EVALUATION OF LIGHT AIRPLANE PERFORMANCE IN STALL AND SPIN – A SURVEY

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The purpose of the paper is to present some approximate criteria for assessment of a light tail-aft aircraft configuration from the point of view of its properties in stall and spin. In some cases, compilation and/or modification of methods presented in literature is proposed. The paper is divided into two parts:
– Stall characteristics. This part is focused on determination the span-wise distribution of a lift coefficient over a wing and setting some additional criteria. Fulfillment of these criteria determines the aircraft performance in stall and pro-spin conditions.
– Spin characteristics. Some rough criteria for evaluation of an aircraft arrangement accounting its spin characteristics are given. A simplified mathematical model of motion in spin with approximate methods for determination of necessary aerodynamic coefficients is presented.

Notations

$\alpha, AA$ – angle of attack
$\beta$ – angle of sideslip (positive when velocity deflects towards the right wing)
$\delta_e$ – elevator deflection
$\delta_{kl}$ – flaps deflection
$\varepsilon$ – geometric twist (angle between $c_l$ and $c_r$)
$\lambda$ – nondimensional angular velocity around the spin axis, $\lambda = \Omega b / 2 V_d$
$\Lambda_{0.25}$ – wing sweep angle
\( \Omega \) – angular velocity about the spin axis
\( \Omega_y \) – pitching angular velocity with respect to the body frame of reference
\( \rho \) – air density
\( A \) – wing aspect ratio
\( b \) – wing span
\( c \) – mean aerodynamic chord
\( c_A \) – total aerodynamic force coefficient
\( c_D \) – drag coefficient
\( c_g \) – mean geometric chord
\( c_l \) – airfoil lift coefficient
\( c_{l\beta} \) – sideslip angle derivative of rolling moment
\( c_l(\zeta) \) – coefficient of aerodynamic moment produced by a deflected rudder, about the central axis of inertia parallel to the spin axis
\( c_l^B(p) \) – coefficient of aerodynamic moment produced by a fuselage, about the central axis of inertia parallel to the spin axis
\( c_l^W(p) \) – coefficient of aerodynamic moment produced by a wing, about the central axis of inertia parallel to the spin axis
\( c_L \) – lift coefficient
\( c_{L\text{max}} \) – maximum lift coefficient
\( c_m \) – pitching moment coefficient
\( c_B \) – pitching moment coefficient produced by fuselage with respect to the body frame of reference
\( c_H \) – pitching moment coefficient produced by fuselage with respect to the body frame of reference
\( c_m \) – coefficient of pitching moment produced by inertial forces with respect to the body frame of reference
\( c_m^B(q) \) – coefficient of pitching moment due to angular velocity about the lateral axis with respect to the body frame of reference, produced by fuselage
\( c_m^W \) – pitching moment coefficient produced by wing with respect to the body frame of reference
\( c_{n\beta} \) – sideslip angle derivative of yawing moment with respect to the body frame of reference
Evaluation of light airplane performance...

\[ g \] — acceleration due to gravity

\[ G_\alpha \] — Glauert autorotation criterion factor

\[ h_h \] — distance between horizontal tail aerodynamical center and mean geometric chord of the wing, along the 0z axis with respect to the body frame of reference

\[ I_x, I_y, I_z \] — airplane moments of inertia about 0x, 0y and 0z axes with respect to the body frame of reference

\[ l_f \] — distance between unshielded rudder aerodynamical center and airplane center of gravity (along the 0x axis of the body frame of reference)

\[ l_h \] — horizontal tail arm (distance between quarter of horizontal tail MAC and airplane center of gravity along the 0z axis of the body frame of reference)

\[ l_s \] — distance from aerodynamic center of fin plus rudder to airplane center of gravity (along the 0x axis of the body frame of reference)

\[ m \] — airplane mass

\[ \bar{q} \] — dynamic pressure, \( \bar{q} = 0.5 \rho V_d^2 \)

\[ R \] — spin radius (the radius of spiral which is a trajectory of airplane center of gravity in spin)

\[ S \] — wing area

\[ S_f \] — fin plus rudder area

\[ S_s \] — unshielded rudder area

\[ V_d \] — descent velocity in spin

\[ V_s \] — stall speed

1. Introduction

During a preliminary design phase it is often necessary to determine some basic airplane characteristics in a fast and relatively confident way. It is often inconvenient (because of time and costs) to use advanced methods (i.e. panel methods, full equations of motion solving). It would be useful to have a set of criteria for fast evaluation of selected parameters. In the paper some of the criteria are reviewed and summarized and a methodology for determination of airplane performance at stall and in spinning is proposed. Most of simplifying
assumptions are mentioned at particular criterion descriptions. The airplane is modeled as a rigid body and steady aerodynamic is applied.

