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MULTIPLE PIEZOELECTRIC SEGMENTS IN STRUCTURAL VIBRATION CONTROL

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This paper presents active damping of structural vibration of a simply supported beam excited by a time-dependent concentrated force. The control is realized by means of multiple piezoelectric patches working as collocated sensors and actuators. The attention is focused is on suppression of low vibration modes. The control low is based on the velocity as well as proportional-velocity feedback. To simplify the analysis, dynamic responses of the system are obtained by adopting the static model of piezoelement-substructure coupling which is formulated assuming the perfect bonding and negligible bending stiffness and mass of distributed sensors and actuators. An influence of spatial arrangement of sensor/actuator pairs on active damping efficiency is numerically investigated. The effective set of sensor/actuators with the controllers incorporating the proportional-velocity actions is proposed. Some results based on a simple model of the mechanical system are compared with those obtained in the case when the activated section is modelled as a laminated beam, and when the dynamic model of actuator-beam interaction, which includes the finite-thickness shear bonding layer, is applied. This comparison shows that a sufficiently stiff glue layer between the actuator and basic structure, and thin piezoelements (when compared to the beam thickness) allow for using of the static coupling model.

Key words: active damping, piezoelement, segmentation

1. Introduction

Recently, there has been observed a tremendous interest in the use of piezoelectric materials as distributed actuators and sensors for controlling the response of flexible structures. The collocated sensor/actuator pairs have been applied successfully to active damping of beams, thin plates and shells (cf

Bailey and Hubbard, 1985; Newman, 1991; Dimitriadis et al., 1991). The modal response of the beam driven by multiple actuator patches was analysed by Clarc et al. (1991). Lee et al. (1991) proposed shaped piezo-film sensor/actuators designed to sense and control a particular mode of the laminated structures. This concept was applied to active damping of simply supported laminated beams e.g., by Pietrzakowski (1994). Tzou and Fu (1992) showed that segmented sensors and actuators improve the efficiency of structural vibration control of plates. The analysis is commonly based on static coupling relations between the perfectly bonded actuator and the substructure. A comprehensive static model including an elastic bonding layer, was analysed by Crawley and de Luis (1987). The dynamic approach the problem of a simply supported beam with perfectly bonded actuators was presented by Jie Pan et al. (1991). Tylikowski (1993) formulated and examined the extended dynamic coupling model including the bonding layer with the shearing stiffness and applied it to the stabilization of beam parametric vibrations (Tylikowski, 1999). Taking into account the bending effect of the actuator this model was developed by Pietrzakowski (1996).

In this study, a theoretical model of a simply supported beam configured with segmented sensor/actuator pairs is analysed. The control concept is based on separate closed-loops with velocity or proportional-velocity feedback. The analysis is focused on the ability to damp low vibration modes, which can be dangerous for the structure. The effects of dislocation of sensor/actuator pairs, taking into account their positions related to modal lines, are numerically investigated. An influence of other parameters of the control system, e.g. dimensions, material properties of piezoelements and also coupling parameters, on the active damping efficiency is not considered. The dynamic analysis is simplified by assuming the static pure bending model of piezoelement-substructure coupling with the perfect bonding and by neglecting the bending stiffness and mass of piezoelements. The results are expressed in terms of frequency response functions. Some of the responses yielded by that simple model of the system are compared with those obtained in the case when each of activated beam sections is modelled as a laminated beam, and when the bending-extensional dynamic model of the actuator-beam interaction which includes the finite shear bonding layer is applied.

2. Formulation of the problem

The considered system is a simply supported Bernoulli-Euler beam loaded by a time-dependant concentrated force $F(t)$. Piezoelectric patches are mounted to both opposite sides of the beam, and form the set of collocated sensor/actuator pairs working in independent closed-loops. In Fig.1 an example of the beam with sensor/actuator segments is shown.

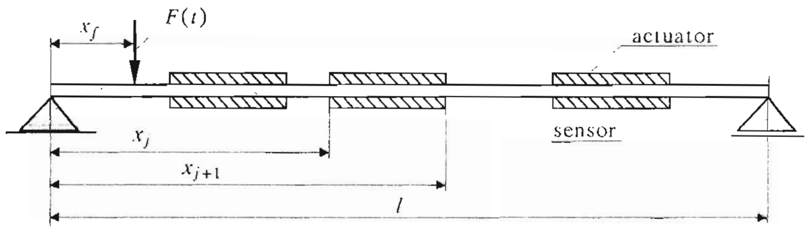

Fig. 1. Beam with a set of sensor/actuator pairs

The sufficiently thin polymer (PVDF) film sensors and beam surface are assumed to be perfectly bonded neglecting the mechanical properties of the glue material. The ceramic (PZT) actuators, depending on the applied coupling model, are perfectly bonded (static model) or mounted by means of a bonding layer with finite shearing stiffness (dynamic model).

