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# GENERALIZATION OF THE KERR FOUNDATION MODEL[1]

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The aim of this paper is to propose a generalized model of the elastic foundation consisting of the $n$ – SBS layers. Each SBS layer (Shear layer – Bending layer – Spring layer) consists at the outmost three layers: a layer capable of bearing shearing loads, a bending layer and a layer consisting of springs. The general formulae describing the response of such a foundation are derived and analyzed.

## 1. Introduction

Since about a hundred years – at least since 1867 when the Winkler's paper [1] was published there have been proposed various physical as well as mathematical models describing response of a real soil subgrade with the help of a model of elastic foundation. The response of the soil subgrade is usually modeled in terms of the layers of springs, membranes or the layers which are capable of bearing the bending or shearing loads. These layers can also be modeled by means of stipulating certain constraints on the continuum layer description.

If one assumes that the foundation is capable of bearing loads normal to its surface then the response can be modelled with the help of one equation of the following general form

$$\mathcal{L}p(x^\alpha) = \mathcal{R}w(x^\alpha) \tag{1.1}$$

Here $p$ represents density of the load normal to the foundation and $w$ stands for the displacement in the $z = x^3$ direction, (cf Fig.1).

The differential operators $\mathcal{L}$ and $\mathcal{R}$ can be expressed as follows

$$\mathcal{L} = a + \sum_{i=1}^{I}(-1)^i a_i \nabla^{2i} \qquad\qquad \mathcal{R} = k + \sum_{j=1}^{J}(-1)^j b_j \nabla^{2j} \tag{1.2}$$

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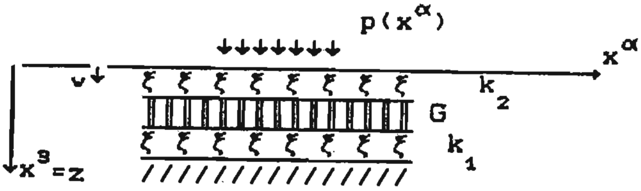


Fig. 1.

where coefficients $a$, $a_i$, $k$, and $b_j$, respectively characterizing the foundation are given in the following units $[a] = 1$, $[a_i] = \mathrm{m}^{2i}$, $[k] = \mathrm{N\,m}^{-3}$, $[b_i] = \mathrm{N\,m}^{2j-3}$.

If we assume $a = 1$, $a_i = 0$ and $b_j = 0$ in Eqs (1.2) for all values of indices $i$ and $j$, we obtain the Winkler model of elastic foundation. If we assume that only $a$, $b_1$ and $k$ is non–vanishing we arrive at the equations for the models of Wieghardt [2], Filonenko–Borodich [3], Schiel [4], Pasternak [5], Vlasov and Leont'ev [6], respectively. It is noteworth that the coefficient $b_1$ has a different mechanical interpretation in each of these models.[1]

One can assume $a = 1$, $a_1 \neq 0$, $a_3 \neq 0$, $k \neq 0$, $b_2 \neq 0$ and put the other coefficients equal to zero. Then we obtain the model of Hetényi (cf [8,9]). The Levinson model [10,11] refers to the case when only $a$, $k$, $b_1$ and $b_2$ are non–vanishing.

In the models proposed by Ratzersdorfer [12], Favre [13,14], Bharatha and Levinson [15],respectively it is assumed that $a = 1$, all coefficients $a_i = 0$, $k \neq 0$ and all $b_i$ are non–vanishing.

Reissner (cf [16,17]) puts forward the model where $a = 1$, $a_1 \neq 0$, $k \neq 0$ and $b_1 \neq 0$.

Kerr and Rhines [18] generalized the model of Bosson [19] and obtained the Eq (1.2) with all coefficients being different from zero. Kerr considered composite models formed by several alternately placed shear–deformable layers and elastic layers.

In that case, according to on the kind and the number of layers one can obtain equations where $a_i \neq 0$, $b_i \neq 0$.

In 1965 Kerr [20] derived an equation describing the response of the foundation model given in Fig.1. However in the Kerr's paper [21] we can find equations for the models given in Fig.2÷4, respectively.

