modulus, and the retardation time. The present paper aims at analysis of the classical stability problem: how does the electric activation change dynamic stability domains of the beam. In order to derive the dynamic stability criteria the Liapunov functional as a sum of the modified kinetic energy and the elastic energy of the beam is chosen. The stability regions as functions of loading characteristics, damping coefficient are given.

Key words: dynamic stability, electrorheological fluids, electrical activation

1. Introduction

By the application of an electric field to electrorheological fluids a dramatic change in viscosity can be achieved due to formation of fibres of semiconducting particles in the fluid. The particles behave as dielectrics and so form chains in the direction of the electric field. When the fluid is sheared additional energy is necessary to break the bonds between particles, as well as to overcome the shear forces due to the viscous effect of the fluid. The rheological changes are fast [10^{-3} s] and reversible, thereby making the fluids suitable for real-time control of vibration. The induced shear resistance τ, i.e. the increase above that for unactivated fluid, is found to be nearly a parabolic function of the electric field strength $\tau \pm \tau_o = k(U \pm U_o)^2$ (Winslow, 1949). The supplemental shear resistance τ_o and the factor k depend on the particle dimension and surfaces, type and amount of a dispersing agent. The fluid is very versatile since it combines properties of solids and liquids. In the activated state the cell rheology is described by the Voigt-Kelvin model. Extensive variation in the mechanical properties of the fluid makes torque drive coupling and clutch systems more appealing than the conventional ones wich suffer considerable mechanical wear. The areas of application lie in the field of vibration isolation, shock absorber, and drive clutches (Shullman et al., 1989). The problems with electrode abrasion, rapid degradation of fluids, segmentation, temperature sensitivity and the necessity to provide high voltage on the order of $2 \div 3$ kV across a gap of 1 mm are the reasons why such a promissing idea is not commercially incorporated.

Recently, preliminary studies have been performed to use dynamic mechanical properties of liquid crystal. Its apparent viscosity depends on the orientation of molecules which can be controlled by the applied electric fields. Liquid crystal is a homogeneous electro-rheological fluid and may be superior to a dispersive electro-rheological fluid in the stability. Tani et al. (1996) presented the experimental results of application of a liquid crystal to the vibration suppresion of rotary shaft and a pendulum.

The controllable rheological behaviour of electrorheological materials is useful in engineering systems and structures, where a variable performance is desired. As a result we can obtain an intelligent or an adaptive structure, which reacts in an appropriate manner so as to meet the defined performance criteria, eg. minimizing vibrational amplitude or damping its response (Choi et al., 1992). The crucial point in developing an adaptive structure is to create a mathematical model of the dynamic structural response. The problem of static buckling of viscoelastic columns under constant axial forces was solved by De Leeuw (1963). One of the first analyses of the dynamic stability of viscoelastic continuous systems with viscoelastic boundary conditions was made by Genin and Maybee (1972). In the next significant study Plaut used (1973) the Liapunov method to determine the stability criteria of viscoelastic columns subjected to a compressive axial force. The dynamic stability analysis of viscoelastic continuous systems under time-dependent deterministic or stochastic forces has been usually applied to a linear Voigt-Kelvin solid. More sophisticated models of material damping such as a linear standard solid (Potapov, 1985) or a non-linear standard model (Życzkowski and Kowalski, 1984) were introduced to the stability analysis of structures but only constant forces and nonconservative time-independent loadings were assumed. The uniform stability of standard material columns under time-dependent random axial loads was determined using the Liapunov method (Tylikowski, 1991). Electrorheological based beam structures in a shear configuration were modeled in the past as a modified Bernoulli-Euler beam by Ross et al. (1959) and as a partial differential equation of the sixth order by Mead and Markus (1969).

In the present paper the symmetrically laminated structure consists of the uniform beam and the multi-cell chains containing the electrorheological fluid in an extensional configuration. The electrorheological properties are described using the standard material and the Oldroyd equation (Oldroyd, 1953). The beam as a real mechanical system is subjected not only to nontrivial initial conditions but also to to permanently acting excitations and a semiactive vibration control should be applied in order to balance the supplied energy by parametric excitation. The electric activation increases the stiffness and damping property described by a retardation time of the adaptive beam. The increase in the applied voltage changes the rheological parameters of the electrorheological fluid usually modelled as the Bingham viscoelastic material (Shiang and Coulter, 1996). We will use the two alternative models: standard viscoelastic body and suspension described by the Oldroyd equation (Oldroyd, 1953). We model the analysed structure as a stright Rayleigh beam of constant cross section. Due to the cell structure the beam is treated as a moderately thick plate and the rotary inertia term is included.

