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MANOEUVRABILITY ANALYSIS OF MODERN FIGHTER AIRCRAFT[1]

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Manoeuvrability in a vertical plane has a great importance in performance assessment of modern fighter aircraft. Multi-dimensional optimization approach for performing some of these manoeuvres e.g. half-loop and split-S is presented in this paper. The problem is formulated and solved as a typical two point boundary value problem (TPBVP). The pay-off function is defined as the minimum time for performing the manoeuvre. The Pontryagin Maximum Principle (PMP) is applied to solve the problem. Some of the results of the simulation performed for the test aircraft PZL I-22 are presented to show effects of different parameters on a half-loop manoeuvre. The computer program worked out based on the proposed methodology be applied to the conceptual design phase, comparison of manoeuvrability properties in the vertical plane for different existing aircraft, to train pilots performing such manoeuvres in flight simulators and finally, to generate the solutions on board to provide the pilot with real-time display projections of trajectory information.

Nomenclature

a	–	speed of sound, [m/s]
b	–	polytropic exponent of standard atmosphere in Eq (3.12), $b = 1.235$
C_D	–	drag coefficient

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C_{D_0} – drag coefficient at zero-lift
C_L – lift coefficient
$C_{L_{max}}$ – maximum usable lift coefficient
D – total aerodynamic drag, [N]
D_0 – zero-lift aerodynamic drag, [N]
E – specific energy, [m]
$\boldsymbol{f}$ – right hand side of the state equation vector
G – performance criterion
g – acceleration due to gravity, [m/s^2]
h – flight altitude, [m]
h_{max} – ceiling, [m]
J – performance index
K – induced drag coefficient
κ – adiabatic constant
L – aerodynamic lift, [N]
m – total aircraft mass, [kg]
Ma – Mach number
n – transversal aerodynamic load factor
n_{max} – structural/physiological load factor limit
$n_{L_{max}}$ – lift-limited load factor
n_c^* – calculated unconstrained optimal load factor
p – atmospheric pressure, [Pa]
p_0 – atmospheric pressure at sea level, [Pa]
q_{max} – maximum structural dynamic pressure limit, [Pa]
R – gas constant, [J/kgK]
S – aerodynamic reference area, [m^2]
F – engine thrust, [N]
F_0 – maximum static thrust at sea level, [N]
F_{max} – max. available thrust, [N]
T – air temerature, [K]
T_0 – air temerature at sea level, [K]
t – time, [s]
$\boldsymbol{u}$ – control variable vector
$\boldsymbol{U}$ – admissible control variable vector
V – aircraft velocity, [m/s]

$\boldsymbol{X}$ – state variable vector
β – coefficient of exponential atmosphere model in Eq (3.10), $\beta = 10^{-4}$ [m^{-1}]
γ – flight-path angle, [deg]
η – throttle parameter
λ^0 – constant in the Hamiltonian
$\boldsymbol{\lambda}^\mathsf{T}$ – row vector of the co-state variables
λ_γ – co-state variable for the flight-path angle
λ_E – co-state variable for the specific energy
λ_h – co-state variable for the altitude
λ_x – co-state variable for the horizontal distance
ρ – air density, [kg/m^3]
ρ_0 – air density at sea level, [kg/m^3]
$*$ – (superscript) indicates optimal values.

1. Introduction

In performance assessment of modern fighter aircraft manoeuvrability analysis is one of the prime concerns. Different figures often practiced by pilots (such as the loop, the Immelmann, the half loop and the split-S etc.) during standard exercise in the fighter pilot training, aerobatics and in air combat both for defensive and offensive purposes involve hard turning in the vertical plane (cf Nguyen (1993), Rahman and Maryniak (1994)). The amount of difficulty and expense experienced in optimization of such manoeuvres depends, among others, upon the complexity of the dynamic model used to describe the aircraft. The models used range from a simple point-mass (Schultz and Zagalsky (1972)) quasi-steady representation to rigid models with six degrees of freedom (cf Rahman (1991)) or even to models that include deformable airframe with heigher number of degrees of freedom (Buttrill et al. (1987)). It is also well practiced in performance prediction an improved approximation which is called the energy-state approximation, where it is assumed that kinetic and potential energy can be traded back and forth in zero time without loss in total energy (cf Rahman and Maryniak (1994), Hedrick and Bryson (1972), Shinar et al. (1978)). This approximation has certain drawbacks, firstly in practice the exchange of energy involves a short duration and never instantaneous as assumed and secondly, manoeuvre optimization using this approximation may often lead to unrealistic discontinuities in velocity and flight altitude. In this paper it is overcome by added complexity i.e. considering an elaborate model than considered in Rahman and Maryniak (1994). As a result of this added complexity the optimization problem has to be solved as a typical TPBVP and not as a simple initial value problem (cf Rahman and Maryniak (1994), Shinar et al. (1978)). At present the availability of high speed computer would allow one to consider a more complex model rather than compromise on a worse result. The exact solution to the TPBVP can be found by means of different methods (cf Jazwinski (1964), [7]). Obviously, one may argue about the convergence of the iterative nature of such a TPBVP. For that purpose the approximate solution of some parameters by the reduced order energy state approximation may be very helpful.