2. Stall behavior of an airplane

2.1. Spanwise lift distribution

Behavior of an airplane in stall conditions is mainly affected by wing characteristics at around-stall angles of attack. Especially, a location of the beginning of flow detachment on a wing and wing pitching moment time history are important. The stall beginning in the ailerons region is definitely disadvantageous. It may lead to loose of roll control and to uncontrolled banking what is especially dangerous during landing approach. At preliminary design stage, information about the detachment beginning location can be obtained from a spanwise lift distribution (e.g. determined by Shrenk, Lotz or Glauert methods or by means of simple panel codes).

![Diagram of spanwise lift distributions](image)

Fig. 1. Spanwise lift distributions (Lotz method)

The Lotz method (cf Nenadovic (1948)) presented below is a simplification of the Glauert method. It is much more exact then the Shrenk method. It is also more convenient for a preliminary designing then panel methods because of computing time costs and tool requirements. Data of airfoils and
basic geometry of a wing are only needed. The Lotz method allows to determine a spanwise lift distribution for trapezoidal wings with small sweep $-10^\circ < A_{0.25} < 10^\circ$, taking into account geometric and aerodynamic twist and deflections of flaps and ailerons.

The point where a local value of $c_l$ is equal to $c_{L_{\max}}$ for the first time is considered as the location of a flow detachment beginning. Fig.1 presents results of calculations made for the wing of a light airplane (aspect ratio 8, taper ratio 0.6, wing twist $-3.5^\circ$, sweep $-1.8^\circ$).

A flight speed value related to the beginning of stall is evaluated for defined altitude, mass of the airplane and airplane lift coefficient for the angle of attack, which corresponds to $c_{l_{stall}}$ for the wing.

2.2. Pitching moment effect

Behavior of an airplane in post-stall conditions is strongly affected by $c_m(\alpha)$ characteristics shapes for the wing and the whole airplane, respectively. Stable and unstable characteristics of the moment coefficient for around-stall AA are given in Fig.2 (cf Roskam (1990)). The unstable characteristics are acceptable for airplanes certified by the FAR\(^1\) 25 and for combat airplanes. The FAR 23 permit unstable pitching moment $c_m = f(\alpha)$ on condition of application of stick-shaker and/or stick-pusher devices.

![Characteristics of $c_m = f(\alpha)$](image)

Fig. 2. Characteristics of $c_m = f(\alpha)$, (a) – stable, (b) – unstable

Mathematical models used for preliminary design do not make possible to evaluate exact pitching moment characteristics for around-stall AA. In this case, the criteria given by Roskam (1990) can be useful. Fig.3 presents

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\(^1\)Federal Airworthiness Regulations
a border line of stable/unstable $c_m$ characteristics for different aspect ratios and sweep angles of wings, respectively. From this figure it is clear that wings of high aspect ratio and large sweep angle are predestined to have unstable $c_m$ characteristics.

Fig. 3. Bound line of wing pitching moment stability depending on wing geometry

There is also an influence of a horizontal tail position on a pitching moment curve shape. Fig. 4 shows four regions of possible position of a horizontal tail relative to the wing mean aerodynamical chord.

Fig. 4. Four regions of horizontal tail location affecting post-stall pitching moment characteristics

The $A$ and $B$ regions are the best ones for low speed airplanes accounting the $c_m$ characteristic. The $C$ and $D$ regions are acceptable only for airplanes with stable $c_m$ characteristics for wings. In another case, a deep-stall can appear (Fig. 5). Deep-stall recovery is possible for high effective longitudinal control only and it is connected with large altitude losses. The deep-stall phenomenon often appears for airplanes with positive wing sweep and a $T$-tail. For such airplanes, large span horizontal tails are used (for deep-stall
recovery) and the aforementioned stall warning devices are employed.

