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SELECTION OF AN OPTIMIZATION MODEL IN OPTIMAL DESIGN OF RIBBED CYLINDRICAL SHELLS

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Two stages precede the solution to optimal design problems. A mathematical model for the structure optimization which shows the heart of the problem is selected at the first stage, its numerical implementation is performed at the second stage. As the selection of mathematical model is very important for the solution to an optimal design problem to achieve the global structure optimum, an attempt has been made in this paper to analyze the use of some new mathematical models. Optimization of the supported cylindrical shells subjected to a stochastic load has been shown as an example. A numerical example is given to show the application of the most adequate, in our opinion, mathematical model.

1. Introduction

Optimal design of mechanical systems is a very complicated and controversial problem since the proper statement of the problem (i.e., the selection of optimization criteria and their interrelations) is subjective for some reasons. Let us show on the example of optimization of one of engineering structure elements – supported cylindrical shell subjected to a stochastic load such as white noise (coverings of flying vehicles exposed to the field of acoustic pressure are often subjected to such a load), some new mathematical models applicable to solving of this group of optimization problems.

To solve the problem of optimal design of a supported cylindrical shell structure the following models broken down conventionally into 2 classes have been proposed:

1. The problems with functional approach comprising the following statements (cf Pochtman and Fridman (1989), (1990a,b))

$$(a) \quad \exists X_{opt} \quad \max_K\Big[\min\Big(\mu(K,V),\mu(K,T_1)\Big)\Big] \qquad \text{or} \tag{1.1}$$

$$(b) \quad \exists X_{opt} \quad \max_K\Big[\min\Big(\mu(K,V),\mu(K,T_1),\mu(K,P)\Big)\Big] \qquad \text{or} \tag{1.2}$$

$$(c) \quad \exists X_{opt} \quad \max_K\Big[\min\Big(\mu(K,V),\mu(K,T_1),\mu(K,P),\mu(K,F)\Big)\Big] \tag{1.3}$$

where X_{opt} is a vector of variable parameters (from the sets of structure states K)

2. The problems with an economical approach comprise two statements (cf Pochtman (1993))

$$(a) \quad \exists X_{opt} \quad H = H_0 + P_f H_f \to \min \qquad \text{or} \tag{1.4}$$

$$(b) \quad \exists X_{opt} \quad U(T,X) = B(T,X) - H_1(X) - L(T,X) \to \max \tag{1.5}$$

$$\wedge \quad P(t) \geq P_*$$

2. Functional approach

Functional approach given by Eqs (1.1)÷(1.3) is based on the solution to an optimal design problem when the object to be studied must meet constraints (objective function), i.e., in this case: volume or weight V, life T_1, reliability P and functional stability F.

Life T_1 means the expected value of the 1st order time during which the system does not go out of the feasible area comprising its stable equilibrium; reliability P is the probability that the system will not go out of it within the preset period of time t. The theory of Markov processes was used to determine the shell life, while Bolotin's theory (cf Bolotin (1984)) was used to determine its reliability. The following equations of life and reliability were obtained in by Pochtman and Fridman (1989), (1990a)

$$
\begin{aligned}
T_1 = {} & \frac{q_*^2}{s\omega_{mn}^2}\Big\{10.27\Big(1 - \frac{w_0^2}{w_*^2} - \frac{\dot{w}_0^2}{\omega_{mn}^2 w_*^2}\Big) + 2\Big[\Big(1 - \frac{w_0^2}{w_*^2} - \frac{\dot{w}_0^2}{\omega_{mn}^2 w_*^2}\Big) \cdot \\
& \cdot \Big(\frac{w_0^2}{w_*^2} + \frac{\dot{w}_0^2}{\omega_{mn}^2 w_*^2}\Big) - 3.6\Big[\Big(\frac{w_0^2}{w_*^2} + \frac{\dot{w}_0^2}{\omega_{mn}^2 w_*^2}\Big)^2 - \Big(\frac{w_0^2}{w_*^2} + \frac{\dot{w}_0^2}{\omega_{mn}^2 w_*^2}\Big)^9\Big]\Big\}
\end{aligned}
\tag{2.1}
$$

$$
\begin{aligned}
P(t) = {} & 0.772\exp\Big(-0.745\frac{t}{T_1}\Big) + 0.227\exp\Big(-1.911\frac{t}{T_1}\Big) + \\
& +0.001\exp\Big(-1.063\frac{t}{T_1}\Big)
\end{aligned}
\tag{2.2}
$$

$$
q_* = \frac{q_{mn}E}{1-\nu^2}
\tag{2.3}
$$

where

s	–	intensity of a white noise
q_{mn}	–	critical value of the static external pressure
ω_{mn}	–	proper vibration frequency of the unloaded system
m, n	–	waves parameters in longitudinal and peripheral directions
E, ν	–	modulus of elasticity and the Poisson ratio of the shell material, respectively
$w_0, \dot{w}_0$	–	initial deflection and initial velocity of the system
w_*	–	maximum admissible deflection of the shell.