The manoeuvrability analysis of fighter aircrafts, in this paper, is limited to turning manoeuvres in the vertical plane only and illustrated on the example of the half loop and the split-S, where the split-S is considered as a mirror image of the half loop (Fig.1).

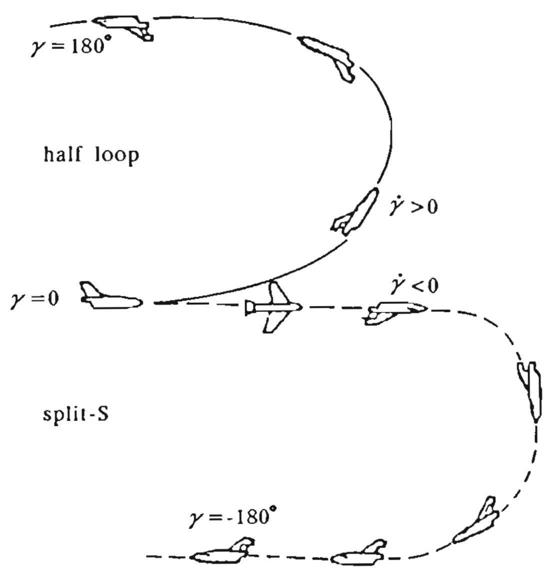

Fig. 1. Split-S as the so called mirror image of half-loop

2. Problem formulation

The problem considered is to find a simple method for the influence of different parameters (structural, aerodynamical, physiological etc.) analysis on manoeuvrability of aircraft in the vertical plane. Having an equivalent mathematical model of the aircraft it can be realized that we have a dynamic system described by the state equation $\dot{\boldsymbol{X}} = \boldsymbol{f}(\boldsymbol{X}, \boldsymbol{u}, t)$ subject to some state constraints to express the fact that the manoeuvre should take place in the aircraft dynamic flight envelope, on which may have been imposed some given initial and final conditions. The problem is to estimate the control parameters $\boldsymbol{u}$ subject to some constraints $\boldsymbol{u} \in U$ such that the performance index $J = \int_0^{t_1} G(\boldsymbol{X}, \boldsymbol{u}, t)$ is minimized. Taking into consideration the combat purpose of such manoeuvres, the performance index is the minimum time i.e. $J = t_1$. For the state variables, it is assumed for simplicity that the accessibility region is coincident with that of admissible values. This will depend on the initial conditions as well as on the control histories. The problem formulated by the previous set of Eqs can be solved applying the PMP (cf Athans and Falb (1966), Hacker (1970)) and accepting the concept put forward by Schultz and Zagalsky (1972), Hedrick and Bryson (1972), Dubiel and Homziuk (1991). The control variables are determined from the necessary conditions of the variational Hamiltonian, $H(\boldsymbol{u}, \boldsymbol{\lambda}, \boldsymbol{X}, t) = \lambda^0 G + \boldsymbol{\lambda}^{\top} \boldsymbol{f}$, which for optimal control is $H\Big|_{max}(\boldsymbol{u}^*, \boldsymbol{\lambda}, \boldsymbol{X}, t) = 0$. The constraints on the control variables will be considered by evaluating the switching functions for the particular case studied in the next part of the paper. The state variables $\boldsymbol{X}$ and co-state variables $\boldsymbol{\lambda}$ are determined by solving a TPBVP of the set of first-order ordinary differential equations $\dot{\boldsymbol{X}} = \boldsymbol{f}(\boldsymbol{X}, \boldsymbol{u}, t)$ and $\dot{\boldsymbol{\lambda}} = -\partial H/\partial \boldsymbol{X}$ with the given set of boundary conditions for the state variables where advantage of the transversality conditions for the co-state variables are also taken in to account (cf Hacker (1970)).

3. System description

3.1. Mathematical model of the aircraft motion

Turning in a vertical plane involves varying flight path angle $\dot{\gamma} \neq 0$ (as shown in Fig.1 and Fig.2). The bank angle ϕ is constant and equals zero. The point mass model of the aircraft turning in a vertical plane (for a small