The aim of the present paper is to put forward a method of constructing the $\mathcal{L}$ and $\mathcal{R}$ operators relevant to the composite model of the foundation consisted of the arbitrary number of the alternately placed: shear–deformable layers, bending layers and the springs layers. This model can be viewed as a generalization of the Kerr models [21,22].

[1] Mechanical interpretation of each these models can be found in the paper [7].

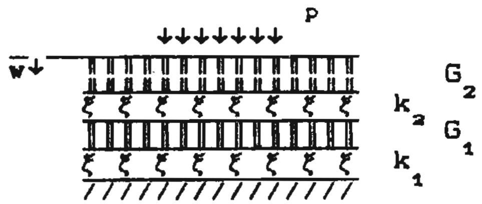


Fig. 2.

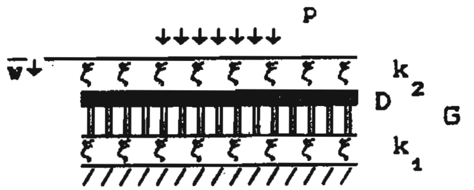


Fig. 3.

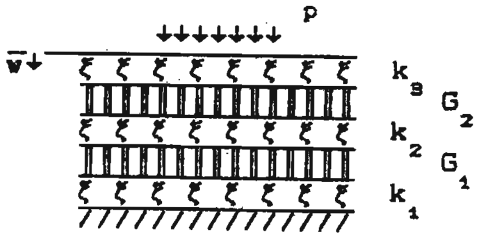


Fig. 4.

## 2. SBS Model

Let us consider a model of a foundation consisted of three layers: the shear layer, the bending layer and the spring layer (cf Fig.5). The equation of this model can be written in the form

$$p(x^\alpha) = \mathcal{H}w(x^\alpha) \tag{2.1}$$

where $\mathcal{H}$ is the differential operator defined by

$$\mathcal{H} = k - G\nabla^2 + D\nabla^4 \tag{2.2}$$

$k$ is a coefficient of the compliance of the elastic layer, $G$ represents the modulus of shear referring to the layer of unit thickness[2] and $D$ stands for the bending stiffness of the layer.

[2]If the layer is of thickness $h$, $G$ should be replaced by $Gh$.

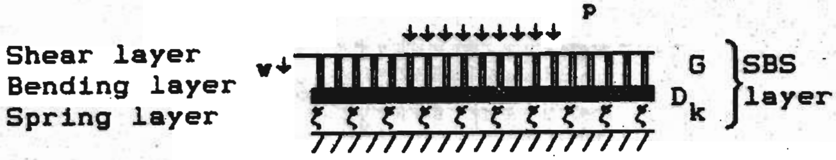


Fig. 5.

The considered model will be called the SBS layer (or the SBS model). SBS means: Shear layer – Bending layer – Spring layer. Let us consider the composite made out of $n$ SBS layers (cf Fig.6).

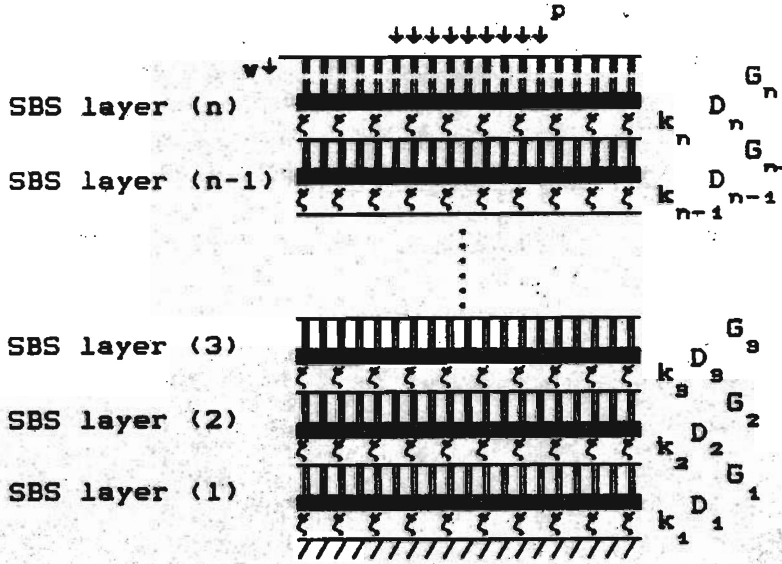


Fig. 6.

