MECHANIKA TEORETYCZNA
I STOSOWANA
Journal of Theoretical
and Applied Mechanics
1, 32, 1994

CRACKING OF CREEPING PLATES IN TERMS OF CONTINUUM DAMAGE MECHANICS

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In the paper the process of cracking of plates subjected to creep conditions is dealt with by means of Continuum Damage Mechanics. In such a way two stages of failure process can be taken into account: the nucleation of a macro-crack as result of the damage field development, and the propagation of this crack throughout the plate body. Two types of plates have been considered: that for which the whole plate thickness is subjected to a deterioration process, and plates with an elastic core which is non-degrading. In the latter case the time to first crack appearance practically terminates time dependent behaviour of a structure, whereas in the former case the analysis can be extended to cover both stages and evaluate the ratio of time to first crack appearance to the time of through-body crack formation.

1. Introduction

Failure of structures in creep conditions results from a time-dependent process of material deterioration. Three stages can be distinguished in this process:

1. Nucleation of a macroscopic defect at the point (or points) of a structure, as a result of dispersed damage growth, referred to as the First Crack Appearance (FCA)

2. Propagation of macro-crack(s) understood as points assemble at which damage parameter reaches its critical value to form a through-body macro-crack

3. Formation of a net of macro-cracks leading to overall collapse of a structure.

Corresponding times will be denoted below by t with subscripts related to the appropriate stage of a failure process (t_1, t_2, t_3).

In our previous papers (cf Bodnar, 1991; Bodnar and Chrzanowski, 1989, 1992) the analysis of plates was limited only to evaluation of t_1, as normally the time interval $(0, t_1)$ is supposed to consume the most of structures lifetime. However, the ratios t_2/t_1 and t_3/t_1 may depend on many factors like material properties, mode of failure, and/or boundary conditions imposed on an analyzed structure.

The aim of this paper is to extend the analysis over evaluation of t_2/t_1 ratio for plates. It will be shown that – depending e.g. on boundary conditions – this ratio may be large enough to motivate the analysis which covers the second stage of failure process, as well.

2. Governing equations

The following assumptions with respect to the constitutive equations were made:

- the total strain is decomposed into two components: elastic and creep ones, respectively;
- steady state creep theory is modified by coupling with a damage variable;
- damage is represented by a scalar parameter, with the appropriate evolution law, given below.

The above assumptions yield the set of equations which describe the state of material

$$\varepsilon = \varepsilon^e + \varepsilon^c \tag{2.1}$$

$$\varepsilon^e = \mathbf{D}^{-1}\sigma \tag{2.2}$$

$$\dot{\varepsilon}^c = \mathbf{\Gamma}(\sigma, \omega)\sigma \tag{2.3}$$

$$\dot{\omega} = A\Big(\frac{\sigma_{eq}}{1-\omega}\Big)^m \tag{2.4}$$

where

$\boldsymbol{\varepsilon}^e$, $\boldsymbol{\varepsilon}^c$, $\boldsymbol{\sigma}$, ω	–	elastic and creep strains, stress, and damage parameter $(0 \leq \omega \leq 1)$, respectively, (dots stand for the time derivative of respective variables),
A, m	–	material constants,
$\mathbf{D}$, $\boldsymbol{\Gamma}$	–	elastic constants matrix and creep material function, respectively, which will be specified later.

The equivalent stress σ_{eq} in Eq (2.4) is given by the formula

$$\sigma_{eq} = \alpha\sigma_1 + (1-\alpha)\sigma_e \tag{2.5}$$

where σ_1 and σ_e stand for the maximum principal tensile stress and von Mises effective stress, respectively. A parameter α $(0 \leq \alpha \leq 1)$ characterizes failure mechanism mode; for $\alpha = 1$ the damage growth occurs on grain boundaries perpendicular to the maximum principal stress, whereas $\alpha = 0$ is associated with slides along grain boundaries. The intermediate values of α correspond to mixed modes of failure. For plane stress condition the respective limit curves for a given time to failure are shown in Fig.1.