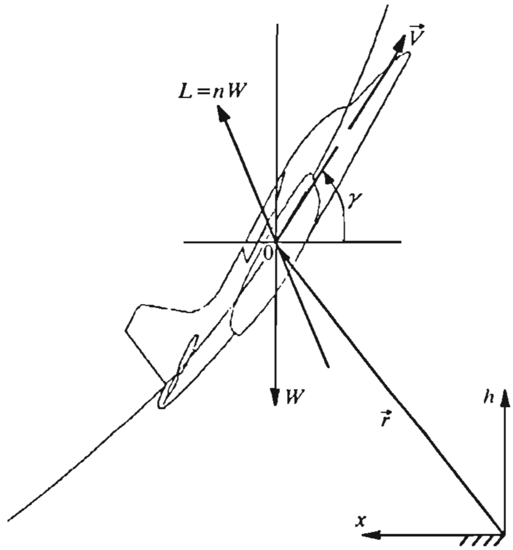

Fig. 2. State variables in a half loop manoeuvre

angle of attack) is governed by the following set of equations (Nguyen (1993))

$$
\begin{aligned}
\dot{V} &= g\Big(\frac{F-D}{W} - \sin\gamma\Big)\\
\dot{\gamma} &= \frac{g}{V}\Big(\frac{L}{W} - \cos\gamma\Big)\\
\dot{h} &= V\sin\gamma\\
\dot{x} &= V\cos\gamma
\end{aligned}
\tag{3.1}
$$

Two new parameters will now be introduced because of their important role in such hard manoeuvres, an example of which is a minimum time vertical turn. The load factor n defined as $n = L/W$ is particularly important for safety purposes of the aircraft or the pilot and hence the structural or physiological limit remains imposed during the manoeuvre. On the other hand, the second parameter known in flight mechanics as energy height, which is proportional to the total energy of dynamical system, $E = h + V^2/(2g)$ is important due to the fact that during such hard turns there is a sharp increase in drag due to lift. This increase in drag causes an important loss of the system energy and hence limits manoeuvrability and controlability of the aircraft, so care has to be taken that the manoeuvre ends with a prescribed energy height E_1.

Taking these facts into consideration, the state equations (3.1) are rewritten as follows

$$
\begin{aligned}
\dot{E} &= V\Big(\frac{F-D}{W}\Big)\\
\dot{\gamma} &= \frac{g}{V}(n-\cos\gamma)\\
\dot{h} &= V\sin\gamma\\
\dot{x} &= V\cos\gamma
\end{aligned}
\tag{3.2}
$$

where the aerodynamic forces drag D, lift L and the engine thrust F in Eqs (3.1) and (3.2) are determined as follows

$$
D = \frac{1}{2}\rho(h)V^2SC_D(\mathrm{Ma},C_L) \tag{3.3}
$$

$$
L = \frac{1}{2}\rho(h)V^2SC_L \tag{3.4}
$$

$$
F = \eta F_{max}(h,V) \tag{3.5}
$$

where, for simplicity, the maximum available thrust is taken to be linearly dependent on flight altitude and velocity, i.e.

$$
F_{max}(h,V) = F_0 + \frac{\partial F}{\partial h}h + \frac{\partial F}{\partial V}V \tag{3.6}
$$

The parabolic drag polar and the weight of the aircraft are given by the following relations

$$
C_D(\mathrm{Ma},C_L) = C_{D_0}(\mathrm{Ma}) + K(\mathrm{Ma})C_L^2 \tag{3.7}
$$

$$
W = mg \qquad\qquad m \cong \mathrm{const} = m_{average} \tag{3.8}
$$

Since in this paper, one of the control parameters is the load factor n, the drag in Eq (3.3) is rewritten as follows

$$
D = \frac{1}{2}\rho(h)V^2SC_{D_0} + n^2\frac{2KW^2}{\rho(h)V^2S} \tag{3.9}
$$

3.2. Mathematical model of the atmosphere

The atmosphere, which is also a part of the system where the manoeuvre takes place, is taken according to the 1962 U.S. Standard Atmosphere. The basic parameters of which are approximately described as follows

— density

$$\rho(h) = \rho_0 e^{-\beta h} \tag{3.10}$$

— temperature

$$T(h) = T_0 + \frac{dT}{dh} h \tag{3.11}$$

— pressure

$$p(h) = p_0 \Big[\frac{\rho(h)}{\rho_0}\Big]^b \tag{3.12}$$

In performance analysis, for flight at high speeds, the aerodynamic forces depend on the Mach number. It can be shown that the speed of sound is related to the pressure and the density, which after some derivation can be expressed as

$$a(h) = \sqrt{\kappa R T(h)} \tag{3.13}$$

4. Constraints

It should be mentioned here that the successful completion of the vertical turning manoeuvre depends on the choice of the initial conditions as well as on the control histories $F(t)$, $L(t)$ or eventually $n(t)$. They have to ensure the functions of $h(t)$, $\gamma(t)$ are the monotonic and avoid violating the state and control constraints.

