EXPERIMENTAL DETERMINATION OF CRITICAL STRESSES IN SANDWICH CYLINDERS UNDER NON-ELASTIC CONDITIONS

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In the present paper both the theoretical analysis of the compression process acting upon cylinders filled with the polyurethanes and the results of experiments have been presented. The experimental tests were performed under substantial plastic strains until the cylinder totally collapsed. Depending on the test bar slenderness ratio different strain patterns have been obtained. It is noteworthy that the critical compressive stresses curve obtained for the mean slenderness ratios neither coincides with the Tetmajer–Jasiński straight line nor with the Johnson–Ostenfeld parabola. A relatively simple function approximating the experimental results has been presented.

1. Introduction

Modern designs and instruments have to meet strict requirements. They should display, for example, high strength or stiffness, be of light weight and the manufacture and exploitation costs of them should be relatively low. Some subassemblies should also meet certain additional requirements, like e.g. the maximal energy absorption condition imposed on the buffers and other shock absorbers.

One should expect that these requirements can be most effectively satisfied by means of the modern sandwich designs application.

The high strength and stiffness requirement together with the deep energy absorption condition imposed on the thin-walled designs could be satisfied by means of e.g. the continuous support with the aid of the polyurethanes foam layer. One can also make the construction materials more useful by applying accurate calculation methods basing e.g. on the physical nonlinearity theory. In the case of buffers one should take into account, for example, the substantial plastic strain. The theory of design analysis by means of the complete plastic conditions application has recently become an interesting subject of many theoretical and experimental researches. The analysis and experiment results obtained for circular sandwich bars
of different lengths filled with the polyurethanes foam and subject to the compression are given in the present paper. The theoretical analysis of critical compressive stresses in slender bars was given by Romanów [2], while the same problem in sandwich shells was considered by Dzielendziak and Romanów [3]. The kinematic analysis of energy absorption in hollow, rectangular columns under compression was shown by Abramowicz [3]. One can also find some solutions to different stability problems in inelastic regions of thin-walled designs in the work of Królak et al. [4].

2. Experimental tests

The tests were carried out on circular tubes of external diameter 30 mm and with the walls 0.75 mm thick made of duraluminium PA-4N-tb according to the Polish Standard PN-7/-H-74592. The following material properties were taken according to experiments: \( E = 72387 \) MPa; \( R_H = 153 \) MPa; \( R_{0.2} = 191 \) MPa; \( R_m = 259 \) MPa; \( A_{10} = 19.8\% \).

The cores were made of polyurethanes foam of densities: \( \rho_r = 100, 125, 150 \) kg/m\(^3\), respectively; the coefficients applied were: Poisson ratio \( \nu_r = 0 \); Kirchoff modulus \( G_r = 19.4 \) MPa; Young modulus \( E_r = 38.8 \) MPa.

![Fig. 1.](image)

The test bar slenderness ratios were calculated for hollow tubes and were equal to: 4.7; 5.8; 6.8; 7.7; 9.5; 20; 50; 75, respectively.

For the more exact determination of the test bar carrying capacity in the plastic region the number of samples of small slenderness ratio was increased.
The test bars of slenderness ratio $\lambda \geq 20$ were compressed with the aid of
sharing shackles (Fig.1), while the remaining samples were compressed on the
testing machine directly between the holders.

The results obtained for tubes filled with the polyurethane foam were compared
to those obtained for the hollow samples under the same conditions. 51 tubes were
tested i.e. 23 hollow and 28 filled with polyurethane foam ones. The results for
hollow tubes are presented in Table 1, while those obtained for the filled ones can
be found in Tables 2 and 3, respectively.

Table 1. Critical loadings and deformation works obtained for the hollow
tubes

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<th>Notation</th>
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<th>$\lambda$</th>
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<th>$P_{1m}$ [kN]</th>
<th>$\sigma_{cr}$ [MPa]</th>
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<th>$L_1$ [J]</th>
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2.1. Discussion of the tests carried out in elastoplastic conditions

Below, one can find some remarks concerning hollow tubes. The results obtained in experiments for slenderness ratios \(4.8 \leq \lambda \leq 75\) are given in Table 1.