2. Dynamics equation of electrorheological fluid-filled beam

2.1. Standard viscoelastic material

Consider a beam-like structure with the electrorheological fluid incorporated in the extensional configuration. The fluid-filled laminate consists of an inner elastic beam of the thickness h and external cells filled with the electrorheological fluid. By incorporating a material with known controllable rheology into an otherwise passive beam, the response of the entire composite system becomes tunable. Therefore, it becomes possible for the structure to adapt to a variable enviroment e.g. variable intesivity of time-dependent axial force. The beam geometry and the cell structure are shown in Fig.1. The voltage applied to the side walls of the cell is denoted by U. The unactivated state corresponds to $U = 0$. If the linear standard material is used to describe the rheological properties the partial equation of transverse displacement $w(x,t)$

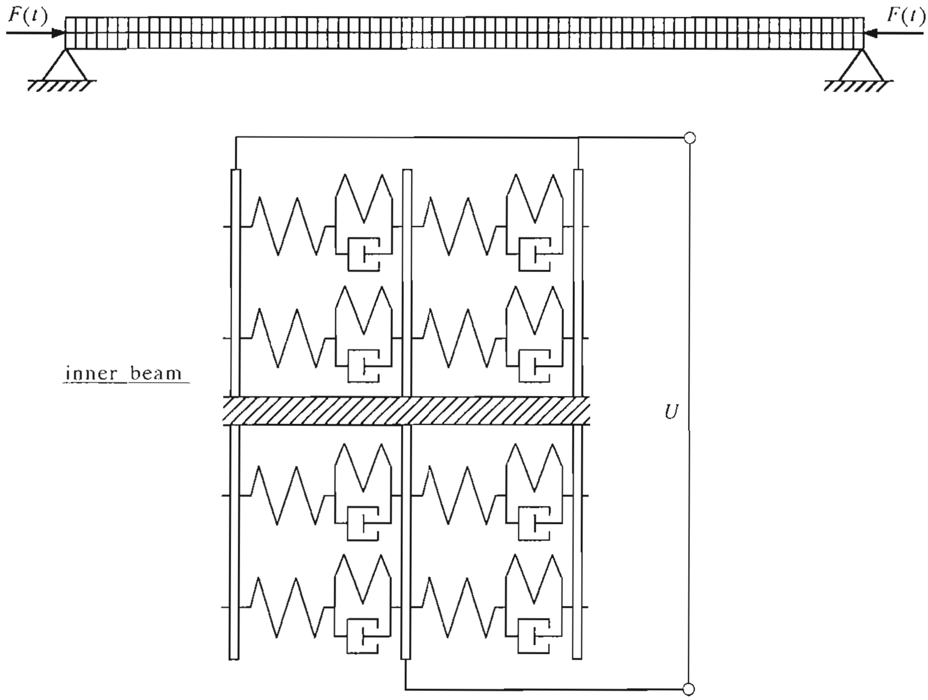

Fig. 1. Electrorheological fluid-filled beam structure

obtained by the correspondence principle has the form

$$
\begin{aligned}
&\rho A w_{,tt} - \rho J w_{,xxtt} + (F_o + F(t,\gamma)) w_{,xx} + \\
&+ EJ\Big(w_{,xxxx} - \frac{E}{\eta}\int\limits_0^t \exp[-\lambda(t-\tau)] w_{,xxxx}\, d\tau\Big) = 0 \qquad x \in (0,\ell)
\end{aligned}
\tag{2.1}
$$

where

- A – cross-sectional area
- J – cross-sectional moment of inertia
- ℓ – beam length
- ρ – mean beam density
- E – Young's modulus
- λ – effective retardation constant of the composite beam
- η – viscosity of the standard solid

$F_o, F(t,\gamma)$	–	constant and time-dependent components of the axial force
x	–	longitudinal coordinate
t	–	time.