4.1. State constraints

In general, if there is a high risk of violating the state constraints the pay-off function considers some additional penalty term. In this paper, as we mentioned in point 2, it is assumed that resultant values of the state variables are admissible. In fact, to be more realistic and sure that the state constraints are not violated, the domain of validity must be known, which is defined in this case by imposing the following constraints on the flight envelope (cf Hedrick and Bryson (1972), Shinar et al. (1978))

$$h > 0 \qquad\qquad V \leq \sqrt{\frac{2q_{max}}{\rho(h)}}$$

$$V \leq a(h)\mathrm{Ma}_{max} \qquad\qquad n(C_{L_{max}}) \geq 1$$

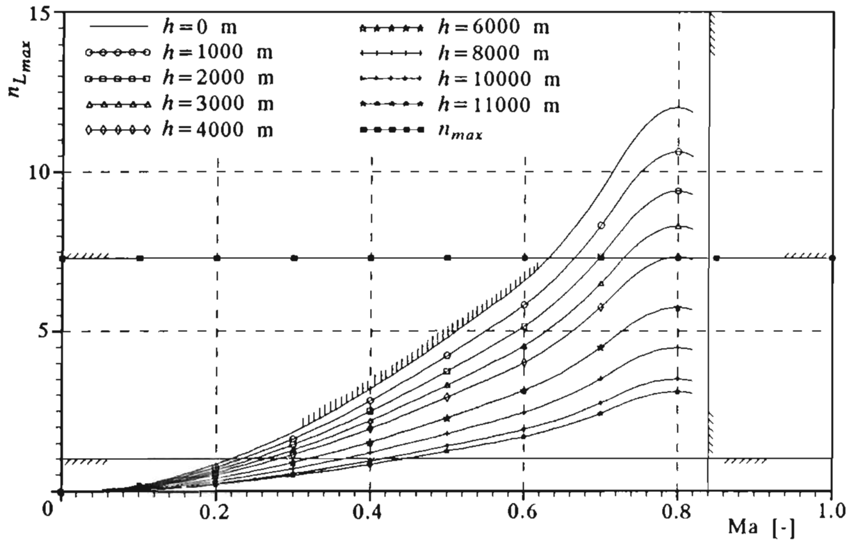

Fig. 3. Domain of the flight for a half loop at various altitudes with structural limit n_{max}

An example of the domain of flight at various altitudes with structural limit is presented in Fig.3. The values presented in this figure correspond to the test aircraft I-22 Iryda, for which the half-loop calculations are performed in this paper.

4.2. Control constraints

The control parameters are piecewise continuous and hence can be changed instantaneously. It is very important to consider the control constraints to find the optimal control histories using the PMP. The point mass aircraft in the vertical plane is controlled by two independent variables namely the throttle parameter η and the lift coefficient C_L. Considering the safety characteristic of a such hard manoeuvre the latter is replaced by the load factor, a fine illustration of which is given by Nguyen (1993). The control parameters are subjected to the following constraints:

— throttle parameter limit, $0 \leq \eta \leq 1$

— physiological/structural limit on the load factor (lower of these two values is taken into consideration), $n \leq n_{max}$

— maximum lift limit on the load factor

$$n \le n_{L_{max}} = \frac{pKS}{2W}\Big(C_{L_{max}}(\mathrm{Ma})\mathrm{Ma}^2\Big)_{max}$$

5. Solution to the problem

The problem is formulated in the way that we should find the optimal control parameters and state variables for a minimum time half-loop or a split-S manoeuvre from a given initial condition ($\gamma_0 = 0, x_0, h_0, E_0$) to a final state ($\gamma_1 = \pm\pi, E_1$). As shown in Fig.2, a half-loop may be mathematically described by $\gamma_0 = 0$, $\dot{\gamma} > 0$, $\gamma_1 = \pi$, whereas a split-S as the mirror image of a half-loop is described by $\gamma_0 = 0$, $\dot{\gamma} < 0$, $\gamma_1 = -\pi$. The test calculations presented in this paper were made for a half-loop. For the case of a split-S described as above, the load factor and respective lift coefficient are negative. Application of the PMP in optimal control begins with defining the performance criterion, the Hamiltonian and the boundary conditions which in our case are

$$G = 1 \tag{5.1}$$

$$H = -1 + \lambda_E V\frac{F-D}{w} + \lambda_\gamma\frac{g}{V}(n - \cos\gamma) + \lambda_h V\sin\gamma + \lambda_x V\cos\gamma \tag{5.2}$$

$$\begin{aligned} &E(t_0) = E_0(V_0, h_0) && E(t_f) = E_1\\ &\gamma(t_0) = \gamma_0 = 0 && \gamma(t_f) = \pi\\ &h(t_0) = h_0 && \lambda_h(t_f) = \lambda_{h_1} = 0\\ &x(t_0) = x_0 = 0 && \lambda_x(t_f) = \lambda_{x_1} = 0 \end{aligned} \tag{5.3}$$

Now it can be seen that we have a set of 8 first order ordinary differential equations (4 state and 4 costate variables) with a total of 8 given boundary conditions at the start and the end point of the manoeuvre which is solved numerically by iterations as a TPBVP. For this purpose a program in Fortran (LOOPBVP) is written using the Math Science Library procedure [7].

Optimal control values are obtained by performing maximization of the Hamiltonian (i.e. partial derivatives of the Hamiltonian with respect to control parameters being equal to zero). There might exist two cases of controls namely singular controls (partial) and "bang-bang" controls (maximum or minimum, intermediate values are excluded) (cf Athans and Falb (1966), Hacker (1970), Dubiel and Homziuk (1991)).