The following notation is used in figures and tables, respectively:
- \(a\) — tube length,
- \(\lambda\) — slenderness ratio,
- \(P_{cr}\) — maximal force which the tube is able to carry, \(P_{cr} = P_1\), corresponding to the point 1 in Figs. 2 and 4,
- \(\sigma_{cr}\) — stresses which one can calculate dividing the critical force \(P_{cr}\) magnitude by the hollow tube cross-section area; \(\sigma_{cr} = \sigma_1\),
- \(L_1\) — work of deformation corresponding to the area under the 0–1 segment of the curve,
- \(L_c\) — total work of deformation,
- \(k_{P1}\) — ratio between the same \(P_1\) force for the hollow and filled tube, respectively,
- \(k_{L1}\) — the same as the foregoing ratio but corresponding to the work of deformation,
- \(k_{Lc}\) — corresponds to the total work of deformation.

For the tested hollow samples the limiting slenderness ratio \(\lambda_{lim}\) calculated with the aid of the Euler formula (for \(R_H = 155\) MPa) equals 68.3.

The control tests carried out for hollow tubes of \(\lambda = 75\) (Table 1) have shown a very good correlation between the experimental test results and those obtained after theoretical calculations, e.g. \(\sigma_{cr}\) from experiments was equal to 125 MPa, while after calculation one can obtain 127 MPa. For samples of slenderness \(\lambda = 75\) the critical compressive stresses have not reached the limit of proportionality, which for the present material is equal to \(R_H = 153\) MPa. The values of critical compressive stresses obtained for the tubes of the slenderness ratio \(\lambda = 20\) have been lying in the range between \(R_H = 153\) MPa and \(R_{0.2} = 191\) MPa. The negligible differences have appeared between the values of the critical compressive stresses in tubes of \(\lambda_{lim} = 1 \leq 10\), which were lying in the range of 207 \(\div\) 220 MPa. The mean value has been calculated as 214 MPa with the standard deviation equal to 4.8.

The experimental results concerning the critical compressive stresses in samples of slenderness ratio

\[10 \leq \lambda \leq \lambda_{lim} = 68.3\]

have been approximated by means of the Tetmajer straight line (Fig. 3)

\[\sigma_{cr} = 226.4 - 1.1\lambda\]  \hspace{1cm} (2.1)

while the value of 214 MPa has been applied to the samples of \(\lambda < 10\).
Fig. 2.

--- according to Engesser - Shanley

\[ \sigma_w = \frac{2264 - 1.1\lambda}{\lambda} \text{ for } 10 < \lambda \leq 68.3 \]

\[ \sigma_w = 214 = \text{const} \text{ for } \lambda = 10 \]

\[ \sigma_w = \frac{\tau_i}{\lambda} \text{ for } \lambda = 68.3 \]

Fig. 3. The critical compressive stresses diagram versus the slenderness ratio how the hollow tubes
The Euler formula holds for \( \lambda \geq \lambda_{\text{lim}} = 68.3 \).

The whole diagram of the critical compressive stresses distribution in hollow tubes versus the slenderness ratio is shown in Fig.3 together with the results obtained with the aid of the Engesser–Shanley theory (broken line). It can be easily seen from Fig.3 that the experimental results do not correspond to with the calculated ones.

The results for tubes filled with the polyurethanes of apparent density \( \rho = 100; 125; 150 \text{ kg/m}^3 \), respectively of slenderness ratio \( \lambda = 20; 50; 75 \), respectively are presented in Table 2.

