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# NEW ENERGY MODEL OF FATIGUE DAMAGE ACCUMULATION AND ITS VERIFICATION FOR 45-STEEL

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A method of fatigue damage accumulation based upon application of energy indicators of the fatigue process is proposed in the paper. The approach is based on the assumption that the constant fatigue damage lines exist and a diagram of the accumulated dissipation energy represents their boundary line being a function of the number of cycles until fatigue failure at a constant level of the total strain. An experimental verification shows a satisfactory agreement between the fatigue life calculation results (based upon the hypothesis) and test results. The method is sensitive to a change of both the sequence of loading cycles and number of cycles on a particular level. Therefore, the "cycle by cycle" calculation course is possible.

Key words: fatigue damage accumulation, random loading, programmed loading

### Notation

- i number of strain programme loop
- j strain level in the programme
- k total number of levels in the programme
- k' level in the programme, at which the calculation process is ended
- n current number of cycles
- $n_o$  total number of cycles in a period of the strain programme (block contents)
- $n_{oj}^{i}$  number of cycles for the jth strain level and ith programme repetition

$n^i_{j-1,j}$	-	number of cycles corresponding to the passage along a
		constant damage line from $j-1$ level to the $j$ th one for
		the ith programme repetition
$n_c$	_	number of cycles until fatigue failure in the case of ran-
		dom or programmed loading
$N_f$	_	fatigue life for constant amplitude loading
$N_{fj}$	-	fatigue life for constant amplitude strain $\varepsilon_{acj}$
$U_D$	_	dissipated energy unit volume corresponding to the area
		of hysteresis loop, [MNm/m <sup>3</sup> ]
$arepsilon_{ac}$	_	amplitude of total strain
$\varepsilon_{acj}$	_	amplitude of total strain for the $j$ th strain level
$\varepsilon_f'$	_	fatigue ductility coefficient
$\lambda$	_	number of the strain programme repetitions
$\sigma_f'$	_	fatigue strength coefficient, [MPa]
ζ	_	coefficient of spectrum density
$\Sigma U_D(n)$	_	cumulated dissipated energy unit volume after $n$ cycles
		of constant amplitude loading, [MNm/m <sup>3</sup> ]
$\Sigma U_D(N_f)$		cumulated dissipated energy unit volume at the moment
- <b>、 ,</b>		of fatigue failure, [MNm/m <sup>3</sup> ].

## 1. Introduction

Despite of many attempts at formulation of the exact physical description, due to a complex nature of the fatigue process of materials and construction parts the hypotheses of fatigue damage accumulation are still applied to calculations of fatigue life of construction elements under random and programmed loading conditions. Among dozens of known hypotheses (cf Puskar and Golovin, 1985; Hwang and Han, 1986), there are many which have only phenomenological background or no physical meaning at all (cf Kocańda and Szala, 1997). Therefore, they are based upon only general macroscopic fatigue properties.

Depending on the test conditions (i.e. type and nature of the loading history etc.) and a choice of the constants, which are used in majority of them – we can obtain the calculation results of fatigue lives which give better or worse approximation of the test results. The simplest method consists in application of the Palmgren-Miner hypothesis and the results can vary from 0.1 to 10 times of the experimental fatigue life (cf Szala, 1980). Despite of this, the method is most frequently used for primary calculations.

The fatigue damage accumulation model based upon the lines of constant fatigue damage (Hashin and Rotem, 1978) called it the equivalent loading postulate reveals a more general character and the results of calculations are closer to the experimental ones. The hypothetical zones, in which the degree of damage is the same, create the lines of constant fatigue damage. There have been some attempts to assign these lines to the boundaries of zones in which the same fatigue/physical mechanism of damage appears: in the case of metals – formation of slips, progress of slip bands, formation of the critical slip bands, microcracks propagation and joining of microcracks; while in the case of polymers – existence of microcracks of specific lengths, occurrence and development of fibre and matrix decomposition, cracks of fibres, etc.

Basing upon the experimental results e.g., Kocańda (1985), Owen (1980), it can be stated that the aforementioned charts have the form of a straight line in the double logarithmic co-ordinate system (loading – number of cycles). Most frequently it is assumed that these lines form a family of lines limited at the top by the Wöhler curve, which is one its elements.