The suitability functional of the system F is considered as a ratio of maximum deflection and the length of the shell w/L. In the first two statements the vector of variable parameters is selected as

$$
\boldsymbol{X} = [h_1, b_1, h_2, b_2, k_1, k_2, h]^\top
$$

where

h_i, b_i	–	dimensions of angular sections of transverse frames and stringers, $i = 1, 2$
k_1, k_2	–	number of transverse frames and stringers, respectively
h	–	shell thickness.

In the statement (1.3), additional variable parameter w_* is involved and this settles an obvious contradiction while selecting the feasible deflection because in order that the system life and reliability be higher at the aforementioned state K, w_* must be taken as close to its higher limit as possible. While for the functional stability to be higher w_* should be taken as close to its lower limit as possible.

2.1. Fuzzy statement of the problem and a method for its solution

The first class of problems is presented in the form of fuzzy statements (i.e., when the aspects of the problem are put unclearly) and is solved in terms of the theory of fuzzy sets (cf Cofman (1982)) which permits effective determining of the membership value of the sets of the structure states K to the admissible sets of the criteria chosen. The describing functions which vary within the interval $[0,1]$ are used for the qualitative determination of the membership value

$$\mu(K,V)=\begin{cases}1 & V\le V_{opt}\\ \frac{1}{2}-\frac{1}{2}\sin\frac{\pi}{V_{max}-V_{opt}}\left(V-\frac{V_{opt}+V_{max}}{2}\right) & V_{opt}<V<V_{max}\\ 0 & V\ge V_{max}\end{cases}$$

$$\mu(K,T)=\begin{cases}0 & T_1\le T_{min}\\ \frac{1}{2}+\frac{1}{2}\sin\frac{\pi}{T_{opt}-T_{min}}\left(T-\frac{T_{opt}+T_{min}}{2}\right) & T_{min}<T<T_{opt}\\ 1 & T_1\ge T_{opt}\end{cases}$$

$$\mu(K,P)=\begin{cases}0 & P(t)\le P_{min}\\ \left(\frac{P(t)-P_{min}}{P_{max}-P_{min}}\right)^n & P_{min}<P(t)<P_{max}\\ 1 & P(t)\ge P_{max}\end{cases}$$

$$\mu(K,F)=\begin{cases}1 & \frac{w_*}{L}\le a_1\\ \left(\frac{a_2-W_*/L}{a_2-a_1}\right)^n & a_1>\frac{w_*}{L}>a_2\\ 0 & \frac{w_*}{L}\ge a_2\end{cases}$$

where V_{opt}, V_{max}, T_{min}, T_{opt}, P_{min}, P_{max}, a_1 and a_2 stand for the limits of respective design criteria which are specified unclearly.

The choice of type of the describing functions is designer's subjective appraisal of the degree of criteria importance when solving a specific optimization problem. In our case, more stringent requirements were imposed upon the light weight and life-time of the shell, reflected by the form of the function $\mu(K,V)$ and $\mu(K,T_1)$. From Eqs (1.1)÷(1.3) it follows that the problem becomes more complicated due to the introduction of additional design criteria in each of the successive statements. Thus, the reliability criterion is introduced in the statement (1.2) which enables the shell reliability to be taken into account that is very important for solving such problems. This criterion also permits stating that reliability plays a more significant role in searching optimal solutions (cf Pochtman and Fridman (1990a)) than life which, in this case, is a non-active constraint. Regarding the statement (1.3), the account of the additional criterion, the functional serviceability of the shell, permits selecting more reasonably the value of the feasible deflection w_*, while in the statements (1.1) and (1.2) it is designated empirically.

3. Economical approach

The second class of problems represented by Eqs (1.4) and (1.5), where economical approach is used, takes into account the independent estimation of the project. Thus, in the statement (1.4) expenses H_{fail} due to the fail of the structure with probability P_{fail} are also taken into account, in addition to the initial cost of the shell structure H_0. In our opinion, this statement is notable for the possibility of not using the reliability constraint, because the decrease of the fail probability (or increase in reliability) has been taken into account. The loss due to the fail of the shell structure was accepted by means of the conversion coefficient $k = H_f/H_0$ and this is a disadvantage of the statement problem solution (1.4). It should be noted that the higher was taken this coefficient, the higher was the degree of reliability of the optimal project.