Table 2 contains also the following additional notations

\[
k_p = \frac{P_{\text{cr-filled}}}{P_{\text{cr-hollow}}} \quad K_{\text{LC}} = \frac{L_{\text{filled}}}{L_{\text{hollow}}}
\]

Table 2. Critical loadings and deformation works obtained for the tubes filled with foams of different densities for the slenderness ratio \( \lambda = 20; 50; 75 \)

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<th>( \lambda )</th>
<th>( P_{\text{cr-m}} )</th>
<th>Confidence interval ( P_{\text{cr}} )</th>
<th>( \sigma_{\text{cr-m}} ) [MPa]</th>
<th>( L_m )</th>
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It can be easily shown that the foam density value of the range 100 \( \div \) 150 kg/m\(^3\) is of negligible influence on the critical compressive stress values since the confidence interval contains all the load values. The mean critical force \( P_{\text{cr-m}} \) together with the corresponding confidence intervals have been calculated by means of the results of four experiments. The critical compressive stresses ratio \( k_p \) for filled tubes is close to unity. Basing on the fact that for \( \lambda = 20 \) the foregoing ratio increases only by 10% one can conclude that below the critical values, practically only the lining (the hollow tube) undergoes the compressive loads. Also the work of deformation performed by the force of the value not exceeding \( P_{\text{cr}} \) is for \( \lambda = 50 \) and 75 practically the same as for the hollow tubes. Only for \( \lambda = 20 \) the work of deformation performed on the filled tubes increases, roughly speaking, by 62% in comparison with the value for the hollow ones \( (k = 1.46) \).

The diagrams of compression for samples of \( \lambda \geq 20 \) are presented in Fig.4. It can be easily seen that particular curves differ from each other depending on the slenderness ratio values. For larger values of slenderness ratio the stability loss...
arrives in "an instant way", shown by the refraction of the diagrams through the smaller angle. For slenderness ratio \( \lambda = 20 \) the sample deforms in a milder way. It is noteworthy that in all three diagrams there exist the region of proportionality between the compression force and the compression. Calculations of bars of this type were given by Romanów [1].

2.2. Discussion of the plastic conditions

The way of sample crushing in the form of "bellows" is characteristic in this case.

Depending on the size, the samples deform in the form of

- ring on the circle diagram (Fig.5),
- fold on the hexagon projection (Fig.6),
- in the mixed form in such away that some folds are of the ring form, while the rest of them takes the form of hexagon (Fig.7).

These patterns of deformation are called: ring, hexagon and ring–hexagon ones, respectively.

The samples of the small slenderness ratio values have deformed in such a way according to the results shown in Fig.3. For larger values of the slenderness ratio, after the loss of stability the samples have broken in one cross–section only (cf Fig.8).
According to the experimental results it can be seen that samples of the "higher stiffness" tend to the ring pattern of deformation.

Having two samples of the same slenderness ratio it can be found that the hollow one deforms in the hexagon pattern, while the filled tube deformation patterns on the ring.

Let us have a look at the diagram shown in Fig. 2 (test bar No. 37, Table 3) being the typical one for all the samples of the series from No. 33 to No. 51 (Table 3), i.e. for \( \lambda < 10 \). The plot of the compressing force \( P \) [kN] versus the sample shortage \( \Delta a \) is presented. Having the initial length \( a = 60 \text{ mm} \) and the final one \( a_1 = 15 \text{ mm} \), respectively one can, after a simple calculation, obtain the sample shortage being the distance from the origin to the point 16 in the abscissa direction since the accepted abscissa scale is 5:1 and the real distance equals to 225 mm. The deformation is equal \( \varepsilon = \Delta a / a = -0.75 \).

After passing the point 1 corresponding to the maximal force \( P_1 = 16.4 \text{ kN} \) which the sample can undergo it begins to lose the stability. The first half-wave arrives on one end at \( \sigma_1 = 238 \text{ MPa} \) which is in fact one of the lowest stresses in the whole group of the tubes, calculated with regard to the hollow tube cross-section only (the core was neglected). Taking the small value of the Young modulus for the core \( (E_r = 38.8 \text{ MPa}) \) into account one can establish the fact that the whole load is carried by the hollow tube, since the force which the core undergoes equals 0.0814 kN.