The possible control for the thrust is

$$F(\eta) = \begin{cases} F_{max}(\eta = 1) & \text{if} \quad \lambda_E > 0 \\ F_{min}(\eta = 0) & \text{if} \quad \lambda_E < 0 \\ \text{partial} & \text{if} \quad \lambda_E = 0 \end{cases} \tag{5.4}$$

The occurrence of partial thrust ($\lambda_E = 0$) is very unlikely during such hard manoeuvres. If there exists a partial thrust for a finite period of time in which case determination of singular control η needs a detailed analyse of the expression λ_E, which can be met in some of the works of Bryson, Shultz, et al. (cf Schultz and Zagalsky (1972), Hedrick and Bryson (1972)). From the example presented here it can be seen that $\lambda_E \neq 0$, for that case $\eta^* = \text{sign}(\lambda_E)\eta_{max}$ and that means $F = F_{max}$ if $\lambda_E > 0$ (i.e. for a half-loop) and $F = 0$ if $\lambda_E < 0$ (i.e. for a split-S).

The optimal load factor is obtained from $\partial H/\partial n = 0$, which ultimately gives (cf Rahman and Maryniak (1994), Shinar et al. (1978))

$$n_c^* = \lambda^* \frac{g\rho(h)}{4K(W/S)} \tag{5.5}$$

where $\lambda^* = \lambda_\gamma^*/\lambda_E^*$.

The value predicted by Eq (5.5) may violate the structural/physiological limit n_{max} for low altitude, on the other hand for higher altitude $n_{L_{max}}$ may be violated (see Fig.3). In those cases the optimal load factor will be on the constraint boundaries and the possible control of the load factor is

$$n^* = \begin{cases} n_{max}\text{sign}(n_c^*) & \text{if} \quad n_{L_{max}} > n_c^* > n_{max} \quad \text{or} \quad n_{max} < n_{L_{max}} < n_c^* \\ n_{L_{max}}\text{sign}(n_c^*) & \text{if} \quad n_{L_{max}} < n_c^* < n_{max} \quad \text{or} \quad n_{L_{max}} < n_{max} < n_c^* \\ n_c^*(\text{partial}) & \text{if} \quad n_c^* < n_{L_{max}} < n_{max} \end{cases} \tag{5.6}$$

6. Numerical analysis of a half-loop

Numerical analysis of an optimal half-loop is carried out for the twin-jet engine trainer aircraft I-22 Iryda. The aerodynamic characteristics $C_{D_0}(\text{Ma})$, $K(\text{Ma})$ and $C_{L_{max}}(\text{Ma})$ of this aircraft are presented in Fig.4 and in Fig.5. The remaining aircraft data needed for the simulation are given in Table 1.

The initial conditions were obtained by choosing ($\gamma_0 = 0$ and a given E_0) for an arbitrary values of V_0 and h_0 from admissible region of the

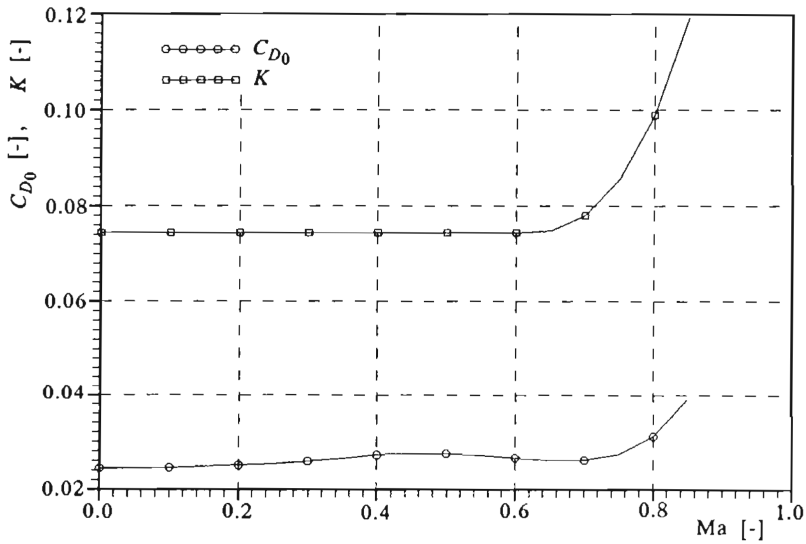

Fig. 4. Zero-lift drag coefficient and induced drag parameter as a function of the Mach number

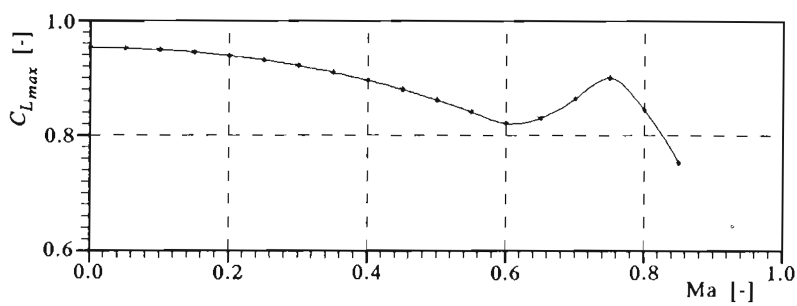

Fig. 5. Maximum lift coefficient as a function of the Mach number

flight envelope. A large number of cases have been studied to see the effects of different parameters on the manoeuvrability of the aircraft. Some of the results for the few cases studied are summerized in Table 1. In Fig.6 comparison of the optimal trajectories in the vertical plane for some cases is presented. In Fig.7 the altitude and energy height history with respect to flight path angle for the basic case is shown.