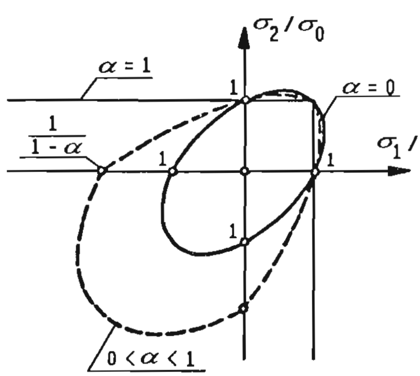

Fig. 1. Failure loci for a constant time to failure

With the equilibrium equation

$$\mathrm{div}\boldsymbol{\sigma} = \mathbf{0} \tag{2.6}$$

and the strain-displacement relationship

$$\boldsymbol{\varepsilon} = \mathbf{L}\boldsymbol{u} \tag{2.7}$$

where $\mathbf{L}$ is differential operators matrix, the equations given in this paragraph form the set of problem governing equations.

3. Method of solution

The Finite Element Method was used to solve the problem of t_1 and t_2 evaluation for plates. The layered shell elements derived from the three-dimensional equations of continuum mechanics by use of degeneration method (cf Ahmad, Irons and Zienkiewicz, 1970; Figureiras and Owen, 1984) were employed here. According to this approach the stress component normal to the midsurface of a plate was neglected and five degrees of freedom were specified at each nodal point corresponding to its three displacements and the two rotations at the node. As the degrees of freedom connected to displacements and rotations are independent, thus this approach is equivalent to the plate theory by Reissner (1945) and Mindlin (1951). It enables the analysis of thin plates as well as moderate ones which consist of layers of different mechanical properties. The particular interest is posed as a problem of an elastic plate shielded by external layers which can absorb all deterioration process (see Sec. 4.2).

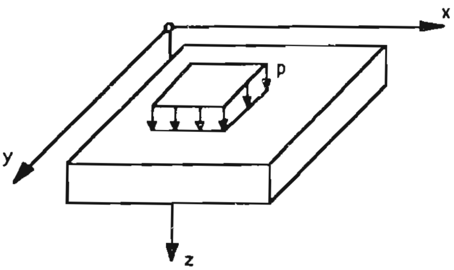

Fig. 2. Plate coordinates

For the theory employed the strain and stress vectors take the form

$$\boldsymbol{\varepsilon} = [\varepsilon_x, \varepsilon_y, \varepsilon_{xy}, \varepsilon_{xz}, \varepsilon_{yz}]^{\mathsf{T}} \tag{3.1}$$

$$\boldsymbol{\sigma} = [\sigma_x, \sigma_y, \sigma_{xy}, \sigma_{xz}, \sigma_{yz}]^{\mathsf{T}} \tag{3.2}$$

(see Fig.2). Elastic constants matrix and creep function $\mathbf{D}$ and $\boldsymbol{\Gamma}$ are assumed to be

$$\mathbf{D} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1-\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{3.3}$$

$$\Gamma(\sigma,\omega) = \gamma\frac{\sigma_e^{n-1}}{(1-\omega)^n}\begin{bmatrix} 1 & -\frac{1}{2} & 0 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix} \tag{3.4}$$

where: E, ν, γ and n are material constants.

In the computer code the layered isoparametric 8-node Serendipity shell elements with reduced integration were employed. Ten layers and two points Gaussian quadrature for volume integration was adopted. For time integration Euler's procedure was used. Details of the algorithm and conditions for numerical stability and accuracy of solution are given by Bodnar and Chrzanowski (1989). The time t_1 is defined by $\omega = 1$ condition fulfilled in any layer and Gaussian point, but the integration procedure proceeds until in all ten layers of a Gaussian point the above condition is met at time t_2.

4. Numerical examples

4.1. Through-thickness degrading plate

Two decisive parameters were chosen out of all these which can influence t_2/t_1 ratio, namely mode of failure reflected by the values of α parameter and boundary conditions. In the latter case two characteristic supports of all edges of a square plate were distinguished: clamped and simply supported one, respectively. Plates side length was 1.0 m, the thickness 0.01 m, uniformly distributed load $p = 0.2$ MPa, and material properties that of Ti-6Al-2Cr-2Mo titanium alloy at 675 K were: $E = 0.102\cdot 10^6$ MPa, $\nu = 0.33$, $n = 6.8$, $\gamma = 1.38\cdot 10^{-24}\,(\mathrm{MPa})^{-n}\,\mathrm{h}^{-1}$, $m = 5.79$, $A = 1.08\cdot 10^{-20}\,(\mathrm{MPa})^{-m}\,\mathrm{h}^{-1}$ (Walczak, 1986). Due to symmetry of the structure only one quadrant of a plate divided into 64 elements was considered.