The beam is assumed to be simply supported at both ends. The boundary conditions corresponding to simply supported edges have the form

$$w(0,t) = w(1,t) = 0 \qquad\qquad w_{,xx}(0,t) = w_{,xx}(1,t) = 0 \tag{2.2}$$

Expanding the transverse displacement into series of functions $\sin i\pi x/\ell$ satisfying the boundary conditions we obtain the infinite system of decoupled integro-differential equations with respect to modes. We can write the governing dynamic equations of the ith mode in the form

$$\ddot{T}_i + k_i^2 T_i - [f_{oi} + f_i(t,\gamma)]T_i - k_i^2 \frac{E}{\eta} \int\limits_0^t \exp(-\lambda\tau) T_i(t-\tau)\; d\tau = 0 \tag{2.3}$$

where

k_i^2	–	natural frequency of the ith mode
$f_{oi}, f_i(t,\gamma)$	–	constant and stochastic components of the axial force divided by $\rho(A + J\alpha_i^2)$, $\alpha_i = i\pi/\ell$
γ	–	element of probability space $\{\varGamma, ß, \mathcal{P}\}$

$$\begin{aligned} k_i^2 &= \frac{EJ\alpha_i^4}{\rho(A + J\alpha_i^2)} \qquad\qquad f_{oi} = \frac{F_o\alpha_i^2}{\rho(A + J\alpha_i^2)} \\ f_i(t,\gamma) &= \frac{F(t,\gamma)\alpha_i^2}{\rho(A + J\alpha_i^2)} \end{aligned} \tag{2.4}$$

If the axial force is a stochastic wide-band Gaussian process with a constant component F_o and intensity σ_F we model it by means of a white-noise process ξ

$$F(t,\gamma) = F_o + \sigma_F\xi \tag{2.5}$$

In order to avoid the integral term in Eq (2.3) we rewrite it in the form of the linear system of Itô diffferential equations

$$\begin{aligned} dT_i &= S_i dt \\ dS_i &= -k_i^2\Big[(k_i^2 - f_{oi})T_i - k_i^2\frac{E}{\eta}R_i\Big]dt + \sigma_i T_i d\mathcal{W} \\ dR_i &= (T_i - \lambda R_i)dt \end{aligned} \tag{2.6}$$

where $\mathcal{W}$ – standard Wiener process with the intensity σ

$$\sigma_i = \frac{\sigma_F \alpha_i^2}{\rho(A + J\alpha_i^2)} \tag{2.7}$$

2.2. Suspension model described by the Oldroyd equation

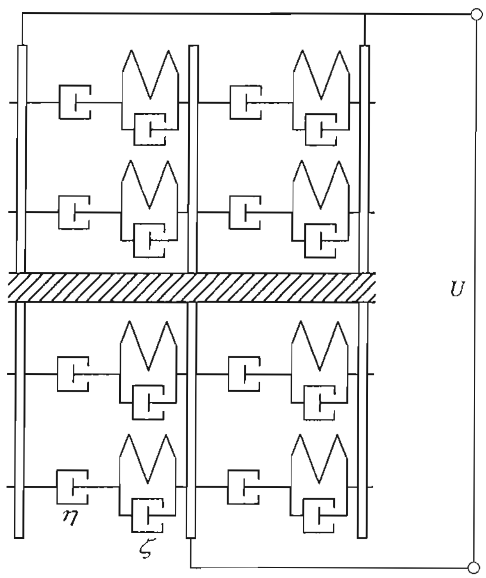

Fig. 2. Oldroyd fluid-filled cell

Consider the cell structure shown in Fig.2. The modified dynamic equation is as follows

$$\begin{aligned} &\rho A w_{,tt} - \rho J w_{,xxtt} + (F_o + F(t,\gamma))w_{,xx} + EJ(w_{,xxxx} + \kappa w_{,xxxxt}) + \\ &- \frac{E_c^2 J \eta^2}{(\eta+\zeta)^3} \int\limits_0^t \exp[-\lambda(t-\tau)] w_{,xxxx}\, d\tau = 0 \qquad x \in (0,\ell) \end{aligned} \tag{2.8}$$

where

η, ζ	–	voltage-dependent viscosities of the Oldroyd fluid
E_c	–	fluid elastic modulus
λ	–	retardation time, $\lambda = E_c/(\eta+\zeta)$

and $\kappa = \eta\zeta/(\eta+\zeta)$. The remaining notation is the same as in Eq (2.3). The beam is assumed to be simply supported at both ends and the boundary conditions are presented by Eqs (2.2). Following the approach taken in Section 2.1 we can write the governing dynamic equation of the ith mode in the form

$$
\begin{aligned}
&\ddot{T}_i + k_i^2 T_i - [f_{oi} + f_i(t,\gamma)]T_i + k_i^2 \kappa \dot{T}_i + \\
&-\delta_i \int\limits_0^t \exp[-\lambda(t-\tau)]T_i(\tau)\, d\tau = 0
\end{aligned}
\tag{2.9}
$$

where

$$
\delta_i = \frac{E_c^2 J \eta^2}{\rho(A + J\alpha_i^2)(\eta+\zeta)^3}
$$

The Itô form is as follows

$$
\begin{aligned}
dT_i &= S_i dt \\
dS_i &= -k_i^2[(k_i^2 - f_{oi})T_i - k_i^2 \kappa S_i - k_i^2 \delta R_i]dt + \sigma_i T_i d\mathcal{W} \\
dR_i &= (T_i - \lambda R_i)dt
\end{aligned}
\tag{2.10}
$$