It can be noticed that high speed airplanes of medium aspect ratio have usually large positive sweep of wings. At the leading edge of such wings strong vortex is generated. It produces an additional lift force and finally moves total lift forward. It causes an increase in pitching moment for high \( AA \) (so called pitch-up moment). This additional pitch-up moment is difficult to trim using the horizontal tail. The pitching moment characteristic is presented in Fig. 5. In such airplanes the control system should automatically put limitation on the operational range of \( AA \) limit.

![Diagram](image)

Fig. 5. Aircraft pitching moment characteristics for different horizontal tail positions

### 2.3. Stall warning

A stall warning is defined as shaking of control devices (a stick or a wheel) and/or a whole aircraft. It should appear in the ranges presented by Smetana et al. (1972). In case of absence of a natural stall warning, the appropriate devices should be applied (sound warning or stick-shaker etc.).

A stalled aircraft should not show spin tendency. Roskam (1990) presents the following criterion (it should be fulfilled for around-stall angles of attack)

\[
c_{n\beta_{dyn}} = \left[ c_{n\beta} - \left( \frac{I_z}{I_x} \right) c_{l\beta} \tan \alpha \right] \cos \alpha > 0 \tag{2.1}
\]

where derivatives \( c_{n\beta} \) and \( c_{l\beta} \) are defined relative the body frame of reference.
3. Evaluation of spin characteristics of an airplane

3.1. General assessment of an airplane configuration

3.1.1. Evaluation of tail configuration

For good airplane characteristics in stall it is substantial to keep as large part of a vertical tail as possible out of the wake of a stalled horizontal tail (see Fig.6).

Stinton (1989) gives rough quantitative criteria for evaluation of:
— spin avoidance (incipient spin recovery)
— steady spin recovery.

They are based on assessment of volume ratios of a whole vertical tail and a rudder alone

\[ C' \frac{S_f l_f}{S b} > 0.005 \]  \hspace{1cm} (3.1)

where \( C' = 0.35 \) for a stabilizer+rudder configuration and \( C' = 0.3 \) for an all-moving rudder and

\[ \frac{S_s l_s}{S c} \geq 0.65 \]  \hspace{1cm} (3.2)

Fig. 6. URVC, TDR and TPDF definitions (cf Bowman (1971))

Bowman (1971) gives the more elaborated criterion, in which takes into account (besides the geometry) inertial characteristics and relative density
of the airplane. The unshielded-rudder volume coefficient (URVC), the tail-damping ratio (TDR) and the tail damping power factor (TPDF) values are calculated (see Fig.6)

$$\text{TDR} = \frac{S_f l_f^2}{S^b_2}$$

$$\text{URVC} = \frac{S_{r1} l_{r1} + S_{r2} l_{r2}}{S^b_2}$$

$$\text{TPDF} = \text{URVC} \cdot \text{TDR}$$

Allowable ranges for the TPDF are presented in Fig.7 (cf Bowman (1971), p.31, fig.8) in terms of the factor $P_{xy} = (I_x - I_y)/(mb^2)$ (which relates to inertial characteristics in rotation about the $0z$ axis of the body frame of reference) and the relative density of an airplane. The diagram is based on the wide experimental data.

![Fig. 7. Required TPDF for different airplane relative density and $P_{xy}$; solid line – recovery by rudder alone, dashed line – recovery by rudder and elevator](image)

The above criteria make possible preliminary assessment of a tail configuration. It is important to remember that these criteria get sense if there is no stall on a vertical tail (e.g. the stall can appear in flat spin).

It should be taken into account that the above criteria were defined for typical tail-aft configurations and for light single-engine airplanes.
3.1.2. Wing evaluation in autorotation

Spin characteristics are strongly dependent on characteristics of a wing in pro-spin conditions. The sine qua non condition of spinning is wing autorotation appearance.

The first factor which affects autorotation characteristics is a shape of airfoil $c_L(\alpha)$ near the critical angle of attack. Its sharp form along with a high $c_{L,m\alpha}$ value is definitely disadvantageous. On the wing of such kind a large autorotative moment appears under conditions of unsymmetrical stall.

In order to assess autorotational characteristics of a wing a graphical representation of, so called, the Glauert criterion can be used (cf Fiszdon (1961), part II, p.222). The criterion is defined as

$$G_{\alpha} = \frac{dc_L}{d\alpha} + c_D \approx \frac{dc_A}{d\alpha} \quad (3.3)$$

If $G_{\alpha} < 0$ then autorotation is possible (rolling moment acts in the direction of rotation). The graphical representation is shown in Fig.8.