One can assign the equation for the $i$th SBS layer as follows (Fig.7)

$$r_{i+1}(x^\alpha) = k_i v_i(x^\alpha) + \widetilde{\mathcal{H}}_i w_i(x^\alpha) \qquad i = 1,2,...,n \tag{2.3}$$

where $v_i(x^\alpha)$ displacement describes the shortening of the springs of $i$th SBS layer, $w_i$ stands for the deflection of the $i$th SBS layer and $\widetilde{\mathcal{H}}_i$ is the differential operator in the form

$$\widetilde{\mathcal{H}}_i = \mathcal{H}_i - k_i = -G_i\nabla^2 + D_i\nabla^4 \tag{2.4}$$

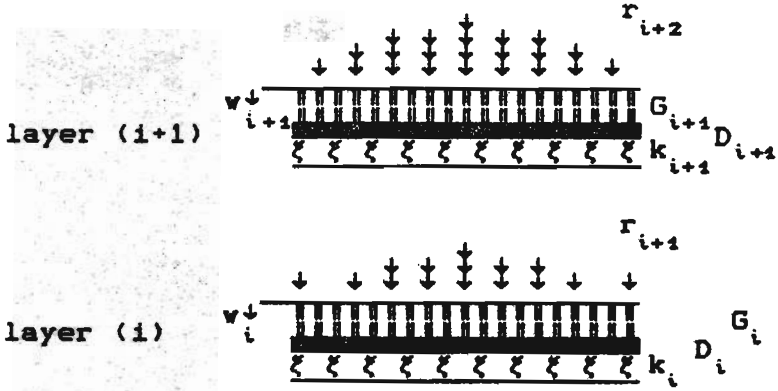

Fig. 7.

The deflection $w_i$ is defined by the formulae

$$w_i(x^\alpha) = \sum_{k=1}^{i} v_k(x^\alpha) \tag{2.5}$$

The quantities $k_i$, $G_i$ and $D_i$ characterize properties of the $i$th SBS layer. The functions $r_{i+1}(x^\alpha)$ for $i = 1, 2, ..., n-1$ represent interactions between the $(i+1)$th SBS layer and the $i$th SBS layer, respectively.

Thus one can determine these interactions by the formula

$$r_{i+1}(x^\alpha) = k_{i+1} v_{i+1}(x^\alpha) \tag{2.6}$$

For $i = n$ we have

$$r_{n+1}(x^\alpha) = p(x^\alpha) \tag{2.7}$$

$$w(x^\alpha) \stackrel{\mathrm{df}}{=} w_n(x^\alpha) = \sum_{k=1}^{n} v_k(x^\alpha) \tag{2.8}$$

where $w(x^\alpha)$ represents the overall deflection of the generalized foundation.

Writing down Eqs (2.3) equation for $i = 1, 2, ..., n$ and using Eqs (2.6)÷(2.8), we obtain the following system of differential equations

$$\mathbf{R}_n \mathbf{V} = \mathbf{J} p \tag{2.9}$$

where

$$\mathbf{V} = \left[v_1\ v_2\ v_3\ ...\ v_n\right]^{\mathrm{T}} \qquad \dim\mathbf{V} = n \times 1 \tag{2.10}$$

$$\mathbf{J} = \left[0\ 0\ 0\ ...\ 0\ 1\right]^{\mathrm{T}} \qquad \dim\mathbf{J} = n \times 1 \tag{2.11}$$