The flight path angle history and the final time to attain $\gamma = 180°$ is easily known from Fig.8. The load factor histories are presented in Fig.9 and in Fig.10, which may be of particular interest for the safety analysis. The first

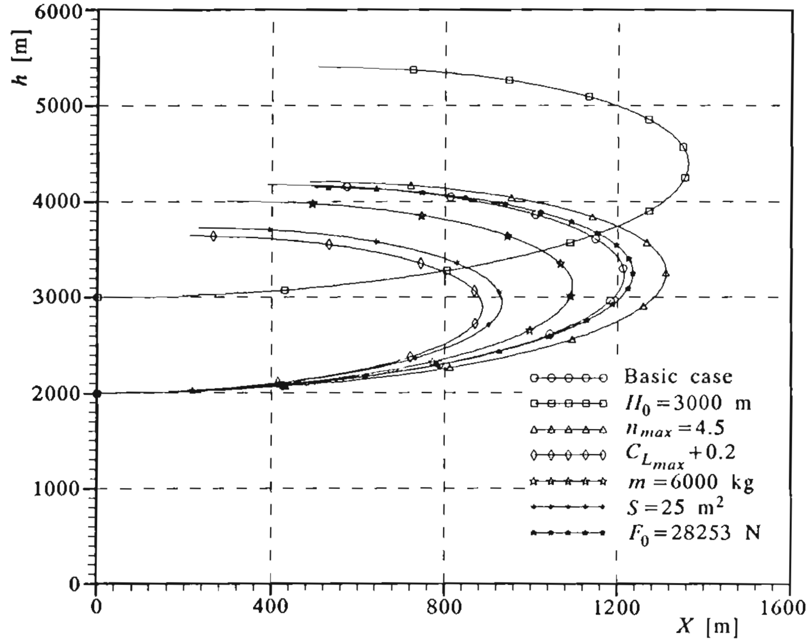

Fig. 6. Comparison of optimal trajectories for different parameters. Basic case parameters: $H_0 = 2000$ m, $S = 20$ m^2, $n_{max} = 7.3$, $F_0 = 35316$ N, $C_{L_{max}} = C_{L_{max}}$, $m = 6500$ kg

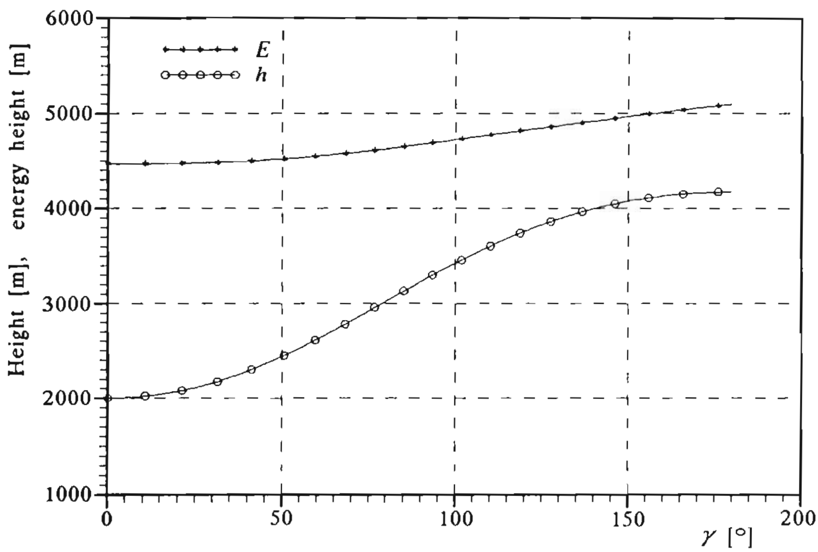

Fig. 7. Flight altitude and energy height versus flight path angle (basic case). Basic case parameters: $H_0 = 2000$ m, $S = 20$ m^2, $n_{max} = 7.3$, $F_0 = 35316$ N, $C_{L_{max}} = C_{L_{max}}$, $m = 6500$ kg