Table 1 gives the values of times t_1 and t_2 and its ratios for analysed plates.

Table 1. Times t_1 and t_2 for simply supported and clamped plates

α	Simply supported plate			Clamped plate		
	t_1 [10^4h]	t_2 [10^4h]	t_2/t_1	t_1 [10^5h]	t_2 [10^5h]	t_2/t_1
0	2.3696	2.4853	1.0488	3.9457	4.7417	1.2017
0.5	2.6800	2.9561	1.1030	2.2354	4.0919	1.8305
1	1.8376	2.1187	1.1530	0.9213	2.0694	2.2462

In Fig.3 and Fig.4 the damage distributions at time t_1 of FCA in outer layers of simply supported plate and clamped one, respectively, are shown for three values of α = 0, 0.5 and 1. The position at which first crack appears is marked by fully blackened area corresponding to $\omega > 0.99$.

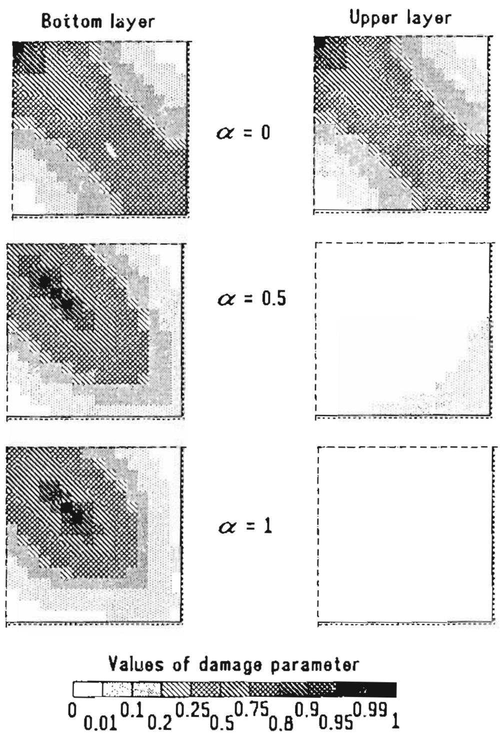

Fig. 3. Damage distribution at time t_1 in outer layers for simply supported plates

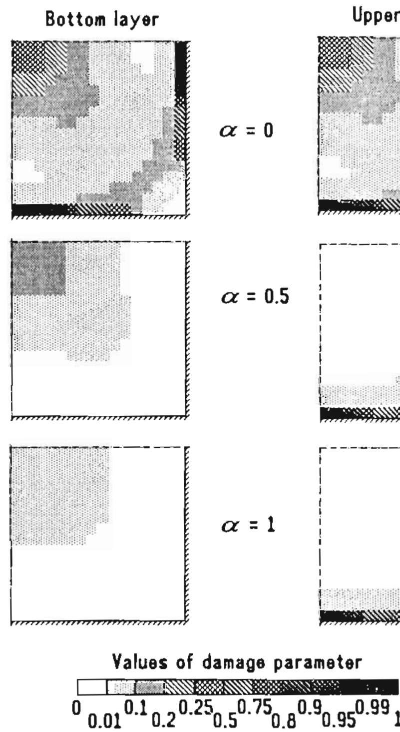

Fig. 4. Damage distribution at time t_1 in outer layers for clamped plates

Fig.5 and Fig.6 show the bottom and top views of plate surfaces at time t_2 for three values of $\alpha = 0$, 0.5 and 1. Number of fractured layers (i.e. these for which $\omega = 1$ condition is fulfilled in a given Gaussian point) is indicated by different shading, as described in the legend.

It is seen from Table 1 that both, incubation time t_1 and the propagation period ($t_1 \leq t \leq t_2$) are shorter for simply supported plates than these for clamped ones. Redundancy of the plates influences essentially the ratio of t_2/t_1. It can be as high as about 225% (for clamped plate and $\alpha = 1$), and as low as about 105% (for simply supported plate and $\alpha = 0$).