The shell structure in the statement (1.5) is considered from the viewpoint of its utility. So, the function of the average expected merit $U(T, X)$ includes: average income $B(T, X)$ expected from the structure operation during its designed period of life T with regard to the probable fail in the moment of time $t_{fail} < T$; initial cost H_1 and loss due to the structure fail $L(T, X)$. The profit is determined, in turn, following Augusti et al. (1984)

$$B(T, X) = \int_0^T B^0(t) p_{fail}(t)\, dt \tag{3.1}$$

where

$$\begin{aligned} p_{fail}(t) &= P'_{fail}(t) = [1 - P(t)]' \\ B^0(t) &= b(1 - e^{-rt}) \qquad\qquad r = \ln(1 + r') \end{aligned} \tag{3.2}$$

and

b – annual profit in a fail-free operation
r' – percentage of the capital

Similarly, the average loss (or damage due to the fail) reduced to the present time is determined (cf Augusti et al. (1984))

$$L(T,X) = \int_0^T L^0(t) p_{fail}(t)\, dt \tag{3.3}$$

where $L^{(}t) = L_t e^{-rt}$ is the loss and L_1 - total damage estimated before the structure operation.

Knowing the function of reliability and having substituted the value p_{fail} into Eqs (3.1) and (3.3) which was found from Eq (3.2), we find the soughtafter expression for the profit and loss due to the fail of the structure

$$\begin{aligned} B(T,X) &= \frac{b}{r}\Big\{-\sum_{i=1}^{3}\frac{a_i}{b_i}\Big[\exp\Big(-\frac{b_iT}{kT_1}\Big)-1\Big] + \\ &+ \sum_{i=1}^{3}\frac{a_i}{b_i+rT_1k}\Big[\exp\Big(-\frac{b_iT}{kT_1}-rt\Big)-1\Big]\Big\} \end{aligned} \tag{3.4}$$

$$L(T,X) = L_t\Big\{-\sum_{i=1}^{3}\frac{a_i}{b_i+rT_1k}\Big[\exp\Big(-\frac{b_iT}{kT_1}-rt\Big)-1\Big]\Big\} \tag{3.5}$$

where

$$\begin{aligned} a_1 &= 0.575 \qquad a_2 = 0.434 \qquad a_3 = 0.001 \\ b_1 &= 0.745 \qquad b_2 = 1.911 \qquad b_3 = 1.063 \end{aligned}$$

k – ratio of the calender time to the time of annual structure operation.

The initial cost of the structure is determined following Lihtarnikov (1979)

$$H_1 = 1.15\Big(C_M G + aT_P + 3.54\frac{G}{1000}\Big)$$

where

C_M	–	specific cost of the structure material
G	–	weight of the shell structure
a	–	average pay per hour
T_P	–	labour input for the structure manufacture, $T_P = 0.904\sqrt{Gn_1}$ where n_1 is the number of main parts of the structure
3.54	–	overhead expenses which do not depend on the labour input for the manufacture.

4. Calculation procedures

Now let us consider the calculation aspect which is common for all the statements considered above. As the life of the shell T_1 depends on the values of the wave formation parameters m and n, optimal parameters of which, in turn, depend on the number of transverse frames k_1 and stringers k_2, costituting the vector of variable parameters of the shell, additionally it is necessary to determine the minimum value T_1 at each stage of the search by an exhaustive method in array $m \times n$. Due to this, the optimization problem one of the criteria of which is life (or reliability) of the supported cylindrical shell, has a local solution and is multi-extremum. The investigations showed that it is advisable to use one of the effective algorithms of a random search method to solve the multi-extremum problems of non-linear programming represented by Eqs (1.1)÷(1.5) (cf Gurvich et al. (1979)). This algorithm based on the global random search incorporates the idea of controllable selection of test points and multiple lowering to a local extremum. The original method for searching direction choice is invented in the present contribution. The process of random search realized in the program is described in the form

$$\bar{X}_{\zeta}^{(k+p)} = \bar{X}^{(k)} \pm \bar{H}_{\Sigma}-$$

$$\bar{H} = \begin{cases} \gamma_1 H & \text{if} \quad F(\bar{X}_{\zeta}^{(k+p)}) < F(\bar{X}_0^{(k)}) \\ \gamma_2 H & \text{if} \quad F(\bar{X}_{\zeta}^{(k+L_p)}) \geq F(\bar{X}^{(k)}) \end{cases}$$

where

Σ	–	single random uniformly distributed vector
γ_1, γ_2	–	constants of tension (contraction) of the searched hypercube H, where $\gamma_1 \geq 1$; $\gamma_2 < 1$ and $\gamma_1\gamma_2 > 1$
p	–	number of random realizations of the vector $\bar{X}_{\zeta}$ at a constant $\bar{H}$, $p = \{1, 2, ..., L_p\}$
$\bar{X}_0^{(k)}$	–	parameters corresponding to the lowest value obtained at the kth stage of the search $F(\bar{X}_0^{(k)})$

and signs $\pm\bar{H}_{\Sigma}$ represent the realization of the double return of the test random point $\bar{X}_{\zeta}$.