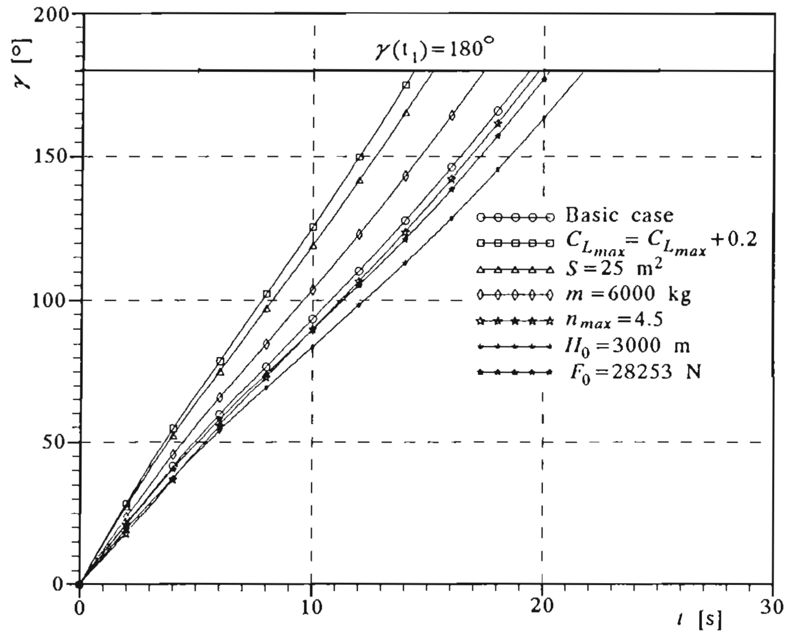

Fig. 8. Comparison of optimal manoeuvre time. Basic case parameters: $H_0 = 2000$ m, $S = 20$ m^2, $n_{max} = 7.3$, $F_0 = 35316$ N, $C_{L_{max}} = C_{L_{max}}$, $m = 6500$ kg

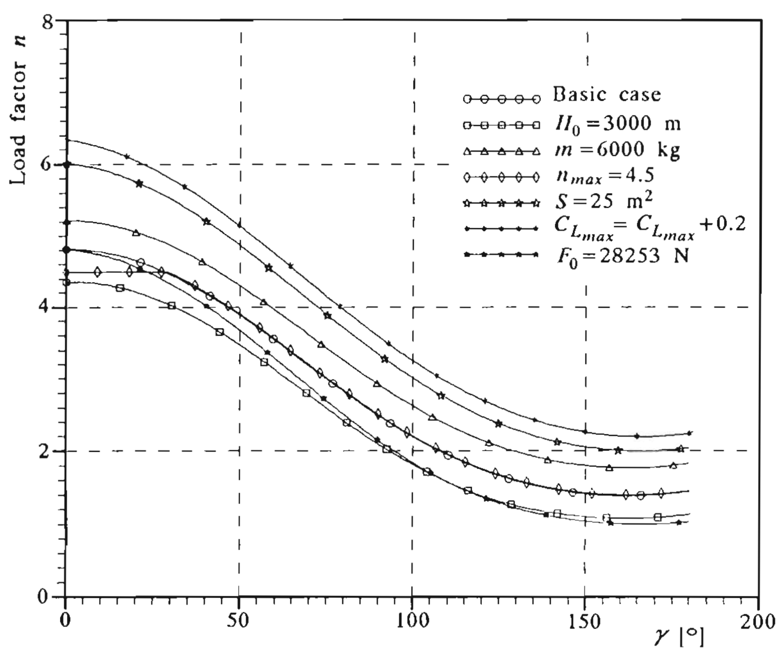

Fig. 9. Comparison of optimal load factors. Basic case parameters: $H_0 = 2000$ m, $S = 20$ m^2, $n_{max} = 7.3$, $F_0 = 35316$ N, $C_{L_{max}} = C_{L_{max}}$, $m = 6500$ kg

one of these figures shows which of the load factor costraints has been active and for which part of the trajectory. The latter shows the duration of the particular load factor acting on the structure and the pilot and it should be analyzed from the physiological viewpoint of the pilot.

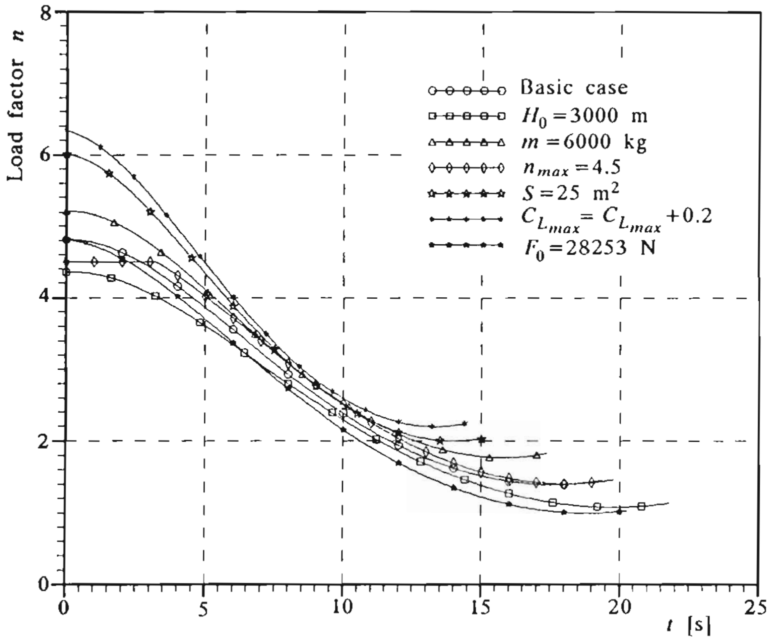

Fig. 10. Comparison of time histories for the optimal load factor. Basic case parameters: $H_0 = 2000$ m, $S = 20$ m^2, $n_{max} = 7.3$, $F_0 = 35316$ N, $C_{L_{max}} = C_{L_{max}}$, $m = 6500$ kg