Basing on the above assumption, several hypotheses have been formulated e.g. for metals Subramanyan (1976), Schott (1977), Szala (1981); for polymers Hashin and Rotem (1978), Daniel and Charewicz (1986), Szala and Topoliński (1990). These hypotheses differ from each other mainly in an origin position of the family of lines, which are considered as the lines of constant fatigue damage. Additionally, they distinguished the damage areas: for example Szala (1981) clearly distinguished the damage areas of low cycle fatigue from those for which the damage accumulation is not taken into consideration. Szala and Topoliński (1990) distinguished between the regions of heat damage and mechanical damage, respectively.

From the early 1980s, the energetic approach to description of the fatigue process has been developed. Furthermore, the attempts at determination of an energetic fatigue criterion had their after-effects on construction of new methods for fatigue life calculation. In the energetic approach the interaction between stresses and strains is taken into account thereby it should result in a more accurate description of the fatigue process with a possibility of using the energetic fatigue criterion within the whole fatigue range (i.e. low-cycle and high-cycle fatigue).

For metals, the energetic approach to hypothesis of constant damage lines was presented by Gołoś (1989) and, in a general form, by Kujawski (1991).

According to these papers, the boundary line should be an energy fatigue curve, i.e. the strain energy density (being a sum of density of dissipation of energy in one cycle due to the occurrence of plastic strain in the saturation

state of material and density of elastic energy in a tension semi-cycle) as a function of the number of cycles. In both papers a satisfactory agreement between the experimental and calculated results was obtained.

For polymer composites, an original method of fatigue life evaluation in energetic approach, also based upon a hypothesis of constant fatigue lines was presented by Topoliński (1997). This hypothesis states that the boundary line represents the accumulated dissipation energy as a function of a number of fatigue cycles until failure and, furthermore, the accumulation of fatigue damages is connected with the accumulation of dissipation energy (increase in the dissipation energy for a particular loading history and a current cycle determinates a new position of the constant fatigue lines). It was assumed that for the group of materials in which substantial non-plastic strains occur, the proposed hypothesis was applicable within the whole range of fatigue failure.

In the paper, the assumption is made that the last of the aforementioned methods should be applicable to metal specimens in the case of substantial plastic strains.

Our considerations aim at presentation of the results of fatigue life calculation (according to the hypothesis) and the experimental results obtained under random and programmed loading conditions (the total strain  $\varepsilon_{ac}$  did not exceed 1.5%).

# 2. Energetic hypothesis of fatigue damage accumulation

Basing on the analysis of energetic parameters variability during the fatigue process of polymer composite elements used in constructions Topoliński (1997), it can be easily seen that in a double logarithmic co-ordinate system the accumulated dissipation energy  $\Sigma U_D(n)$  charts are linear and take the positions characteristic for particular loading levels.

The end points of the charts of accumulated dissipation energy  $\Sigma U_D(n)$  determinated the energetic fatigue curve i.e. variability of total cumulative dissipation energy against number of cycles to a failure  $\Sigma U_D(N_f)$ . This chart can be presented as a straight line, what was done for polymer composites by Topoliński (1997) and for metals, e.g. by Goss (1982). For the assumed hypothesis the chart  $\Sigma U_D(N_f)$  will be a boundary line.

The starting points of the charts  $\Sigma U_D(n)$  are calculated from the dissipation energy for the first loading cycles  $\Sigma U_D(1)$  for each strain level. In practice, they correspond to the first measurable loops for particular loading levels.

The charts  $\Sigma U_D(n)$  make it possible to estimate the fatigue damage accumulation stage using its natural connection with the cumulative dissipation energy.

The investigations into fatigue damage (cf Topoliński, 1997) using the linear hypotheses, in which the ratios of number of cycles  $n_{oj}/N_{fj}$  or ratios of dissipation energy  $\Sigma U_D(n_{oj})/\Sigma U_D(N_{fj})$  are summed, allows for determination of a hypothetical position of the constant damage line indicating the energetic levels, for which the damage is equal.