$$\mathbf{R}_n = \begin{bmatrix} \mathcal{H}_1 & -k_2 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \widetilde{\mathcal{H}}_2 & \mathcal{H}_2 & -k_3 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \widetilde{\mathcal{H}}_3 & \widetilde{\mathcal{H}}_3 & \mathcal{H}_3 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \widetilde{\mathcal{H}}_i & \widetilde{\mathcal{H}}_i & \widetilde{\mathcal{H}}_i & \cdots & \widetilde{\mathcal{H}}_i & \mathcal{H}_i & -k_{i+1} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \widetilde{\mathcal{H}}_{n-1} & \widetilde{\mathcal{H}}_{n-1} & \widetilde{\mathcal{H}}_{n-1} & \cdots & \widetilde{\mathcal{H}}_{n-1} & \widetilde{\mathcal{H}}_{n-1} & \widetilde{\mathcal{H}}_{n-1} & \cdots & \widetilde{\mathcal{H}}_{n-1} & \mathcal{H}_{n-1} & -k_n \\ \widetilde{\mathcal{H}}_n & \widetilde{\mathcal{H}}_n & \widetilde{\mathcal{H}}_n & \cdots & \widetilde{\mathcal{H}}_n & \widetilde{\mathcal{H}}_n & \widetilde{\mathcal{H}}_n & \cdots & \widetilde{\mathcal{H}}_n & \widetilde{\mathcal{H}}_n & \mathcal{H}_n \end{bmatrix} \tag{2.12}$$

$$\dim\mathbf{R}_n = n \times n$$

$$\mathcal{H}_i = k_i + \widetilde{\mathcal{H}}_i = k_i - G_i\nabla^2 + D_i\nabla^4 \tag{2.13}$$

On using formally the Cramer's formulae, we find

$$\det\mathbf{A}_{in}\Big(p\Big) = \det\mathbf{R}_n\Big(v_i\Big) \tag{2.14}$$

where $\mathbf{A}_{in}$ matrix is constructed by replacing the $i$th column of $\mathbf{R}_n$ by the column $J$ of right-hand sides

$$\mathbf{A}_{in} = \begin{bmatrix} \mathcal{H}_1 & -k_2 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \widetilde{\mathcal{H}}_2 & \mathcal{H}_2 & -k_3 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \widetilde{\mathcal{H}}_3 & \widetilde{\mathcal{H}}_3 & \mathcal{H}_3 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \widetilde{\mathcal{H}}_i & \widetilde{\mathcal{H}}_i & \widetilde{\mathcal{H}}_i & \cdots & \widetilde{\mathcal{H}}_i & 0 & -k_{i+1} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \widetilde{\mathcal{H}}_{n-1} & \widetilde{\mathcal{H}}_{n-1} & \widetilde{\mathcal{H}}_{n-1} & \cdots & \widetilde{\mathcal{H}}_{n-1} & 0 & \widetilde{\mathcal{H}}_{n-1} & \cdots & \widetilde{\mathcal{H}}_{n-1} & \mathcal{H}_{n-1} & -k_n \\ \widetilde{\mathcal{H}}_n & \widetilde{\mathcal{H}}_n & \widetilde{\mathcal{H}}_n & \cdots & \widetilde{\mathcal{H}}_n & 1 & \widetilde{\mathcal{H}}_n & \cdots & \widetilde{\mathcal{H}}_n & \widetilde{\mathcal{H}}_n & \mathcal{H}_n \end{bmatrix} \tag{2.15}$$

$$\dim\mathbf{A}_{in} = n \times n$$

Performing summation over limits $i = 1$ to $i = n$ at the both sides of Eq (2.14) and using Eq (2.8), upon appropriate rearranging the determinants $\det\mathbf{R}_n$ and $\det\mathbf{A}_{in}$ one can write down the equation of the model in the form

$$\mathcal{L}_n\Big(p(x^\alpha)\Big) = \mathcal{R}_n\Big(w(x^\alpha)\Big) \tag{2.16}$$