![Fig. 8. Graphic interpretation of the Glauert criterion](image)

Limits of a range of autorotation are for $dc_A/d\alpha = 0$.

The first border point of the autorotation range (for $\alpha = 20^\circ$ in Fig.8) is practically independent of sideslip. The ending border point of the range (for $\alpha = 30^\circ$ in Fig.8) is strongly dependent of sideslip. In spin, sideslip (even for small values of a sideslip angle, $\beta = 10^\circ \div 20^\circ$) towards the leading wing
widens the range of autorotation. The beginning of the autorotation is very important. Actually, the ending point of the range is almost of no importance because of the spin flow conditions (sideslip occurrence). It comes from strict relation between an increase in $AA$ and an increase in angle of sideslip (i.e. widening the autorotation range). Finally, the wing almost never achieves the end of the autorotation range.

An additional disadvantageous factor could be the shape of a wing which affects the flow separation far from the fuselage, e.g. relatively large wing span length (what usually comes along with a high wing aspect ratio value). Motion in spin causes large differences in local velocities and $AA$ between wing tips (cf Bihrlé and Hultberg (1979b), for a sample flat spin – on the inner wing tip $\alpha = 43.3^\circ$, on the outer wing tip $\alpha = 122.5^\circ$, the global $AA$ (i.e. taken for a whole aircraft) is equal to $80^\circ$). An increase in span usually results in increasing autorotation moment.

3.1.3. Evaluation of inertial characteristics of an airplane

The important factors which affect parameters and a form of spin motion (flat or steep spin), and can even make the spin impossible, are inertial characteristics of the airplane. Their importance increases in the case of weak aerodynamic damping of motion, e.g. for low efficiency of a vertical tail (a small unshielded part of the vertical tail).

Simplifying, the balance in spin motion is affected strongly by relation of mass distribution along longitudinal and lateral axes. It can be illustrated by changing continuous mass distribution into point masses located at $0x$ and $0y$ axes, as shown in Fig.9 (cf Stinton (1989), p.469, fig.13.5).

From Fig.9 it can be seen that the moment produced by wing inertial forces is greater than the moment produced by body inertial forces what means that sideslip towards outer wing is increased (pro-spin tendency).

A small value of $I_y - I_x$ (the configurations, for which mass distributed along the lateral axis is relatively great) is unfavorable. It results in sideslip towards the outer wing (the leading wing) what results in widening of an autorotation range (pro-spin tendency).

It should be mentioned that, according to Fiszdon (1961), it is related to airplanes spinning with the outer wing raised. If the inner wing is raised (this can happen if autorotation is difficult to start for the wing) the conclusions for $I_y - I_x$ are opposite. Majority of airplanes are of the first kind (i.e. with outer wing raised) and this case is given as a rule by Stinton (1989).

The difference $I_x - I_y$ is also important. It determines the pitching moment produced by inertial forces. The value of the difference is always positive for
airplanes. It produces a positive (nose-up) pitching moment, increases AA and may result in flat spin (disadvantage).

Preliminary evaluation of inertial characteristics of an airplane can be based on values of ratios $I_x : I_y$ and $I_z : I_x$. Typical values for light single piston engine airplanes are presented in Table 1\(^2\).

**Table 1.** Sample moment of inertia relations for single engined propeller driven light airplanes (cf Raymer (1989))

<table>
<thead>
<tr>
<th></th>
<th>range</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_x : I_y$</td>
<td>0.596 ÷ 1.240</td>
<td>0.850</td>
</tr>
<tr>
<td>$I_y : I_z$</td>
<td>0.554 ÷ 0.776</td>
<td>0.642</td>
</tr>
<tr>
<td>$I_z : I_x$</td>
<td>1.038 ÷ 2.670</td>
<td>1.978</td>
</tr>
</tbody>
</table>

When $I_x : I_y$ and/or $I_z : I_x$ values are higher than typical ones it may be necessary to introduce high degree of damping of yaw and pitch as well as to increase the effectiveness of rudder under spin conditions.

Preliminary evaluation of tail unit geometry, accounting inertial characte-

\(^2\)These values should be considered as rough, orientational ones, *not* being statistical data
ristics, is presented in 3.1.1.

More accurate assessment of inertial parameters can be made if simplified motion parameters (AA, rate of descent, spin angular velocity) are calculated.