7. Conclusions

A half-loop as an example of the vertical plane turning manoeuvre with multi-dimensional optimal control problem has been solved using the PMP. The problem is formulated as a genaral TPBVP, not as a simplified initial value problem (cf Rahman and Maryniak (1994), Shinar et al. (1978)). The initial value problem was based on energy state approximation, the drawback of this model is the risk of violation of continuity of the flight velocity and altitude as they are not obtained by integration of differential equation.

Table 1. Comparison of results for few cases studied in the paper (half-loops for I-22)

No.	Case studied	t_1 [s]	h_1 [m]	x_1 [m]	$n_{t=t_1}$
1	Basic case: $m = 6500$ kg; $S = 20$ m^2 $F_0 = 35316$ N $\partial F/\partial V = 27.17$ N/(m/s) $\partial F/\partial h = -1.47$ N/m $H_0 = 2000$ m; $V_0 = 220$ m/s; $n_{max} = 7.3$; $C_{L_{max}}$ as in Fig.5.	19.4	4176.7	388.5	1.45
2	$h_0 = 3000$ m	21.8	5403.1	504.1	1.14
3	$n_{max} = 4.5$	19.8	4204.5	484.7	1.46
4	$C_{L_{max}}$ = case1 + 0.2	14.4	3635.6	209.0	2.24
5	$m = 6000$ kg	17.3	4006.8	296.2	1.83
6	$S = 25$ m^2	15.2	3721.0	229.5	2.04
7	$F_0 = 28253$ N	20.3	4143.8	496.2	1.03

This violation is not always so mild as it was in the case mentioned by Shinar et al. (1978). In some cases studied in this paper, there was a strong violation of continuity of the mentioned parameters due to the existence of vertical zoom climb resulting from the simplified model of the dynamic system. At present the availability of high speed, large memory computers made it possible to solve such a TPBVP for real time simulation purpose. Any way, solution of the simplified initial value problem can be taken as the expected first approximation (crude solution) of the TPBVP. A large number of cases have been numerically simulated to see the effects of different parameters (mass, geometric, aerodynamic, engine, safety and initial conditions) on the half-loop manoeuvre of the test aircraft I-22. Some of the results presented in the earlier paragraph are self-explanatory. Presented methodology may be treated as an attractive and effective way for the aircraft optimal performance assessment of such manoeuvres in the vertical plane. It can be said on the basis of the obtained results that lower wing loading or higher n_{max} needs less time for such turning, also turning is faster at lower altitude. It has also been observed that in most problems for low altitude turn a large part of the optimal trajectory is flown with load factor where the constraint n_{max} remains active, on the other hand, at higher altitude the max lift limited load factor constraint remains active. Attention has to be paid not only to n_{max} but also the time during which the load factor is on the constraint boundary due to the physiological limit of the pilot. In the future an attempt will be made to include the longitudinal moment equation into the model to obtain directly the elevator deflection, the pilot stick force or the hinge moment for such manoeuvres.

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Analiza manewrowości współczesnych samolotów bojowych

Streszczenie

W analizie osiągów samolotów akrobacyjnych lub szybkich samolotów bojowych duże znaczenie ma manewrowość w płaszczyźnie pionowej. W pracy tej przedstawiono wielowymiarową optymalizację wykonania niektórych manewrów jak pół-pętla czy wywrót. Do rozwiązania postawionego zadania z podanymi ograniczeniami oraz funkcjonałem jakości, zastosowano zasadę Maksimum Pontriagina. Funkcjonał jakości przyjęto jako najkrótszy czas wykonania takiego manewru. Wyniki przeprowadzonej symulacji dla samolotu I-22 Iryda, przedstawiono na wykresach, które pozwalają przeanalizować wpływ różnych parametrów na manewrowość samolotu. Opracowany program numeryczny można wykorzystać np:

1. na etapie projektu wstępnego, gdyż można sprawdzić różne koncepcje projektu i ich wpływ na manewrowość samolotu w płaszczyźie pionowej,
2. do porównania własności manewrowych w płaszczyźnie pionowej dla różnych istniejących konstrukcji,
3. do szkolenia pilotów wykonujących takie manewry na symulatorach,
4. jako źródło informacji dla pilota w czasie rzeczywistego wykonywania takich manewrów.

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