3. Uniform stability analysis

In order to examine the parametric vibration excited by a time-dependent longitudinal force $f(t)$ the stability of trivial solution (equilibrium state) of Eq (2.3) and/or Eq (2.9) has to be analysed. To estimate deviations of the solution from the trivial solution we introduce the uniform stability definition. If the force is a stochastic wide-band Gaussian process we will discuss the conditions implying equality

$$
\bigwedge_{\epsilon>0} \bigwedge_{\delta>0} \bigvee_{r>0} \quad \|w(\cdot,0)\| < r \quad \Rightarrow \quad \mathcal{P}\{\sup_{t\geqslant 0} \|w(\cdot,t)\| > \epsilon\} < \delta \tag{3.1}
$$

where $\|\cdot\|$ is the scalar measure or the distance between a solution with nontrivial conditions and the trivial solution.

Using the function derived for stability analysis of viscoelastic columns made of the standard material (Tylikowski, 1991) we choose the positive-definite Liapunov function in the matrix form

$$
V = \boldsymbol{z}^\top \mathbf{B} \boldsymbol{z} \tag{3.2}
$$

where

$$\boldsymbol{z}^\top = [T_i, S_i, R_i]$$

$$\mathbf{B} = \begin{bmatrix} b(k_i^2 - f_{oi} + \lambda^2)\lambda & 0 & -\dfrac{k_i^2 E\lambda}{\eta}b \\ 0 & \lambda^3 + b & \lambda^2 k_i^2 \dfrac{E}{\eta} \\ -\dfrac{k_i^2 E\lambda}{\eta}b & \lambda^2 k_i^2 \dfrac{E}{\eta} & \lambda^2 k_i^4 \dfrac{E}{\eta} \end{bmatrix} \tag{3.3}$$

and

$$b = \Big[k_i^2\Big(1 - \frac{E}{\lambda\eta}\Big) - f_{oi}\Big]$$

Using Sylvester's conditions we notice that the matrix $\mathbf{B}$ is positive-definite if the following inequality is fulfilled for $i = 1, 2, ...$

$$f_{oi} < k_i^2\Big(1 - \frac{E}{\eta}\Big) \tag{3.4}$$

Substituting the denotations (2.4) the positive-definiteness of matrix $\mathbf{B}$ is implied by the static criterion of stability

$$F_o < EJ\alpha_1^2\Big(1 - \frac{E}{\eta}\Big) \tag{3.5}$$

Therefore, the distance is chosen as a square root of the Liapunov function $\|z(t)\| = \sqrt{V}$. Calculating the infinitesimal generator LV (Khasminski, 1969) along the solution of Eqs (2.6) we have

$$\begin{aligned} LV &= \Big\{-2k_i^2 \frac{E}{\eta}\lambda\Big[k_i^2\Big(1 - \frac{E}{\lambda\eta}\Big) - f_{oi}\Big] + \\ &+ \lambda\Big[\lambda^2 + k_i^2\Big(1 - \frac{E}{\lambda\eta}\Big) - f_{oi}\Big]\sigma_i^2\Big\}T_i^2 \end{aligned} \tag{3.6}$$

where the first and second terms correspond to the deterministic and stochastic parts of the dynamic equations (2.6), respectively. Therefore, the trivial solution $z = 0$ is uniformly stochastically stable if the intensity of stochastic force is sufficiently small, i.e.

$$\sigma_i^2 < \frac{2k_i^2 \frac{E\lambda}{\eta}\Big[k_i^2\Big(1 - \frac{E}{\lambda\eta}\Big) - f_{oi}\Big]}{\lambda^2 + k_i^2\Big(1 - \frac{E}{\lambda\eta}\Big) - f_{oi}} \tag{3.7}$$

Having the stability problem of the ith mode solved we proceed to the stability analysis of the beam with the viscoelastic electrorheological fluid. Substituting Eqs (2.4) into the condition (3.7) we obtain the infinite system of inequalities for $i = 1, 2, \dots$

$$\sigma_F^2 < 2\rho(A + J\alpha_i^2)\frac{E}{\eta}\frac{EJ\Big(1 - \frac{E}{\lambda\eta}\Big)\alpha_i^4 - F_o\alpha_i^2}{\lambda^2\rho(A + J\alpha_i^2) + EJ\Big(1 - \frac{E}{\lambda\eta}\Big)\alpha_i^4 - F_o\alpha_i^2} \tag{3.8}$$

Minimizing the result with respect to i we arrive the sufficient condition for the uniform stochastic stability of the electrorheological fluid filled beam as all beam modes are uniformly stable.