3.2. General criterion for spin recovery assessment

Stinton (1989) presents a simplified criterion for determination of possibility of spin recovery. It is based on an equation of moments acting about the axis going through center of gravity of the airplane and parallel to the axis of rotation in spin. Pitching moments in Eq (3.4) include influences of:

— fuselage with a tail unit
— unshielded part of ruder deflected for recovery
— wing.

The criterion is

\[ c^B(p) + c_l(\zeta) + c^W_l(p) = k_s \]  \hspace{1cm} (3.4)

where \( k_s \) depends on the ratio \( I_y : I_x \) values and moment coefficients are defined as follows

\[
\begin{align*}
  c^B_l(p) &= \frac{2l^B(p)}{qSb} \\
  c^W_l(p) &= \frac{2l^W(p)}{qSb} \\
  c_l(\zeta) &= \frac{2l(\zeta)}{qSb}
\end{align*}
\]

where \( l^B(p), l^W(p), l(\zeta) \) are moments of fuselage, wing and deflected ruder.

The acceptable range of \( k_s \) in terms of \( 1 - I_y/I_x \) is presented in Fig.10 (cf Stinton (1989), p.477, fig.13.7).

![Fig. 10. Ranges of \( k_s \) coefficient for the spin recovery criterion](image)

For airplanes with the \( I_y : I_x \) value equal approximately to unity (typical for modern light, single engined airplanes) \( k_s \) should be greater then \( 13 \cdot 10^{-3} \). The value of \( k_s \) less then approximately \( 2.5 \cdot 10^{-3} \) is unacceptable.

The way for determining particular components in equation (3.4) is described by Stinton (1989):
• Coefficient of moment produced by wing, acting about the central vertical axis is determined in simplified way from a diagram (cf Stinton (1989), p.480, fig.13.9). The diagram presents values of the moment in terms of an airfoil thickness and a nondimensional angular velocity around the spin axis.

• Coefficient of moment produced by a fuselage, acting about the central vertical axis is determined from equation

\[ c_i^B(p) = \lambda \frac{1}{S b^2} \sum \epsilon_k x_i^2 h_i \Delta x_i \]  

(3.5)

where \( \epsilon_k \) is determined for particular fuselage segments and depends on the geometry, \( x_i \) is a distance between the centroid of the segment and airplane center of gravity, \( h_i \) is a height of the segment and \( \Delta x_i \) is a length of the segment. The values of \( \epsilon_k \) are given by (cf Stinton (1989), p.460, tab.13-3).

• Coefficient of moment produced by a deflected unshielded rudder is calculated (cf Stinton (1989), p.478) as

\[ c_l(\zeta) = \frac{S_r l_r}{S b} \]  

(3.6)

where \( S_r \) is area of an unshielded part of a rudder, \( l_r \) is the distance (along the 0x axis) from airplane center of gravity to aerodynamic center of the unshielded part of the rudder.

It should be noticed that the position of airplane center of gravity affects the spin (the most unfavorable case is of course the most aft location). Some results obtained for a light, single-engined, business-tourist aircraft are presented in Fig.11. Coefficient \( k_s \) versus \( AA \) for different values of angular velocity about the central axis parallel to the spin axis is shown below.

The described criterion, similarly as the previous ones, is valid for spin without stall on a vertical tail.

3.3. Approximated evaluation of airplane motion parameters under steady spin conditions

Quantitative criteria described above do not make possible to determine which kind of spin occurs for a particular airplane. Evaluation of motion
parameters is necessary to assess whether an airplane will perform flat or slope spin and for determination of balance conditions in spin.

According to Roskam (1990) there is no exact analytical methods for such type of calculations. In the results presented below the approximated method based on assumptions given by Babister (1961), Blajer (1982), Fiszdon (1961) was used along with methods of some airplane parameters estimation (cf Stinton (1989)).

The following assumptions were accepted (cf Babister (1961)):

1. The spin axis is vertical
2. AA in steady spin is constant
3. Vertical velocity and angular velocity about the spin axis are constant
4. Velocity component in the plane of symmetry is vertical
5. Horizontal velocity component is small comparing to vertical velocity of an airplane
6. Sideslip angle is small
7. Bank angle is small.
From these assumptions it follows that:

1. AA and pitch angle are approximately equal

2. Aerodynamic forces lay in the airplane plane of symmetry \((0xz)\).

Form the above assumptions it follows that:

- a centrifugal force is balanced by lift
- weight is balanced by the drag force

\[ mg = \frac{1}{2} \rho V_d^2 S c_D \]
\[ m\Omega^2 R = \frac{1}{2} \rho V_d^2 S c_L \]  
(3.7)

Additionally, it is assumed that the total aerodynamic force is normal to a wing chord (what is almost true for large post-stall AA).