2.1. Equations of motion for the static coupling model

Due to the static coupling model the action of the perfectly bonded actuator on the beam can be reduced to a bending moment $M_a(x,t)$ distributed along the beam. In the simplified model of the system under consideration the bending stiffness and mass of piezoelements are negligible when compared with that of the beam. Therefore, the transverse beam motion $w(x,t)$ excited by the point force $F(t)$ is described by the equation

$$E_bJ_b\Big(\frac{\partial^4 w}{\partial x^4} + \mu\frac{\partial^5 w}{\partial x^4 \partial t}\Big) + \rho_b b t_b \frac{\partial^2 w}{\partial t^2} = F(t)\delta(x-x_f) - \frac{\partial^2 w M_a(x,t)}{\partial x^2} \qquad (2.1)$$

where

- E_b – beam Young modulus
- ρ_b – mass density
- b – beam width

t_b – beam thickness
J_b – cross-sectional moment of inertia
μ – viscous internal damping parameter (serves as a basis to create a finite response at resonance)
x_f – co-ordinate of the point force
$\delta(x)$ – Dirac function.

The beam deflection develops the strain in the sensor. Due to the constitutive equation of piezoelectric material the electric displacement is as follows

$$D_3 = -d_{31}^s \sigma_s \tag{2.2}$$

where
d_{31}^s – piezoelectric constant
σ_s – sensor stress given by relation

$$\sigma_s = E_s \frac{t_b + t_s}{2} \frac{\partial^2 w}{\partial x^2} \tag{2.3}$$

where
E_s – sensor Young modulus
t_s – sensor thickness.

After integrating the charge over the sensor electrode area and applying the charge/voltage relation, the voltage produced by the sensor is determined by

$$V_s = -C_s \int\limits_0^l \frac{\partial^2 w}{\partial x^2} b_s(x)\, dx \tag{2.4}$$

where $b_s(x)$ denotes the sensor width distribution written by means of the Heaviside function $H(x)$ in the form

$$b_s(x) = b_s\Big[H(x - x_i) - H(x - x_{i+1})\Big]$$

C_s is the sensor constant

$$C_s = d_{31}^s E_s \frac{(t_b + t_s)t_s}{2A_s e_{33}}$$

where
A_s – sensor electrode area, $A_s = b_s(x_{i+1} - x_i)$
e_{33} – permittivity of sensor material.

Assuming that the sensor and actuator are electrically coupled with proportional-velocity feedback, the voltage applied to the actuator is given by

$$V_a = k_p V_s + k_d \frac{\partial V_s}{\partial t} \tag{2.5}$$

where k_p, k_d are the proportional-velocity gain factors of the control loop, respectively.

The actuator constitutive strain-voltage relation has the form

$$\lambda = \frac{d_{31}^a}{t_a} V_a \tag{2.6}$$

where
d_{31}^a – piezoelectric constant
t_a – actuator thickness.

The extensional forces produced by the actuator are transmitted to the basic structure by equivalent moments, which effectively act at the actuator edges so the pure bending of the beam occurs. Since the actuator layer is thin, the normal stress σ_a is assumed to be uniform in the cross-section

$$\sigma_a = E_a(\varepsilon_b - \lambda) \tag{2.7}$$

where
E_a – actuator Young modulus
ε_b – beam surface strain.