It is assumed that these lines constitute the part of the family of lines, origin of which lies at the intersection point of the top boundary line and the number of cycle axis. Obviously, in a double logarithmic co-ordinate system, the intersection point of number of cycles axis and the boundary line for the zero energy level can not be considered. Therefore, in practice, it is possible to assume the level, at which the fatigue life determined by the Wöhler curve is greater than 10<sup>10</sup> cycles, i.e., we obtain the intersection point at a very small, almost equal to zero, value of dissipation energy. The courses of the lines are presented in Fig.1.

Basing on the above presented considerations, the basic assumptions of the proposed hypothesis of fatigue damage accumulation, have been formulated (using the notation after Szala (1981)):

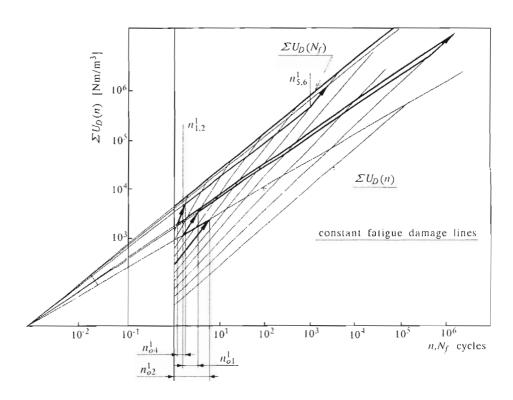
1. The boundary line is described by the formula

$$\Sigma U_D(N_f) = aN_f^b \tag{2.1}$$

- 2. The point corresponding to the lowest measurable level of dissipation energy is the origin of the constant fatigue lines
- 3. The increment of dissipation energy due to performance of  $n_{oj}^i$  loading cycles for the  $\varepsilon_{acj}$  level depends upon the present stage of fatigue process (in the meaning of energetic level determined by the constant fatigue line relevant to situation after performance of  $n_{0(j-1)}^i$  cycles for  $\varepsilon_{ac(j-1)}$ ) and the increase takes place along the line  $\Sigma U_{Dj}(n)$ . The superscript i indicates the number of the current loading cycle and it does not show the change of number of loading cycles, therefore e.g.  $n_{o1}^5 = n_{o1}^7$
- 4. The fatigue failure occurs if the number of cycles on a given loading level reaches the critical value, i.e.

$$n_{j-1,j}^{i} + n_{oj}^{i} \geqslant N_{fj}$$
 (2.2)

and at that moment k'=j and  $\lambda=i$ 



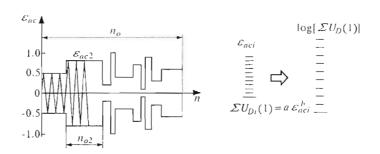


Fig. 1. Schemes of the fatigue damage cumulation process and the loading programme

5. The fatigue life for the block type loading including k loading levels is equal to the sum of cycle numbers of particular loading levels which had been performed until condition (2.4) was fulfilled, therefore

$$n_c = (\lambda - 1)n_o + \sum_{j=1}^{k'-1} n_{oj}^{\lambda} + n_c^*$$
 (2.3)

where

$$n_c^* = N_{fk'} - n_{k'-1,k'}^{\lambda} \tag{2.4}$$

In Fig.1, the beginning of damage accumulation algorithm (for five first levels of loading – from  $\varepsilon_{ac1}$ ,  $n_{o1}^1$  to  $\varepsilon_{ac5}$ ,  $n_{o5}^1$ ) for the described hypothesis in the case of 10-level block of loading and irregular sequence of levels – is presented. This block of loading was used in calculations and investigations. The so-called, coefficient of spectrum density was  $\zeta = 0.56$ . The lines  $\Sigma U_p(n)$  correspond to characteristics of increase of dissipate energy for all loading levels. In Fig.1, it is lack of an evidence of the case, which existence is possible, when a constant damage line does not cross the line  $\Sigma U_{D(j+1)}(n)$  in the range of fatigue life greater than one cycle. It could occur in the case of transition from low level j to high level j+1. It means that the degree of fatigue damage obtained previously is lower than that obtained in the preceding loading cycle  $\varepsilon_{ac(j+1)}$ . It was assumed that accumulation of the dissipation energy starts once again at the current level of loading.