The steady spin motion parameters can be found

\[ V_d = \sqrt{2 \frac{mg}{S \rho c_D}} \]  
(3.8)
\[ R = \frac{c_L \rho b^2 S}{8m} \left( \frac{2V_d}{\Omega b} \right)^2 \]  
(3.9)

To apply the above equations it is necessary to know the values of \( c_L \) and \( c_D \) (cf Blajer (1982)).

To determine the spin radius, the angular velocity about the spin axis should be calculated first. Two equations of moment can be used:

- simplified equation of pitching moment (in body frame of reference)

\[ c^W_m + c^B_m + c^B_m(q) + c^H_m + c^I_m = 0 \]  
(3.10)

- simplified equation of moments acting about the vertical axis (here, equal \(0x_a\) of the flow frame of reference), like the one used in Section 3.2

\[ c^W_l(p) + c^B_l(p) + c_l(\zeta) = 0 \]  
(3.11)

where

\[
\begin{align*}
c^W_m &= \frac{m^W}{\dot{q} Sc} \\
c^B_m &= \frac{m^B}{\dot{q} Sc} \\
c^B_m(q) &= \frac{m^B(q)}{\dot{q} Sc} \\
c^H_m &= \frac{m^H}{\dot{q} Sc} \\
c^I_m &= \frac{m^I}{\dot{q} Sc} = \frac{-(I_z - I_x)rp}{\dot{q} Sc}
\end{align*}
\]

and \( m^W, m^B, m^H \) – pitching moments due to wing, body and horizontal tail.
3.3.1. Approximated equations of moments

The first equation which determines the steady spin mode is pitching moment equation. However, it only determine the ranges of spin (cf Blajer (1982)).

The pitching moment equation about the $0z$ axis (relative the body frame of reference) has the form

$$-(I_z - I_x)\tau p = \frac{\tau}{\rho S c}c_m$$

(3.12)

Projection of total angular velocity $\Omega$ onto the body frame of reference axes gives

$$p = \Omega \cos \alpha \cos \beta'$$

(3.13)

$$r = \Omega \sin \alpha$$

(3.14)

Assuming that $\beta'$ is small we can obtain from Eqs (3.13) ÷ (3.15) the following equation

$$-\frac{1}{2}\Omega^2(I_z - I_x)\sin 2\alpha = \frac{1}{2}\rho SV_d^2 cc_m$$

(3.15)

Blajer (1982) assumes the moment coefficient $c_m$ to be constant while the angular velocity is variable. It is a rough assumption but for low and mid wing configurations it can be accepted.

In the case of lack of the wind tunnel experimental data $c_m$ can be calculated in a simplified way, including wing fuselage and horizontal tail. The following components contribut to pitching moment coefficients

- fuselage contribution

The total pitching moment coefficient produced by a fuselage can be written in the form

$$c_{m, tot}^B = c_m^B + c_m^B(q)$$

(3.16)

The first component is a coefficient for steady motion without rotations and the second one is a coefficient induced by rotation about the $0y$ axis of the body frame of reference.

The first component in Eq (3.16) can be calculated on the assumption that it is independent of $AA$. The methods of its calculations are well known (e.g. Perkins and Hage (1949), pp.227-228).

The second component of Eq (3.16) can be determined in a similar way as the one presented in section 3.2 for damping moment about the central
axis parallel to the spin axis. The modified equation is of the form

\[ c_m^B(q) = \nu \frac{1}{2Sc^2} \sum_i \varepsilon_k x_i^2 S_i \]  \hspace{1cm} (3.17)

where

\[ \nu = \frac{\Omega_y c}{V_d} \]  \hspace{1cm} (3.18)

and \( S_i \) is the area of projection of an \( i \)th fuselage element onto \( 0xy \) plane.