The bending moment $M_a(x,t)$ distributed along the actuator is calculated from the moment equilibrium of the resultant forces acting in the cross-section of the beam (cf Bailey and Hubbard, 1985)

$$M_a = C_b b_a(x) V_a(t) \tag{2.8}$$

where $b_a(x)$ describes the actuator width distribution

$$b_a(x) = b_a\Big[H(x - x_i) - H(x - x_{i+1})\Big]$$

which in general case can be different from the sensor width, C_a is the actuator constant

$$C_a = \frac{E_a E_b d_{31}^a (t_b^2 + t_a t_b)}{2(E_b t_b + E_a t_a)}$$

After substituting Eqs (2.5) and (2.4) into Eq (2.8), the control moment $M_a(x,t)$ can be rewritten as

$$M_a = C_a C_s b_a(x)\Big(k_p \int\limits_0^l \frac{\partial^2 w}{\partial x^2} b_s(x)\, dx + k_d \int\limits_0^l \frac{\partial^3 w}{\partial x^2 \partial t} b_s(x)\, dx\Big) \tag{2.9}$$

In the modified model of the system based on the static coupling the beam is divided into parts due to its geometry, for which the dynamic behaviour is described by different equations. The activated section is modelled as a laminated beam with mechanically isotropic layers symmetrically oriented about the midplane. Hence, the bending stiffness E_bJ_b can be replaced by the equivalent stiffness $(EJ)^*$ defined as follows

$$(EJ)^* = \sum_{k=1}^{3} E_kJ_k \tag{2.10}$$

where

E_k – Young modulus of the kth layer
J_k – cross-sectional moment of inertia of the kth layer about the midplane.

Assuming the same width of the layers ($b_a = b_s = b$), the equivalent mass density becomes

$$\rho = \frac{1}{t_b}(\rho_bt_b + \rho_at_a + \rho_st_s) \tag{2.11}$$

where the subscripts b, a, s indicate the beam, actuator and sensor, respectively.

The equations of system motion can be written in the form
— for activated sections

$$(EJ)^*\Big(\frac{\partial^4 w}{\partial x^4} + \mu\frac{\partial^5 w}{\partial x^4\partial t}\Big) + \rho bt_b\frac{\partial^2 w}{\partial t^2} = 0 \tag{2.12}$$

— for classical beam sections

$$E_bJ_b\Big(\frac{\partial^4 w}{\partial x^4} + \mu\frac{\partial^5 w}{\partial x^4\partial t}\Big) + \rho_bbt_b\frac{\partial^2 w}{\partial t^2} = 0 \tag{2.13}$$

The governing equations, the boundary conditions corresponding to the simply supported edges at $x = 0$ and $x = l$, the continuity conditions of deflection, slope, bending moment and transverse force between particular sections of the beam formulate the boundary value problem. The continuity of transverse force in the cross-section $x = x_f$, where the point force $F(t)$ is applied, has the form

$$\frac{\partial^3 w}{\partial x^3}\Big|_{x_f^-} = \frac{\partial^3 w}{\partial x^3}\Big|_{x_f^+} + \frac{F}{E_bJ_b} \tag{2.14}$$

2.2. Equations of motion for the dynamic coupling model

The dynamic model of interaction between the actuator and the beam is formulated by imposing the actuator extension to which the inertial forces also contribute and by assuming the pure one-dimensional shear in the elastic bonding interlayer of glue (cf Tylikowski, 1993). Due to its geometry the beam is divided into sections with separately formulated dynamic relations. The motion of the activated section is described by two coupled equations expressed in the beam transverse displacements $w(x,t)$ and the pure longitudinal displacements $u_a(x,t)$ of the actuator

$$E_bJ_b\Big(\frac{\partial^4 w}{\partial x^4}+\mu\frac{\partial^5 w}{\partial x^4\partial t}\Big)+\rho bt_b\frac{\partial^2 w}{\partial t^2}-\frac{Gbt_b}{2t_g}-\frac{\partial}{\partial x}\Big(u_a+\frac{t_b}{2}\frac{\partial w}{\partial x}\Big)=0$$

$$E_at_a\frac{\partial^2 u_a}{\partial x^2}-\rho_at_a\frac{\partial^2 u_a}{\partial t^2}-\frac{G}{t_g}\Big(u_a+\frac{t_b}{2}\frac{\partial w}{\partial x}\Big)=0 \tag{2.15}$$

where G and t_g are the shear modulus and bonding layer thickness, respectively.

The motion of other beam sections is described by the Bernoulli-Euler equation for visco-elastic beam, Eq (2.13).