The differential operators $\mathcal{L}_n$ and $\mathcal{R}_n$ are given by the formulae

$$\mathcal{L}_n \stackrel{\text{df}}{=} \sum_{i=1}^{n}\Big(\det\mathbf{A}_{in}\Big) = \sum_{i=1}^{n}(-1)^{i+n}\det\mathbf{B}_{in} \tag{2.17}$$

$$\mathcal{R}_n = \det\mathbf{R}_n \tag{2.18}$$

where

$$\mathcal{R}_n = \det \begin{bmatrix} \mathcal{K}_1 & -k_2 & 0 & 0 & \cdots & 0 & 0 & 0 \\ -k_2 & \mathcal{K}_2 & -k_3 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -k_3 & \mathcal{K}_3 & -k_4 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -k_{n-1} & \mathcal{K}_{n-1} & -k_n \\ 0 & 0 & 0 & 0 & \cdots & 0 & -k_n & \mathcal{H}_n \end{bmatrix} \tag{2.19}$$

$$\mathbf{B}_{in} = \begin{array}{cc} \begin{array}{cccccccc} & i-4 & i-3 & i-2 & i-1 & i+1 & i+2 & \end{array} & \\ \left[\begin{array}{cccccccc} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & k_{i-3} & \widetilde{\mathcal{K}}_{i-3} & -k_{i-2} & 0 & 0 & 0 & \cdots \\ \cdots & 0 & -k_{i-2} & \widetilde{\mathcal{K}}_{i-2} & -k_{i-1} & 0 & 0 & \cdots \\ \cdots & 0 & 0 & -k_{i-1} & \mathcal{H}_{i-1} & 0 & 0 & \cdots \\ \cdots & 0 & 0 & 0 & \widetilde{\mathcal{K}}_i & -k_{i+1} & 0 & \cdots \\ \cdots & 0 & 0 & 0 & -k_{i+1} & \widetilde{\mathcal{K}}_{i+1} & -k_{i+2} & \cdots \\ \cdots & 0 & 0 & 0 & 0 & -k_{i+2} & \widetilde{\mathcal{K}}_{i+2} & \cdots \\ \cdots & 0 & 0 & 0 & 0 & 0 & -k_{i+3} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{array}\right] & \begin{array}{c} \\ i-3 \\ i-2 \\ i-1 \\ i \\ i+1 \\ i+2 \\ i+3 \\ \\ \end{array} \end{array} \tag{2.20}$$

$$\begin{aligned} &\dim B_{in} = (n-1) \times (n-1) \\ &\mathcal{K}_i = \mathcal{H}_i + k_{i+1} \\ &\widetilde{\mathcal{K}}_i = \widetilde{\mathcal{H}}_i + k_{i+1} \end{aligned}$$

## 3. SBVS model

Eq (2.16) found in the previous section holds when the foundation is viscoelastic and composed of the (SBVS) layers given in Fig.8 and 9, respectively.

In such a case the quantity $k_i$ should be replaced by $k_i + \eta_i \frac{\partial}{\partial t}$. The operator $\mathcal{H}_i$ assumes the form

$$\mathcal{H}_i = k_i + \eta_i \frac{\partial}{\partial t} + \widetilde{\mathcal{H}}_i = k_i + \eta_i \frac{\partial}{\partial t} - G_i \nabla^2 + D_i \nabla^4 \tag{3.1}$$

The foregoing above model of the foundation composed of the SBS or SBVS layers can by generalized to the foundation having anisotropic properties and being capable of bearing all three components of the loading $p_1, p_2, p_3 = p$.

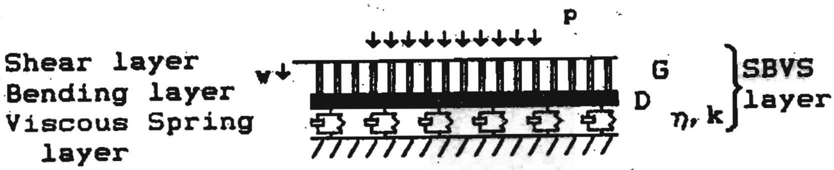


Fig. 8.