- wing and horizontal tail contribution

Moments produced by a wing and a horizontal tail are constant on assumption that the total aerodynamic force on a lifting surface is normal to its chord and approximately constant. The moment coefficient can be written as

\[ c_m^H = c_m^H - c_A^H \frac{l_H}{c} \]  \hspace{1cm} (3.19)

\[ c_m^W = c_m^W - c_A^W \frac{l_W}{c} \]

As the second moment equation, the equation of moment about the central axis parallel to the spin axis can be employed. It is similar to the one given in Section 3.2. The only difference is the coefficient of moment produced by an unshielded part of deflected rudder. In this case, we assume that a rudder is undeflected what makes \( c_l(\zeta = 0) = 0 \). Parameters of spin motion for a deflected rudder can be calculated as well.

3.3.2. Algorithm for the calculations

The spin motion parameters can be found in a graphical form on the plane \((c_m - \alpha)\). The algorithm is as follows:

1. Calculate vertical velocity from Eq (3.8) in terms of \( \alpha \). According to the first, fourth and fifth assumptions from Section 3.3 this velocity is equal to the total center of gravity velocity of the airplane.

2. Solve Eq (3.11) of moments about the central axis parallel to the spin axis with respect to a nondimensional angular velocity in spin. It should be done for the same series of as the one assumed in the previous point and for the series of velocities related to these \( \alpha \). The values of nondimensional angular velocity about the spin axis in terms of are achieved as the result.
3. Pitching moment (in body frame of reference) produced by inertial forces is to be calculated next (see the left-hand side of Eq (3.15). It is calculated for the values of nondimensional angular velocity obtained at the previous step. The course of the moment in terms of $\alpha$ is obtained.

4. Aerodynamic pitching moment (in body frame of reference) is calculated versus $AA$.

5. Intersections of the curves obtained at steps 3 and 4 determine the $AA$ value for balance in spin motion. The intersection located on the positive slope part of the curve obtained at stage 3 is not the stable equilibrium. The stable one is related to the intersection which is located in the region of negative slope.

6. The spin radius can be calculated from Eq (3.9).

The most important case corresponds to the aft center of gravity position and for maximum $T-O$ weight. However, several combinations of center of gravity positions and weights should be examined to have some information of possible ranges of spin parameters.

Some results for a light, business-tourist airplane are presented below.

![Graph](image_url)

Fig. 12. Sample results for steady spin $c_m$ versus $AA$ (for a light business-tourist airplane); $\alpha = 69.5^\circ$, $V_d = 28.7$ m/s, $\Omega = 2.1$ rad/s, $\bar{R} = 0.91$ m

4. Concluding remarks

The paper reviewed and summarized a methodology for determination an airplane performance in stall and spin. Some criteria described above can be
used for preliminary design. They can be applied as simple numerical codes for personal computers or included in conceptual design systems. They are easy to use but can be treated rather as necessary but not sufficient conditions.

Most of the presented criteria were verified on flying airplanes. However, it should be taken into account that most of the geometrical criteria are sensible for configurations without vertical tail stall and they were formulated for typical light, tail-aft configurations.

It would be useful to formulate out similar criteria for complex configurations, however, in this case more parameters should be included. Some of presented methods can be used for particular components of the configurations (e.g. evaluation of wing stall characteristics). Majority of the presented methods should be modified and verified for other then typical configurations.

The gradual elimination of simplifications can be done to achieve more exact spin motion analysis also.

References


4. Blajer W., 1982, Badanie dynamiki samolotu w korkociagu, Rozprawa doktorska, Politechnika Warszawska, Warszawa


Ocena samolotu lekkiego z punktu widzenia własności w przeciągnięciu i korkociągu

Streszczenie

Celem pracy jest przedstawienie podstaw do przybliżonej oceny samolotu lekkiego o układzie klasycznym z punktu widzenia własności w przeciągnięciu i w korkociągu. Praca ma charakter przeglądowy. W niektórych przypadkach zaproponowano połączenie przytoczonych metod lub ich niewielkie modyfikacje.

Pracę podzielono na dwie zasadnicze części:
- charakterystyki w przeciągnięciu. Zasadniczym elementem tej części jest wyznaczanie rozkładu współczynnika siły nośnej wzdłuż rozpiętości plata oraz przedstawienie pewnych kryteriów dodatkowych, których spełnienie determinuje zachowanie się samolotu na około i zakrytycznych kątach natarcia;
- charakterystyki korkociągowe. W części tej zawarte kryteria oceny konfiguracji samolotu z punktu widzenia własności korkociągowych, uproszczony model matematyczny ruchu w korkociągu wraz z przybliżonymi metodami obliczania niezbędnych współczynników aerodynamicznych.

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