Equations of motion have to satisfy boundary conditions at the beam edges at $x = 0$ and $x = l$, continuity of beam deflection, slope, curvature and transverse force at the borders of the sections, and free stress condition for the ends of actuator. The external force $F(t)$ acting at the cross-section $x = x_f$ imposes the continuity condition of transverse force given by Eq (2.14). The continuity of transverse force at the border co-ordinate x_j is found taking into account the shear stresses transmitted by the bonding layer

$$E_bJ_b\frac{\partial^3 w}{\partial x^3}\Big|_{x_j^-}=E_bJ_b\frac{\partial^3 w}{\partial x^3}\Big|_{x_j^+}-\frac{bt_b}{2}\frac{G}{t_g}(u_a-u_b)\Big|_{x_j^+} \tag{2.16}$$

where u_b is the beam surface longitudinal displacement, $u_b=-\frac{t_b}{2}\frac{\partial w}{\partial x}$.

The free stress condition at the actuator borders is based on the stress-strain relation

$$\sigma_a=E_a(\varepsilon_a-\lambda) \tag{2.17}$$

and has the form

$$\varepsilon_a(x_j)=\frac{\partial u_a}{\partial x}\Big|_{x_j}=\lambda \tag{2.18}$$

where the piezoelectric strain λ is given by Eq (2.6) with the voltage supplying of the actuator determined by the feedback rule, Eq (2.5).

The steady-state responses are analysed so the force loading the beam is assumed to be a harmonic single frequency function, $F(t) = F_0 \exp(\mathrm{i}\omega t)$. On this assumption the solutions of dynamics equations are harmonic with the same angular velocity as the excitation

$$\begin{aligned} w(x,t) &= W(x)\exp(\mathrm{i}\omega t) \\ u_a(x,t) &= U_a(x)\exp(\mathrm{i}\omega t) \end{aligned} \tag{2.19}$$

The governing equations and the boundary conditions mentioned above lead to a boundary value problem. By solving this problem, the transverse displacement response to the force loading the beam are obtained and expressed in terms of frequency characteristics.

3. Results

Calculations are performed for the simply supported steel beam of length $l = 380\,\mathrm{mm}$, width $b = 40\,\mathrm{mm}$ and thickness $t_b = 2\,\mathrm{mm}$. The collocated sensor/actuator pairs are of the same length, $l_p = 38\,\mathrm{mm}$, and electromechanical properties. The thickness of piezoceramic actuator is $t_a = 0.2\,\mathrm{mm}$ and piezofilm sensor $t_s = 0.04\,\mathrm{mm}$. The amplitude of loading force F is assumed to be equal to unity. Material parameters of the system used in calculation are listed in Table 1.

Table 1. Material parameters

Material parameter	Beam	Actuator PZTG-1195	Sensor PVDF
ρ [kg/m^3]	7800	7280	4500
E [N/m^2]	$2.16 \cdot 10^{11}$	$6.3 \cdot 10^{10}$	$2 \cdot 10^{9}$
d_{31}^a [m/V]	–	$1.9 \cdot 10^{-10}$	–
d_{31}^s [C/N]	–	–	$3.3 \cdot 10^{-11}$

A small passive damping due to the beam material friction in terms of the Kelvin-Voigt model with the viscous constant $\mu = 10^{-7}\,\mathrm{s}$.

Assuming the simplified static coupling model, the influence of a number and dislocation of sensor/actuator pairs on the structural vibration control is numerically investigated. The control system ability to damp low modes is analysed by considering the following spatial arrangements of sensor/actuator segments:

1 – one segment located between $x_1 = 76$ mm and $x_2 = 114$ mm;
2 – two segments between: $x_1 = 76$ mm and $x_2 = 114$ mm, $x_3 = 266$ mm and $x_4 = 304$ mm;
3 – three segments between: $x_1 = 76$ mm and $x_2 = 114$ mm, $x_3 = 171$ mm and $x_4 = 209$ mm, $x_5 = 266$ mm and $x_6 = 304$ mm;
4 – four segments between: $x_1 = 76$ mm and $x_2 = 114$ mm, $x_3 = 133$ mm and $x_4 = 171$ mm, $x_5 = 209$ mm and $x_6 = 247$ mm, $x_7 = 266$ mm and $x_8 = 304$ mm.