# 3. Experimental verification

## 3.1. Specimens for fatigue tests

The specimens were made of the normalised 45-steel which chemical constitution was as follows: C - 0.476%; Mn - 0.593%; P - 0.015%; S - 0.027%; Cr - 0.16%; Fe - 98,1%. The following mean values of ultimate tensile strength  $S_u = 700$  MPa and yield point  $S_y = 430$  MPa were assumed. The size and shape of the specimen prepared according to the standard [18] are presented in Fig.2.

## 3.2. Loading programmes

The fatigue tests for irregular sequence of loading were preceded by the constant amplitude sinusoidal loading tests. These tests were performed for

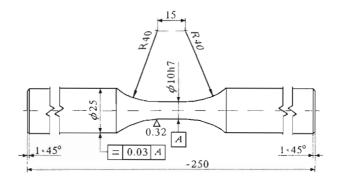
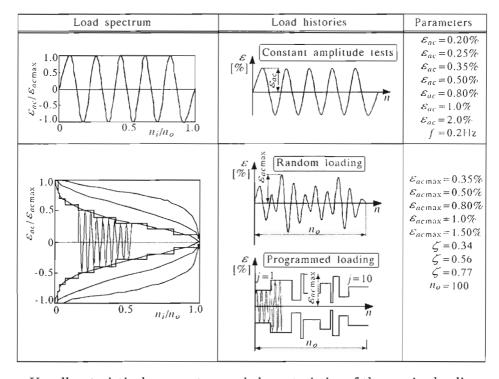


Fig. 2. Shape and size of the specimen for the fatigue tests

a constant total strain amplitude  $\varepsilon_{ac}$ , according to the standard [18]. The parameters of constant amplitude loading are given in Table 1.

Table 1. Parameters of the loading programmes



Usually, statistical parameters and characteristics of the service loading or the loading spectrum create initial data for construction of a loading program for the programmed test. In the present work, the irregular loading was taken into consideration. The assumed loading is represented by the following betadistribution

$$f(\varepsilon_{ac}) = \frac{1}{B(\alpha, \beta)} \varepsilon_{ac}^{\alpha - 1} (1 - \varepsilon_{ac})^{\beta - 1}$$
(3.1)

where

B – beta function in terms of the gamma function  $B(lpha,eta)=\Gamma(lpha)\Gamma(eta)/\Gamma(lpha+eta)$ 

 $\alpha, \beta$  - parameters of the beta distribution.

When changing values of the parameters  $\alpha$  and  $\beta$ , different distributions of the strain amplitudes were obtained. Two different forms of the loading spectrum were applied to the tests (depending on the beta distribution):

- Pseudo-random loading,
- Blocks of cycles, i.e. an irregular sequence of loading levels (basing upon the analysis of pseudo-random loading).

Both the loading histories consisted only of sinusoidal cycles (mean value equal to zero, R=-1). Different histories were determined by the maximal strain amplitude  $\varepsilon_{ac\,\text{max}}$  and the coefficient of spectrum density calculated as follows

$$\zeta = \sum_{j=1}^{k} \frac{\varepsilon_{acj}}{\varepsilon_{ac\,\text{max}}} \frac{n_j}{n_o} \tag{3.2}$$

The characteristic parameters of loading histories are given in the Table 1.

The fatigue tests for constant amplitude and variable (irregular) amplitude tests were performed by means of the testing machine INSTRON 8501. During the constant amplitude tests the first three cycles of loading were recorded, and then, those which characterised the change of cyclic properties. For irregular loading histories, specific blocks of loading (100 cycles) were recorded.

#### 4. Test results

## 4.1. Results of constant amplitude tests

Forces and strains, recorded during the constant amplitude tests, make it possible to calculate the areas of hysteresis loops which determine the elementary dissipation energy (of plastic strain)  $U_D$ . The course of changes  $U_D = f(n)$  until fatigue failure for particular constant levels of total strain  $\varepsilon_{ac}$  is presented in Fig.3.

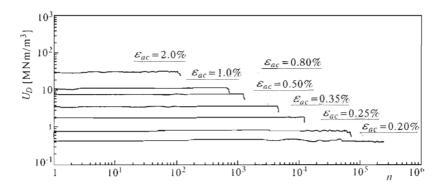


Fig. 3. Changes of elementary plastic strain energy versus the number of cycles

Fig.4, in turn, presents the process of plastic strain energy accumulation as a function of the number of cycles during fatigue process  $[\Sigma U_D = f(n)]$ . In both figures only selected, characteristic histories (for the assumed levels of strain  $\varepsilon_{ac}$ ) are shown.