It is noteworthy that the values of these stresses lie in the region between the yield strength \( R_{0.2} = 191 \text{ MPa} \) and the ultimate strength \( R_m = 259 \text{ MPa} \). Basing on the results presented in Table 3 one can calculate the mean stress for this group of samples \( \sigma_m = 245 \text{ MPa} \).

The stress \( \sigma_1 \) one should take for the critical one corresponding to the moment of the first half-wave occurrence. It is the maximal stress the sample can undergo.

The diagram shows that the maximal forces corresponding to the points 5, 8, 10, 12 and 14, respectively do not exceed the value of \( P_1 \). The differences between
Table 3. Results for filled tubes of slenderness ratio \( \lambda < 10 \)

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the successive maximal force are relatively small with the mean value equal to $P_{m5} \cong 13$ kN.

The ratio between the force $P_1 = P_{max}$ and the mean one $P_{m5}$ causing the successive half-wave equals to 1.26. The compression tests were stopped at the point 16, corresponding to the deformation level at which all the halfwaves were completely compressed and the contact appear. The forces at points 1 and 16, respectively correspond to each other.

The compressed bellows could possibly stand for the homogeneous material (the new tube of thick wall), but the experiments were stopped at this point.

The total work of deformation (the proper area below the curve 0,1,2,...,16) equals 452.2 J. The mean work of deformation corresponding to one half-wave is equal to 65 J. The mean compressive force for the total work of deformation is equal to $P_{mt} = 10.04$, thus their ratio $P_1/P_{mt}$ equals 1.64. It can be seen (Fig.3) that starting from the point 8 the appropriate segments of the curve pass in a regular way. The additional extrema arriving at points 3,4 and 7, respectively are situated between points 2 and 5 or 6 and 8 only.

Applying the diagram presented in Fig.9 one can easily explain this effect.

In this figure the points 1,2,... and 25, respectively are assigned to the deformed sample.

The first tube blister arrives at the point 1. After the first half-wave complete forming the force decreases to the point 2, at which the first half-wave remains incompressed. At the point 2 a concavity arrives which become deeper and deeper
till; the force reaches the point 3. When the force decreases to the point 4 the first tube blister becomes hold down, while the concavity remains almost undeformed.

At point 5 another tube blister arrives and the course begins to repeat, so at the point 8 one can see two strongly compressed blisters together with the concavity. This way of the blister forming continues to the point 16.

This region is characteristic, since for one blister shaped enough and one new concavity to arrive (on the diagram – force versus shortage) one should pass through the two half-waves, i.e. through the points 0, 1, 2 and 2, 3, 4, respectively. For one half-wave to create one should pass through the two minima (points 2 and 4, respectively) and through the two maxima (points 1 and 3, respectively). The similar process repeats till the point 16 is reached.

Starting from point 16 there is one to one correspondence between the the tube blister and the half-wave on the diagram.

It is noteworthy that the force necessary for the next blister to occur on the test bar should be, roughly speaking, greater than the previous one $P_{25} > P_{23} > ...$ and so on.

The characteristic swings of the curve which can be seen in the segments 3–4, 7–8 and 11–12, respectively vanish at point 15 and the next half-waves (starting from the point 16) are already smooth.

The experimental test results for the critical compressive stresses (denoted in Fig. 10 by crosses) can be easily divided into three groups. For the small values of slenderness ratio $\lambda < 10$, the stresses obtained are of the values close to each other with the mean value equal to 245 MPa while the standard deviation about it equals 8.5. The experimental results for this group of tubes are given in Table 3.