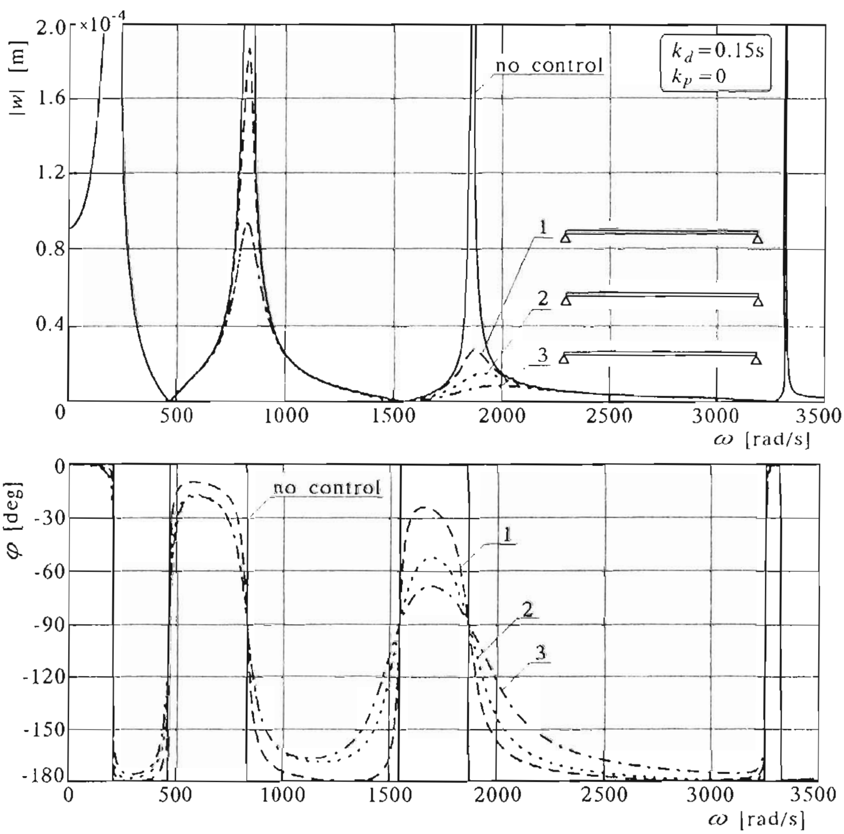

Fig. 2. Influence of the sensor/actuator system arrangement

The considered systems, except arrangement No. 1, are symmetrical about the central point of the beam. The near edges piezoelements have their centers on the fourth modal lines. In the case of layout No. 4 each of the inner sensor/actuator pairs is mounted symmetrically about the fifth modal line. Active damping attained by the independent control loops with the same control function and parameters. The results of computation are presented in terms of transverse displacement frequency responses calculated at the actuator field point $x = 90\,\mathrm{mm}$.

At first, the velocity feedback with the gain factor $k_d = 0.15\,\mathrm{s}$ is assumed. In Fig.2 the amplitude and phase of beam deflection w obtained for sensor/actuator arrangements No. 1, No. 2, and No. 3 are shown.

The effect of active damping can be noticed only for the second and third modes. The actuators located on the modal lines result in non-controllability of the fourth vibration mode. In layout No. 3 the middle actuator location causes that its influence on the second mode damping is negligible. These observations are proved by the phase frequency functions.

Active damping of low modes, including the fourth one, becomes more effective when arrangement No. 4 is applied (Fig.3). In this case, because of the location of inner actuators in the area of sufficient strains, the fourth mode amplitude is reduced considerably.

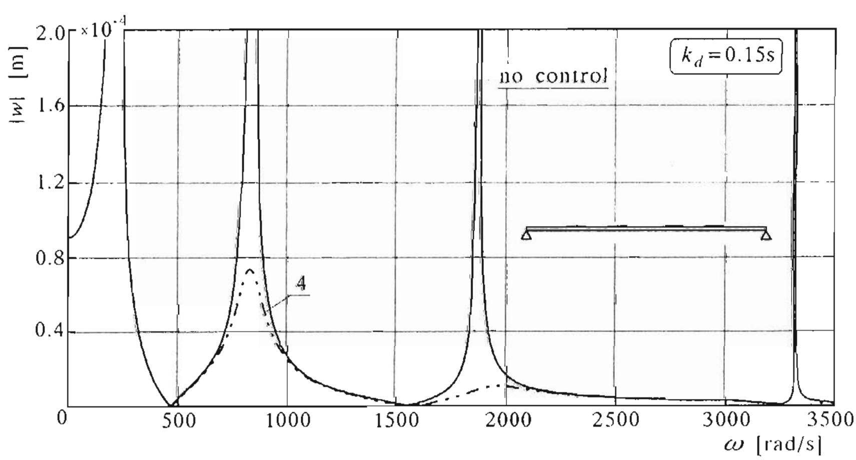

Fig. 3. Damping efficiency of sensor/actuator pairs arrangement No. 4

The effect of active damping of the hardest to suppress first mode is shown in Fig.4. As expected, damping is much more intensive for the systems with multiple activated segments. Comparing the plots obtained for arrangements No. 3 and No. 4 it can be noticed that, in the considered case, replacing of the middle actuator by two symmetrically located actuators results in a relatively small reduction of the resonant amplitude.