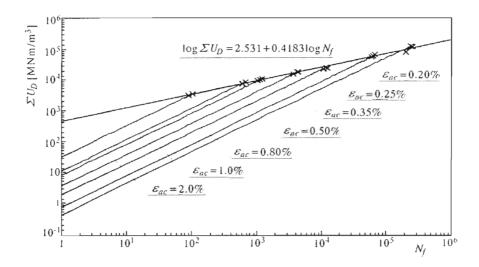


Fig. 4. Process of plastic strain energy accumulation as a function of the number of cycles

In Fig.4 the points representing the values of cumulated dissipation energy at the moment of fatigue failure are also presented for all the constant amplitude tests made. The results (including the end points of energy accumulation lines) were approximated by means of the straight line represented by the

following regression formula

$$\log U_D = 2.531 + 0.4183 \log N_f \tag{4.1}$$

where the correlation coefficient was  $r^2 = 0.99343$ .

For the investigated 45-steel, this line is considered as the boundary line of fatigue damage accumulation.

Basing on the obtained test results, the relationship between  $U_D$  in the first measured cycle of loading  $\Sigma U_D(1)$  and the total strain  $\varepsilon_{ac}$  was established. The results obtained from the fatigue tests together with the approximation curve are presented in Fig.5. The obtained function is represented by the formula

$$U_D = 67173\varepsilon_{ac}^{1.902} \tag{4.2}$$

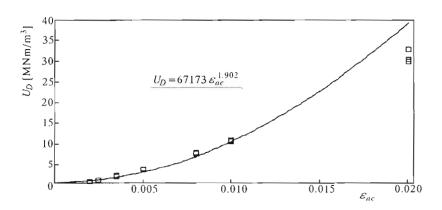


Fig. 5. Relationship between the elementary plastic strain energy in the first loading cycle and the total strain

## 4.2. Results of variable amplitude tests

The test results for variable amplitude investigations are presented using the fatigue curves in the co-ordinate system  $\varepsilon_{ac \max} = f(2N_f)$ .

Due to slight differences between the fatigue lives obtained form the tests with the two loading histories (pseudo-random and block) applied and for the sake of clarity, only the test results for the pseudo-random loading are presented in Fig.6. Comparative analysis of test results for both the loading types will be the subject matter of a separate paper.

In Fig.6, the test results obtained from the constant amplitude tests are presented. These results can be approximated by means of the Morrow equ-

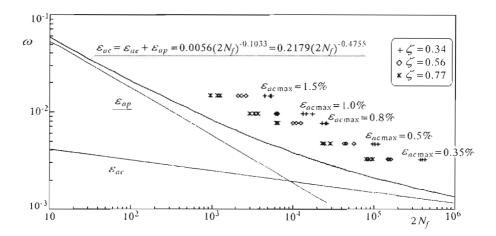


Fig. 6. Fatigue lives obtained from the for constant amplitude and variable irregular amplitude tests, respectively

ation (cf Morrow, 1965) in the following form

$$\varepsilon_{ac} = \frac{1176.3}{E} (2N_f)^{-0.1033} + 0.2179(2N_f)^{-0.4755}$$
(4.3)

As it was to be expected, for all considered loading histories of the programmed loading (i.e. determined by a sequence of cycles, maximal amplitude of total strain  $\varepsilon_{ac\,\text{max}}$  and coefficient of spectrum density  $\zeta$ ) the fatigue life decreases and tends to the value obtained from the constant amplitude tests (i.e. when  $\zeta = 1$ ).

# 5. Results of fatigue life calculations

The calculations of fatigue life were made basing on the presented energetic hypothesis of fatigue damage accumulation, using the data obtained from fatigue tests of 45-steel i.e. the boundary line  $\Sigma U_D(N_f)$ , changes of dissipation energy for the first cycle  $\Sigma U_D(1) = f(\varepsilon_{ac})$  and the Morrow curve.