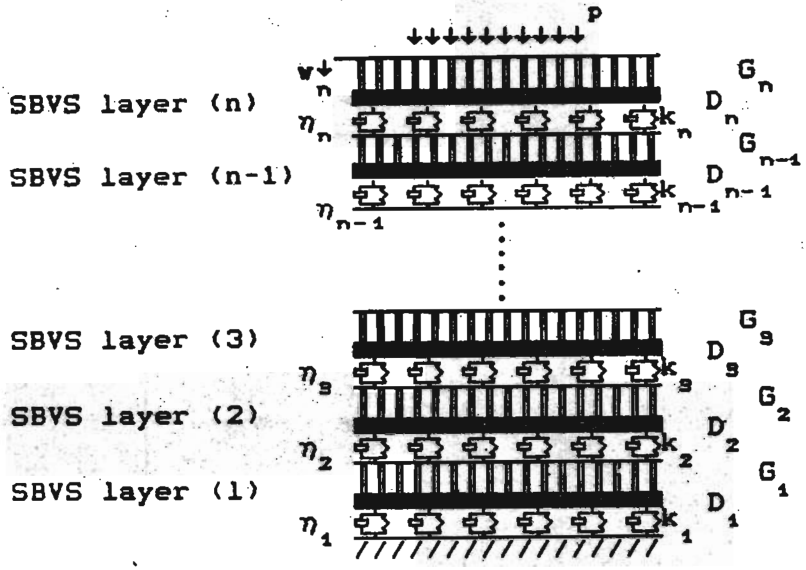


Fig. 9.

## 4. Example

Let us consider a foundation composed of the three SBS layers (cf Fig.10). Using Eqs (2.16)÷(2.20) for $n = 3$, we obtain respectively

$$\mathcal{L}_3\Big(p(x^\alpha)\Big) = \mathcal{R}_3\Big(w(x^\alpha)\Big) \tag{4.1}$$

$$\begin{aligned}
\mathcal{L}_3 &= \sum_{i=1}^{3}\Big(\det \mathbf{A}_{i3}\Big) = \sum_{i=1}^{3}(-1)^{i+3}\det \mathbf{B}_{i3} = \\
&= \det\begin{bmatrix} -k_2 & 0 \\ \mathcal{H}_2 + k_3 & -k_3 \end{bmatrix} - \det\begin{bmatrix} \mathcal{H}_1 & 0 \\ \widetilde{\mathcal{H}}_2 + k_3 & -k_3 \end{bmatrix} + \det\begin{bmatrix} \mathcal{H}_1 + k_2 & -k_2 \\ -k_2 & \mathcal{H}_2 \end{bmatrix} = \\
&= k_1k_2 + k_1k_3 + k_2k_3 - \Big[(k_2 + k_3)G_1 + (k_1 + k_2)G_2\Big]\nabla^2 +
\end{aligned} \tag{4.2}$$

$$+\Big[G_1G_2+(k_2+k_3)D_1+(k_1+k_2)D_2\Big]\nabla^4-(G_1D_2+G_2D_1)\nabla^6+D_1D_2\nabla^8$$

$$\mathcal{R}_3=\det\mathbf{R}_3=\det\begin{bmatrix}\mathcal{H}_1+k_2 & -k_2 & 0\\ -k_2 & \mathcal{H}_2+k_3 & -k_3\\ 0 & -k_3 & \mathcal{H}_3\end{bmatrix}=$$

$$=k_1k_2k_3-\Big[k_2k_3G_1+k_3(k_1+k_2)G_2+(k_1k_3+k_1k_2+k_2k_3)G_3\Big]\nabla^2+$$

$$+\Big[(k_2+k_3)G_1G_3+(k_1+k_2)G_2G_3+k_3G_1G_2+k_2k_3D_1+k_3(k_1+k_2)D_2+$$

$$+(k_1k_2+k_1k_3+k_2k_3)D_3\Big]\nabla^4-\Big[G_1G_2G_3+\Big(k_3D_2+(k_2+k_3)D_3\Big)G_1+ \tag{4.3}$$

$$+\Big(k_3D_1+(k_1+k_2)D_3\Big)G_2+\Big((k_2+k_3)D_1+k_1D_2+k_2D_3\Big)G_3\Big]\nabla^6+$$

$$+\Big[G_2G_3D_1+G_1G_3D_2+G_1G_2D_3+k_3D_1D_2+(k_1+k_2)D_2D_3+$$

$$+(k_2+k_3)D_1D_3\Big]\nabla^8-\Big(G_1D_2D_3+G_2D_1D_3+G_3D_1D_2\Big)\nabla^{10}+$$