The ability of control system to damp low vibration modes can be improved

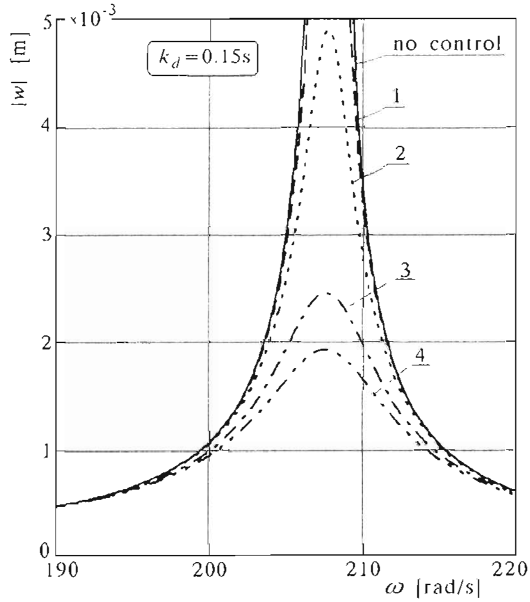

Fig. 4. First mode damping efficiency

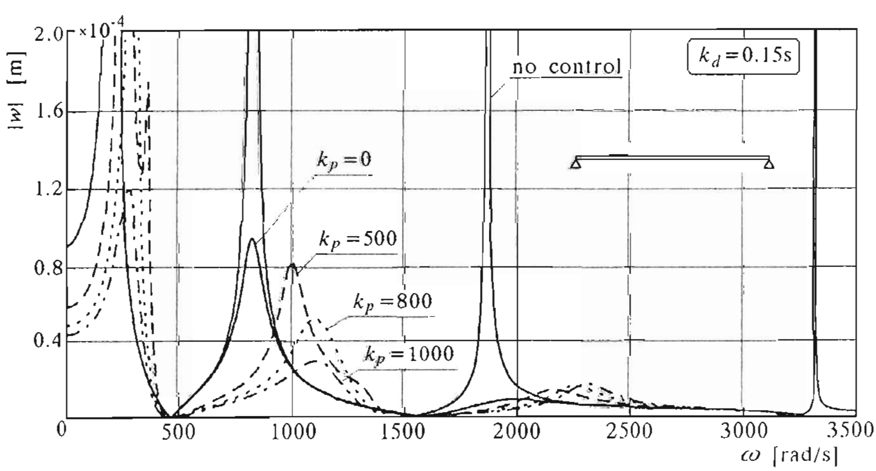

Fig. 5. Effect of the proportional-velocity feedback

by applying the proportional-velocity feedback. The results obtained for the modified control function with the constant coefficient $k_d = 0.15$ s and varying proportional gain factor k_p are presented in Fig.5.

The proportional term of control function causes noticeable suppression of the first and second vibration modes. The desired effect is achieved for coefficients $k_d = 0.15$ s and $k_p = 1000$. Moreover, the proportional term results in an increase in the stiffness of vibrating system so the resonant amplitudes appear at relatively higher frequencies. Certainly, sufficiently high values of amplification coefficients k_d and k_p create a hazard of exposing the actuators to high voltages.