The calculations of fatigue life were made for all the types of loading applied, taking into account the sequence of cycles in block loading and pseudo-random loading, for the three values of coefficient of spectrum density  $\zeta=0.34;\ 0.56;\ 0.77$  and for the five levels of  $\varepsilon_{ac}=0.35;\ 0.5;\ 0.8;\ 1.0$  and 1.5%.

The results of calculations are presented in Table 2 in terms of the of  $n_{ccal}/n_{cex}$  ratios, where:  $n_{ccal}$  stands for the calculated fatigue life and  $n_{cex}$  represents the experimental fatigue life.

**Table 2.** Values of  $n_{ccal}/n_{cex}$  ratios for the irregular loading for  $n_o = 100$  cycles

	$n_{ccal}/n_{cex}$							
$\varepsilon_{ac { m max}}$	$\zeta = 0$	.34	$\zeta = 0.56$		$\zeta = 0.77$			
[%]	random	block	random	block	random	block		
0.35	1.70	1.44	1.15	1.21	0.88	0.89		
0.5	1.15	1.43	0.92	0.94	0.82	0.85		
0.8	1.23	1.15	0.89	0.82	0.75	0.70		
1.0	0.99	1.08	0.78	0.74	0.76	0.55		
1.5	0.96	0.91	0.69	0.56	0.76	0.55		

Additionally, the calculations were made for the considered types of loading when  $\zeta=0.56$  when changing the value of total number of cycles in a period of a programme  $n_o=500$ ; 1000 and 5000 cycles. The results are presented in Fig.7, where  $n_{ccal}$  means the calculated fatigue life in the case of block irregular loading for  $n_o=100$  cycles, whereas  $n'_{ccal}$  denotes the calculated fatigue life for a variable width of blocks of the irregular loading (Fig.7a) or variable sequence of cycles in the block of a constant width i.e.  $n_o=1000$  cycles (Fig.7b).

# 6. Analysis of the results and conclusions

From the data presented in Table 2 it is clearly seen that the fatigue life calculated using the proposed hypothesis is greater than the experimental fatigue life (maximal 70%) in the case of a significant percentage of low strain levels i.e. for  $\zeta = 0.34$  and  $\varepsilon_{ac} = 0.35$ ; 0.5 and 0.8% as well as for  $\zeta = 0.56$ , 0.77 and  $\varepsilon_{ac} = 0.35$ . It is due to that in fatigue life calculation the Morrow curve was applied within the range of fatigue lives greater than  $10^6 \div 10^7$  cycles. The Morrow curve in this range is determined by extrapolation of test results for the range up to  $10^5$  only. For other experimental conditions, the calculated fatigue lives are very close to the experimental ones (maximal differences do not extend 45%) with a tendency to increase the differences as  $\varepsilon_{ac\,max}$  grows. The above conclusions are correct for the pseudo-random and block type of loading. The relationship between  $n_{ccal}$  and  $n_{cex}$  is illustrated in Fig.8a for the pseudo-random loading and in Fig.8b for the block type.

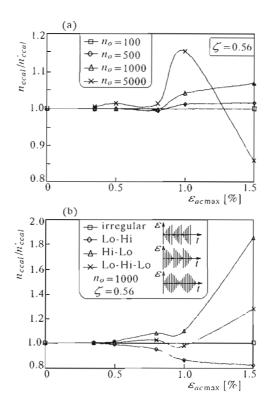


Fig. 7. Influence of loading block form on the calculated fatigue life

From the figures a good agreement between the results obtained from calculation and experiment, respectively, can be clearly seen within the range of  $4\times10^4$  to  $7\times10^5$  cycles. Overestimated fatigue lives (greater than the experimental ones), except for high strain levels for  $\zeta=0.34$ , were obtained only for fatigue lives greater than  $7\times10^5$  cycles i.e. for the blocks of loading of a big number of cycles within the high cycles fatigue range.

In the case of variable number of cycles in the loading block (from 100 to 500 cycles) when changing the sequence of the loading levels for the assumed number of cycles in the block (Fig.7) it can be proved that the proposed method is sensitive to these changes.