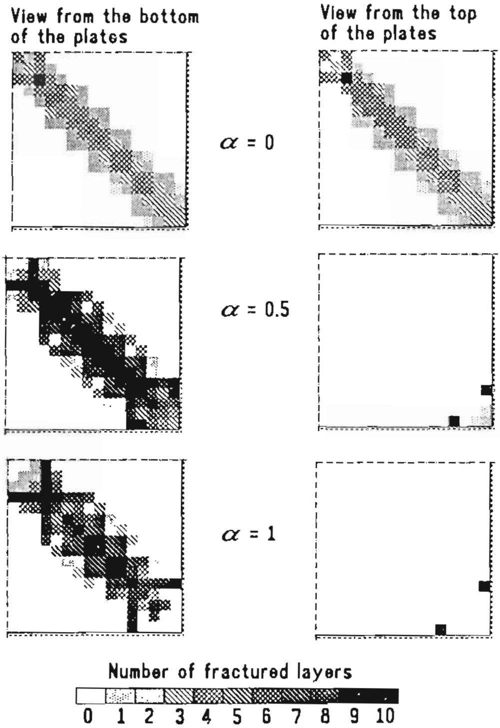

Fig. 5. Fractured layers of simply supported plates at time t_1 as seen from the bottom and from the top

The value of α influences in regular manner the ratio of t_2/t_1: smaller the value of α lower the ratio of t_2/t_1. It is obvious in the light of fact that $\alpha = 0$ implies fracture process initiating and progressing from both outer surfaces of a plate.

Locations of FCA at time t_1 and that of through-plate failure at time t_2 do not coincide for all analysed plates and all values of α. The influence of α on this discrepancy can be significant (cf the case of $\alpha = 0$ for clamped plate). The only case in which this shift is small is $\alpha = 0$ for simply supported plate.

The phenomenon of failure zone branching is observed for both types of plate supports and for $\alpha = 0.5$ and $\alpha = 1$. It takes place in the vicinity of plate corners where interaction of boundary effects on perpendicular edges prevents the damage development along diagonals.

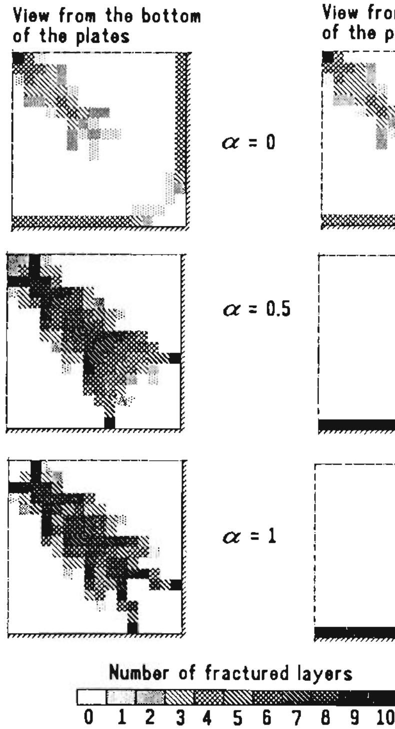

Fig. 6. Fractured layers of clamped plates at t_2 as seen from the bottom and from the top

4.2. Shielded plate

The plates with boundary conditions the same as in the Section 4.1 have been analysed here, but composed of an elastic core, and two external layers (each of 10% plate thickness) with mechanical properties described by Eqs (2.1) through (2.4). Material constants are the same as in the previous Section. For such a structure the whole process of deterioration is confined only to the shielding external layers, and the time to first crack appearance t_1 coincides with time t_2.

The values of time to first crack appearance for $\alpha = 0.5$ and $\alpha = 1.0$ and two types of plate support are given in Table 2, and Fig.7 and Fig.8 show the damage distribution at time of first crack appearance t_1.

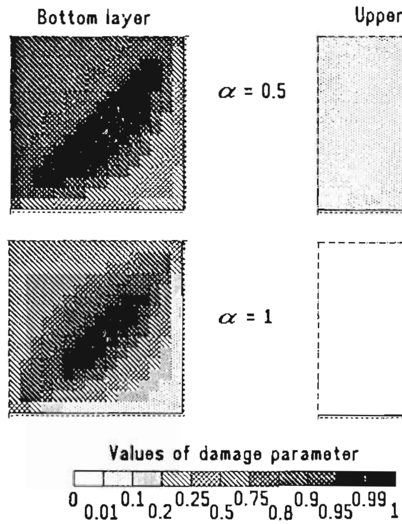

Fig. 7. Damage distribution at time t_1 in external layers for simply supported plates with shielding layers