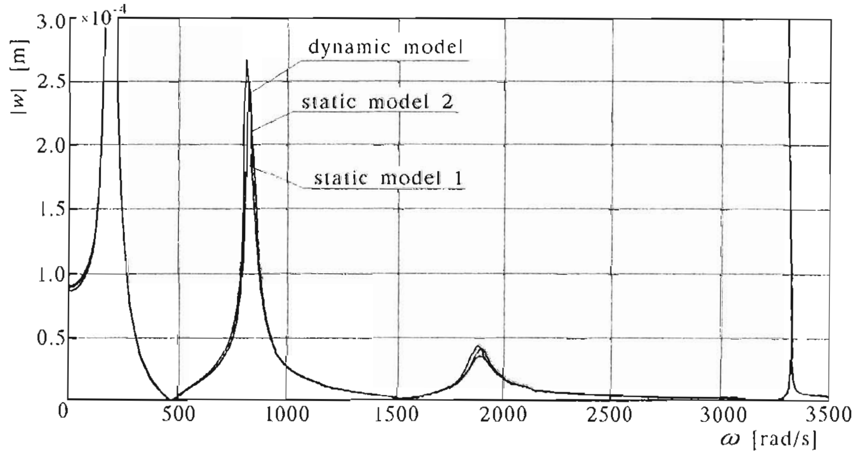

Fig. 6. Influence of the coupling model on the controlled beam response.
Arrangement No. 1

The interesting problem is how the model of actuator-substructure coupling influences the dynamic response. The comparison is performed for the beam with sensor/actuator arrangement No. 1. For the dynamic model the shearing stiffness of bonding layer measured by the ratio G/t_g is assumed to be of relatively large value $G/t_g = 5 \cdot 10^{12}\,\mathrm{N/m^3}$. The velocity feedback with the gain factor $k_d = 0.15$ s is applied. The plots of transverse displacement within the whole observed frequency range are shown in Fig.6, and for the first mode – in Fig.7.

As it is seen, both the modified static model (type 2) with the activated segments treated as a laminated beam and the dynamic bending-extensional model with a sufficiently stiff bonding layer yield transverse displacements and also resonant amplitudes of similar values. The activated segments modelled

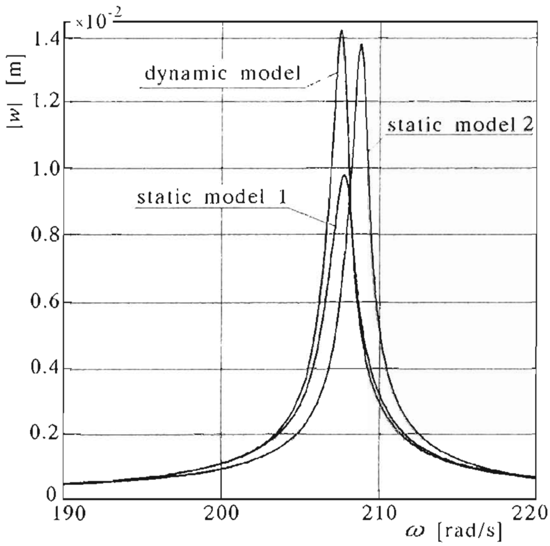

Fig. 7. Damping of the first mode depending on the applied coupling model. Arrangement No. 1

as a laminated beam have a greater bending stiffness then other beam sections, therefore, peaks appear at higher frequencies. The diagrams resulting from the simple static model show the lowest resonant amplitudes. They appear at frequencies very close to those of the peaks referring to the dynamic model because in both these cases the bending stiffness of the actuator layer is neglected.

4. Conclusions

The sensor/actuator pairs arrangement can be adapted to obtain sufficient damping of the particular vibration modes. The efficiency of control system strongly depends on the sensor/actuator pairs location in relation to the modal lines. The most preferable locations correspond to the fields of highest surface strains. The sensor/actuator pair positioned in the area of modal line, where a relatively small curvature and antisimmetric strain distribution exist (because of displacement inflexion), significantly decreases observability and controllability of this mode. The suppression of low modes (first and second) can be improved by applying the control law, which includes not only the velocity but also the proportional-velocity actions. The use of multiple relatively short actuators instead of a longer one allows for avoiding of the control system damage by bending cracks of piezoceramic material. The comparison between the actuator-substructure coupling models shows that for a sufficiently stiff bonding layer and relatively thin piezoelectric patches the dynamic analysis can be performed adopting the static model of interaction and the perfect bonding assumption. A quite good agreement with the dynamic model is obtained for the activated section modelled as a laminated beam. It should be emphasized that the assumptions of perfect bonding and also constant beam stiffness and mass density result in a noticeable increase in the active damping efficiency when compared with those yielded by the more advanced models.