In a general case, an increase in the number of cycles in a block reduces the fatigue life (except for the situation when the number of cycles in the loading block is greater then fatigue life – in this case calculations of only a small part of a block is used). The differences between the obtained fatigue lives, for the assumed conditions, do not exceed 15% and it can be stated that the greater

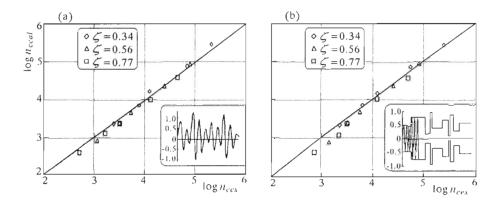


Fig. 8. Differences between the calculated and experimental fatigue lives

the value  $\varepsilon_{ac\, max}$  is, the greater these differences are.

When changing the sequence of cycles from the irregular one to "Lo-Hi" the fatigue life increases whereas the change to the "Hi-Lo" sequence decreases the fatigue life. The best agreement between the experimental and calculated results of fatigue lives in the case of irregular loading was obtained, what seems to be obvious, for the "Lo-Hi-Lo" sequence. Like in the previous cases, greatest differences were observed for the maximal  $\varepsilon_{ac\, max}$  level.

Fig.9 shows the fatigue damage accumulation for specific conditions (random in Fig.9a and irregular in Fig.9b,c,d). From the figures it is seen that courses of damage accumulation processes are different, which is clearly visible at the beginning of accumulation process. They are functions of the block type (Fig.9b,d) and the sequence of loading (Fig.9b,c). Since the boundary line can be reached along different paths the obtained value of cumulated dissipation energy depends on the test conditions.

Basing on the presented analysis we can draw the following conclusion: the proposed energetic hypothesis of fatigue damage accumulation, which originates from the fatigue investigations of polymer composites and bases on the assumption that the constant-damage lines exist, can be applied to fatigue life calculation of the specimens and parts made of steels 45, particularly, in the ranges of loading for which substantial plastic strains occur.

According to the presented hypothesis, the fatigue lives can be calculated for the block type of loading (for different sequences and types of loading blocks) as well as for the random loading using the "cycle by cycle" method.

The stabilisation of density energy  $U_D$  was the characteristic property of the material used during the tests (45-steel). In practice, it means that the

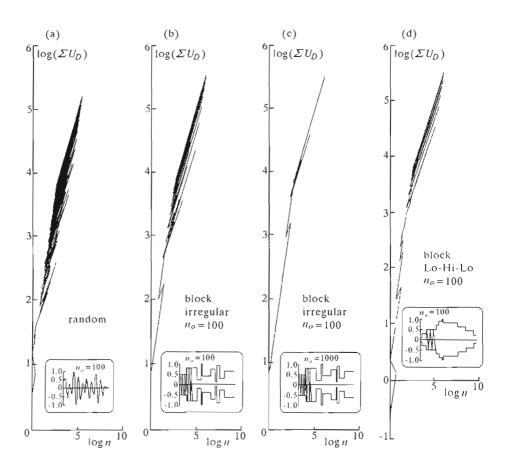


Fig. 9. Fatigue damage accumulation for  $\epsilon_{ac\,max}=1.5\%$ 

energy  $U_D$  was constant as a function of number of cycles (see Fig.3). For extending the range of the proposed hypothesis applicability, new experimental verification is needed based upon the results of investigations of materials of different characteristics – particularly of these materials in which the evident cyclic hardening and softening phenomena are present.

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# Nowy energetyczny model kumulacji uszkodzeń zmęczeniowych i jego weryfikacja dla stali 45

### Streszczenie

W pracy omówiono propozycję kumulowania uszkodzeń zmęczeniowych w oparciu o energetyczne wskaźniki procesu zmęczenia. Propozycja wykorzystuje założenie istnienia linii stałych uszkodzeń zmęczeniowych, z których linię graniczną stanowi wykres zmienności kumulowanej energii dyssypacji w funkcji liczby cykli do zniszczenia zmęczeniowego przy stałym poziomie odkształcenia całkowitego. Weryfikacja doświadczalna wykazała zadowalającą zgodność wyników obliczeń trwałości z wykorzystaniem tej hipotezy z wynikami badań. Wykazała także jej czułość na zmianę sekwencji obciążenia i pojemności bloku obciążenia, dając jednocześnie możliwość określenia trwałości przy obliczeniach "cykl po cyklu".

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