$$+D_1D_2D_3\nabla^{12}$$

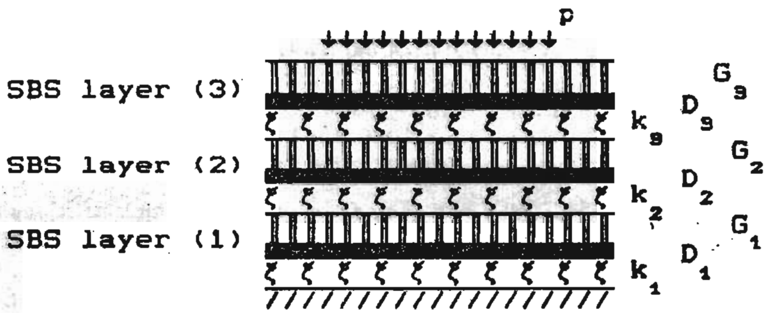


Fig. 10.

With the help of the equations given above one can obtain the equations of the Kerr models given in Fig.1 ÷ 4.

On putting $D_1=D_2=D_3=0$, $G_3=0$, in Eqs (4.2)÷(4.3) one arrives at the equation of the Kerr foundation in Fig.4

$$\Big\{1+\frac{k_1}{k_2}+\frac{k_1}{k_3}-\Big[\Big(\frac{1}{k_2}+\frac{1}{k_3}\Big)G_1+\frac{1}{k_3}\Big(1+\frac{k_1}{k_2}\Big)G_2\Big]\nabla^2+\frac{1}{k_2k_3}G_1G_2\nabla^4\Big\}p= \tag{4.4}$$

$$=\Big\{k_1-\Big[G_1+\Big(1+\frac{k_1}{k_2}\Big)G_2\Big]\nabla^2+\frac{1}{k_2}G_1G_2\nabla^4\Big\}w$$

On putting $G_1 = G$, $D_1 = D$, $D_2 = D_3 = 0$, $G_2 = G_3 = 0$, $k_3 = \infty$ (or $G_1 = G$, $D_1 = D_3 = 0$, $D_2 = D$, $G_2 = G_3 = 0$, $k_2 = \infty$, $k_3 = k_2$), one obtains the equation of the foundation given in Fig.3

$$\Big(1 + \frac{k_1}{k_2} - \frac{G}{k_2}\nabla^2 + \frac{D}{k_2}\nabla^4\Big)p = \Big(k_1 - G\nabla^2 + D\nabla^4\Big)w \tag{4.5}$$

For $D_1 = D_2 = D_3 = 0$, $G_2 = G_3 = 0$ and $k_3 = \infty$ one finds the equation characterizing the model given in Fig.2

$$\Big(1 + \frac{k_1}{k_2} - \frac{G_1}{k_2}\nabla^2\Big)p = \Big\{k_1 - \Big[G_1 + \Big(1 + \frac{k_1}{k_2}\Big)G_2\Big]\nabla^2 + \frac{1}{k_2}G_1G_2\nabla^4\Big\}w \tag{4.6}$$

The equation of the model in Fig.1 can be arrived at upon substituting for $D_1 = D_2 = D_3 = 0$, $G_1 = G$, $G_2 = G_3 = 0$ and $k_3 = \infty$, respectively

$$\Big(1 + \frac{k_1}{k_2} - \frac{G_1}{k_2}\nabla^2\Big)p = \Big(k_1 - G_1\nabla^2\Big)w \tag{4.7}$$

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**Uogólniony model podłoża Kerra**

**Streszczenie**

W pracy przedstawiono uogólniony model podłoża sprężystego składający się z układu $n$ warstw SBS (rys.6). Każda warstwa SBS (Shear layer – Bending Layer – Spring layer) składa się co najwyżej z trzech warstw: warstwy czułej na ścinanie, warstwy przenoszącej zginanie oraz z warstwy sprężyn (rys.5). Wyprowadzono ogólne równania takiego modelu podłoża (2.16).

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