5. Numerical example

As a numerical example, let us consider the solution to the problem of optimization of the supported cylindrical shell using statements (1.5) since they reflect the main point of designing. Regarding the statements (1.1)÷(1.3), it is advisable to use them at the stage of preliminary estimation of the project from the viewpoint of meeting the different quality requirements imposed upon by a designer and the results obtained can be used at the final stage of designing.

The initial data of the problem

$R = 160$ mm	$L_1 = 800$ mm	$E = 6.67 \cdot 10^9$ n/m^2
$\rho = 0.26 \cdot 10^4$ ns^2/m^4	$\nu = 0.3$	$w_0 = h$
$w_0 = 0$	$w_* = 5h$	$P_* = 0.99$
$T = 5y$	$k = 120$	$a = 10$
$C_M = 2.828$	$L_T = 300$	$b = 500$
$r' = 5\%$		

Four different values of the intensity of white noise were considered

$$s_1 = 0.25 \text{ ns}^2/\text{m}^4 \qquad s_2 = 0.5 \text{ ns}^2/\text{m}^4$$
$$s_3 = 0.75 \text{ ns}^2/\text{m}^4 \qquad s_4 = 1 \text{ ns}^2/\text{m}^4$$

The parameter constraints were

$$3 \text{ mm} < h_1 < 20 \text{ mm} \qquad 0.2 \text{ mm} < b_1 < 1.5 \text{ mm} \qquad 4 < k_1 < 9$$
$$3 \text{ mm} < h_2 < 20 \text{ mm} \qquad 0.2 \text{ mm} < b_2 < 1.5 \text{ mm} \qquad 4 < k_2 < 32$$
$$0.2 \text{ mm} < h < 1.5 \text{ mm}$$

The following structure limitations constraint were taken

$$o_1 = 15b_1 - h_1 \geq 0 \qquad o_2 = 15b_2 - h_2 \geq 0$$
$$o_3 = h - 0.7b_1 \geq 0 \qquad o_4 = h - 0.7b_2 \geq 0$$

The results of the numerical experiment for the shell optimization are given in Table 1. As can be seen, the increase in the intensity of white noise results in a gradual decrease of the utility of the structure due to the increase of its initial cost H_1. The income $B(T, X)$ and loss due to the fail $L(T, X)$ remain unchanged as well as the level of the reliability of the structure $P(t)$. The reliability of the optimal project of the shell indicates that the constraint in Eq (1.5) was active when searching the optimal solutions. As both the income and the loss due to the fail, similar to the reliability of the structure in this case, depend on T_1 (the remaing parameters are constant), then it is clear from Eqs (3.4) and (3.5) why $B(T, X)$ and $L(T, X)$ remain constant.

As for the change of the optimal vector parameters of the structure, most stable is the optimal number of the shell ribs (both transverse frames k_1 and stringers k_2); while the remaing optimal parameters increase with the increase of the level of white noise on the average by $12 \div 17\%$.

Table 1

s [ns^2/m^4]	h_1 [mm]	b_1 [mm]	h_2 [mm]	b_2 [mm]	k_1	k_2	h [mm]	$P(t)$	$B(T,X)$	H_1	$L(T,X)$	$U(T,X)$
0.25	3.3	0.252	3.6	0.34	9	11	0.24	0.99	11.51	1.96	2.66	6.89
0.50	4.16	0.387	4.1	0.39	9	11	0.28	0.99	11.51	2.4	2.66	6.45
0.75	4.98	0.394	5.6	0.43	9	11	0.32	0.99	11.51	2.81	2.66	6.04
1.00	5.46	0.447	6.4	0.51	9	11	0.36	0.99	11.51	3.27	2.66	5.58

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Wybór modelu optymalizacji w optymalnym projektowaniu użebrowanych cylindrycznych powłok

Streszczenie

Rozwiązywanie zadań optymalizacyjnych zwykle poprzedzają dwa etapy. Po pierwsze należy sformułować model matematyczny optymalizacji danej struktury, uwypuklający główne aspekty problemu. Następnym etapem jest zastosowanie numeryczne przyjętego modelu. Ponieważ w rozwiązywaniu optymalnego zadania projektowego podstawową rolę odgrywa wybór modelu, w pracy podjęto próbę oceny zastosowania kilku różnych modeli matematycznych. Rozważono optymalizację powłoki cylindrycznej poddanej losowym obciążeniom, dołączając wyniki symulacji numerycznej.

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