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References

1. BAILEY T., HUBBARD J.E., 1985, Distributed Piezoelectric-Polymer. Active Vibration Control of a Cantilever Beam, *Journal of Guidance, Control and Dynamics*, **8**, 605-611

2. CLARC R.L., FULLER CH.R., WICKS A., 1991, Characterization of Multiple Piezoelectric Actuators for Structural Excitation, *J. Acoust. Soc. Am.*, **90**, 346-357

3. CRAWLEY E.F., DE LUIS J., 1987, Use of Piezoelectric Actuators as Elements of Intelligent Structures, *AIAA J.*, **25**, 1373-1385

4. DIMITRIADIS E., FULLER C.R., ROGERS C.A., 1991, Piezoelectric Actuators for Distributed Vibration Excitation of Thin Plates, *Journal of Applied Mechanics*, **113**, 100-107

5. JIE PAN, HANSEN C.H., SNYDER S.D., 1991, A Study of the Response of a Simply Supported Beam to Excitation by a Piezoelectric Actuator, *Proceedings of the Conference on Recent Advances in Active Control of Sound and Vibration*, Virginia, 39-49

6. LEE C.K., CHIANG W.W., O'SULLIVAN T.C., 1991, Piezoelectric Modal Sensor/Actuator Pairs for Critical Active Damping Vibration Control, *J. Acoust. Soc. Am.*, **90**, 374-384

7. NEWMAN M.J., 1991, Distributed Active Vibration Controllers, *Proceedings of the Conference on Recent Advances in Active Control of Sound and Vibration*, Virginia, 579-592

8. PIETRZAKOWSKI M., 1994, Modal Control of One-Dimensional Composite Plates, *Machine Dynamics Problems*, **9**, 83-98

9. PIETRZAKOWSKI M., 1997, Dynamic Model of Beam-Piezoceramic Actuator Coupling for Active Vibration Control, *J. of Theoretical and Applied Mechanics*, **35**, 1, 3-20

10. TYLIKOWSKI A., 1993a, Dynamics of Laminated Beam with Active Fibers, *Proceedings of the 3-rd Polish-German Workshop on Dynamical Problems in Mechanical Systems*, R. Bogacz, and K. Popp (edit.), Wierzba, IPPT PAN, 67-78

11. TYLIKOWSKI A., 1993b, Stabilization of Beam Parametric Vibrations, *J. of Theoretical and Applied Mechanics*, **31**, 3, 657-670

12. TYLIKOWSKI A., 1999, Stabilization of Beam Parametric Vibrations by means of Distributed Piezoelectric Elements, *J. of Theoretical and Applied Mechanics*, **37**, 2, 241-254

13. TZOU H.S., FU H.Q., 1992, A Study on Segmentation of Distributed Piezoelectric Sensors and Actuators; Part 1 – Theoretical Analysis, *Active Control of Noise and Vibration, ASME*, DSC-**38**, 239-46

Segmenty piezoelektryczne w sterowaniu procesem drgań

Streszczenie

Praca dotyczy aktywnego tłumienia drgań swobodnie podpartej belki obciążonej zmienną w czasie skupioną siłą. Sterowanie jest realizowane za pomocą rozłożonych elementów piezoelektrycznych tworzących zespół par pomiarowo-wykonawczych. Głównym celem jest ograniczenie amplitud niskich postaci drgań. Jako funkcję sterowania przyjęto prędkościowe oraz proporcjonalno-prędkościowe sprzężenie zwrotne. W analizie dynamicznej, w celu uproszczenia obliczeń, zastosowano statyczny model oddziaływania piezoelementów idealnie połączonych z belką, zaniedbując ich masę i sztywność zginania. Na podstawie wyników symulacji cyfrowej pokazano wpływ rozmieszczenia par pomiarowo-wykonawczych na skuteczność aktywnego tłumienia wybranych postaci drgań. Zaproponowano efektywny, segmentowy układ pomiarowo-wykonawczy z proporcjonalno-prędkościową funkcja regulacji. Wyniki odpowiadające uproszczonemu modelowi układu mechanicznego porównano z rezultatami uzyskanymi przy zastosowaniu teorii belek laminowanych do opisu właściwości odcinków belki z piezoelementami, a także, opisując oddziaływanie elementu wykonawczego

przez dynamiczny model uwzględniający ścinaną warstwę łączącą. Przedstawione porównanie uzasadnia stosowanie uproszczonego modelu obliczeniowego w przypadku dostatecznie sztywnej warstwy kleju i stosunkowo cienkich, w odniesieniu do grubości belki, elementów piezoelektrycznych.

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