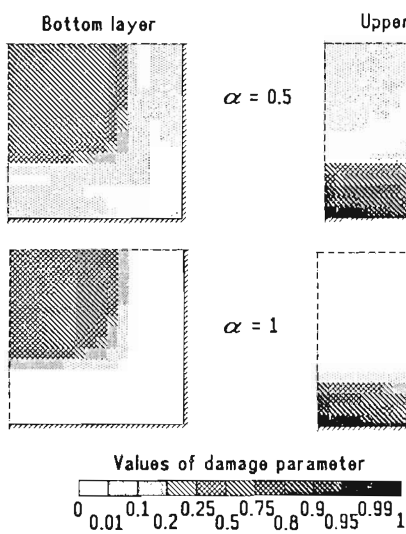

Fig. 8. Damage distribution at time t_1 in external layers for clamped plates with shielding layers

Table 2. Times t_1 for simply supported and clamped plates with shielding layers

α	Simply supported plate	Clamped plate
0.5	$1.2665 \cdot 10^{18}$ [h]	$9.9954 \cdot 10^{13}$ [h]
1	$1.8007 \cdot 10^{12}$ [h]	$6.6362 \cdot 10^{9}$ [h]

The general observation is, that time t_1, is much more higher than that resulting from analysis performed in Section 4.1. For $\alpha = 0$ the state of first crack appearance was not obtained for times of a comparable order. This phenomenon was the result of damage parameter distribution which caused a material to soften evenly to such extent that time-dependent properties become negligible in structure analysis. The same applies to the observation that t_1 for clamped plate is shorter then that for a simply supported one, in the contrary to the through-thickness degrading plates.

Acknowledgement
The work reported in this paper has been performed partially under project No. 3/0917/91/01 granted by KBN (Polish Committee for Scientific Research).

The assistance of Mr. K. Nowak in elaboration of the results of numerical calculations is highly appreciated.

References

1. Ahmad S., Irons B.M., Zienkiewicz O.C., 1970, *Analysis of thick and thin shell structures by curved finite elements*, Int. J. Num. Meth. Eng., **2**, 419-451
2. Bodnar A., 1991, *The influence of failure mechanism mode on the damage field in creeping plates*, Trans. SMIRT-11, **Vol. L**, 391-396
3. Bodnar A., Chrzanowski M., 1989, *Numerical analysis of damage growth in transient creep of plates subjected to bending*, (in Polish), Mechanika i Komputer, **9**, 101-115
4. Bodnar A., Chrzanowski M., Latus P., 1992, *Life-time evaluation of creeping structures*, Proc. 5th Int. Conf. on Creep of Materials, 461-469
5. Figureiras J.A., Owen D.R.J., 1984, *Analysis of elasto-plastic and geometrically nonlinear anisotropic plates and shells*, Finite element software for plates and shells, ed. E.Hinton, D.R.J.Owen, Pineridge Press Ltd, 235-326
6. Mindlin R.D., 1951, *Influence of rotary inertia and shear on flexural motions of isotropic elastic plates*, J.Appl.Mech., **18**, 31-38
7. Reissner E., 1945, *The effect of transverse shear deformation on the bending of elastic plates*, J.Appl.Mech., **12**, 69-77
8. Walczak J., 1986, *On an energy creep rupture criterion*, Int.J.Mech.Sci., **2**, 71-81

Pękanie pełzających płyt w ujęciu kontynualnej mechaniki uszkodzeń

Streszczenie

Zastosowanie Kontynualnej Mechaniki Uszkodzeń do opisu pękania płyt w warunkach pełzania pozwoliło na opis dwóch etapów tego procesu: nukleacji pierwszego makro-pęknięcia, a następnie jego propagacji aż do czasu uformowania pęknięcia przez całą grubość płyty. Rozważono dwa przykłady płyt: płytę jednolitą podlegającą procesowi uszkodzania w całej swojej objętości oraz płytę sprężystą osłoniętą zewnętrznymi warstwami podlegającymi degradacji. W tym ostatnim wypadku pojawienie się pierwszego pęknięcia wyznacza praktycznie kres zależnego od czasu zachowania się konstrukcji, podczas gdy w przypadku płyty o własnościach zależnych od czasu w całej jej objętości możliwe jest wyznaczenie stosunku czasu pierwszego pęknięcia do czasu pęknięcia przez całą grubość płyty.

Manuscript received October 1, 1993; accepted for print October 13, 1993