Numerical calculations based on the formula presented are performed for a wide range of electric fields the applied $0 \div 4\,\mathrm{kV/mm}$ and when changing the constant component of axial force. The static critical force is equal to $F_o = 3200\,\mathrm{N}$. The dimensions of the steel beam are: length $\ell = 500\,\mathrm{mm}$, width $b = 40\,\mathrm{mm}$ and thickness $h = 5\,\mathrm{mm}$. The hight of fluid - filled cells is $H = 20\,\mathrm{mm}$. The wall thickness is $0.5\,\mathrm{mm}$, and the distance between the walls is $1\,\mathrm{mm}$. The cells are filled with the LORD VersaFlo ER-200 fluid with the properties taken from the paper by Shiang et al. (1996). The influence of the applied electric field U on the critical force intensity is shown in Fig.3 for given values of the constant component of axial force.

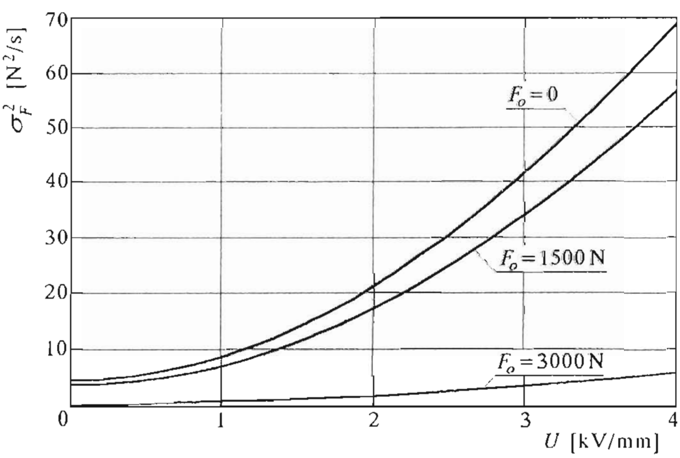

Fig. 3. Critical axial force intensity versus the electric field applied

If the cells contain the Oldroyd fluid the Liapunov function is generated in the same way. Crucial for the stability analysis Sylvester's conditions lead to the following stability condition

$$\begin{aligned} \sigma_i^2 &< 2k_i^2\Big\{\lambda(k_i^2 - f_{oi})[\delta_i + \kappa(\lambda^2 + k_i^2)] - k^4\delta_i\lambda\kappa^2 + \\ &- k_i^2[\delta_i^2 - \kappa^2 k_i^2 + \delta\kappa(\lambda^2 + k_i^2)]\Big\}\Big[\lambda^2(\kappa k_i^2 + \lambda) + \lambda(k_i^2 - f_{oi}) - k_i^2\delta_i\Big]^{-1} \end{aligned} \tag{3.9}$$

4. Conclusions

The paper is concerns the stabilization of an elastic beam subjected to a time-dependent axial forcing. The direct Liapunov method is proposed to establish criteria for the uniform stochastic stability of the unperturbed (trivial) solution for the structure with the semi-active control. The fluctuating axial force is modelled by the gaussian wide-band process. The effective stabilization conditions implying the almost sure stability are the main results. Inequalities (3.8) and (3.9) give a possibility to obtain the maximal force intensity σ_F guaranteeing the uniform stochastic stability of the stright column. Having in mind that the viscosity η in Eq (3.8) and the viscosities η and ζ in Eq (3.9) are the electric field dependent we can enlarge the uniform stability domains. Increasing the voltage applied to the cells we increase the intensity of axial force.

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Dynamiczna stateczność laminatu z cieczą elektroreologiczną

Streszczenie

Praca dotyczy analizy klasycznego zagadnienia staeczności belki wypełnionej cieczą elektroreologiczną w celu odpowiedzi na pytanie: jak elektryczna aktywacja cieczy zamienia obszary stateczności. Ciecz elektroreologiczną opisano modelem standardowym i modelem cieczy Oldroyda o współczynnikach zależnych od przyłożonego pola elektrycznego. W celu wyprowadzenia kryteriów stateczności posłużono się bezpośrednią metodą Lapunowa.

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