Neither the Tetmajer straight line nor the Johnsen curve overlap the measurements results (crosses in Fig. 10) for $\lambda = 20; 50; \lambda_{im} = 68.3$ (Table 2). One can approximate the results with the aid of the curve being in fact almost the straight line ($\sigma_{cr} = 188.6 - 0.492\lambda$, for $\lambda = 43.8 \div 68.3$).

The following function can approximate the experimental results close enough

$$\sigma_{kr} = \frac{b(\lambda)E}{\lambda^2} \quad (2.3)$$

Applying the mean critical values for the slenderness ratio $\lambda = 20; 50$ and 68.3, respectively (Table 2) one can calculate the corresponding parameters with the aid of Eq (2.3)

$$b_{20} = \frac{\sigma_{cr20} \lambda_{20}^2}{E} = \frac{211 \cdot 20^2}{72387} = 1.16$$

$$b_{50} = \frac{167 \cdot 50^2}{72387} = 5.7 \quad (2.4)$$

$$b_{68.3} = \frac{153 \cdot 68.3^2}{72387} = 9.8$$
where $\sigma_{cr68.3} = 153$ MPa represents the Eulerian stresses calculated for the limiting slenderness ratio $\lambda_{lim} = 68.3$. Relative to the $b_i - \lambda$ coordinate system one can approximate these values by means of the following parabola

$$a_i = c\lambda^2 + d\lambda$$

\[ (2.5) \]

![Diagram](image)

Material: PA-4Ntb; #30 x 0.75mm (duralumin)

- $E = 72387$ MPa; $R_{0,2} = 191$ MPa; $R_m = 259$ MPa
- $R_{cr} = 153$ MPa

$x$ - experimental results
$e$ - according to Euler

$$\sigma_{cr} = \frac{b(\lambda) \cdot E}{\lambda^2}$$

1. for $13.8 < \lambda < 68.3$; $b(\lambda) = 0.00181\lambda^2 + 0.0218\lambda$
2. for $\lambda < 68.3$; $b(\lambda) = \pi^2$
3. for $\lambda > 13.8$; $\sigma_{cr} = 265$ MPa

Calculating the parameters $c$ and $d$ in terms of the corresponding values of $a_i$ and $\lambda_i$ one obtains

$$b = 0.00181\lambda^2 + 0.0218\lambda$$

\[ (2.6) \]

The final form of the function approximating the critical compressive stresses distribution in filled tested tubes ($13.8 \leq \lambda \leq 68.3$) is as follows

$$\sigma_{cr} = \frac{(0.00181\lambda^2 + 0.0218\lambda)E}{\lambda^2}$$

\[ (2.7) \]

We can assume that the expression

$$\frac{(0.00181\lambda^2 + 0.0218\lambda)E}{\lambda^2}$$

\[ (2.8) \]

stand for the equivalent Young modulus. The formula for the critical compressive stresses takes then the well-known form

$$\sigma_{cr} = \frac{\pi^2E_s}{\lambda^2}$$

\[ (2.9) \]
The DBEF curve in Fig.10 represents the graph of the foregoing formula. The Eulerian hiperbola bounds this curve from the right-hand side (point \( E \)). The slenderness ratio corresponding to the point \( E \) stands for the limiting slenderness ratio \( \lambda_{lim} \) according to the Eulerian formula application limits for the testing tubes and for \( R_{ef} = 153 \) MPa, \( \lambda_{lim} = 68.3 \).

For the small values of slenderness ratio (\( \lambda \leq 10 \)) one can assume the critical compressive stresses to be constant and equal to 245 MPa. This is the reason why the \( AB \) straight line crosses the curve at the point \( B \), being the bounding point of the \( DF \) curve application from the left-hand side. The limiting slenderness ratio \( \lambda_{lim,1} = 13.8 \) corresponds to the point \( B \). The \( BE \) curve approximates the experimental results close enough since the maximal deviation about it does not exceed the value of 2.4%.

Eq (2.7) can be rewritten as

\[
\sigma_{cr} = \left( 0.00181 + \frac{0.0218}{\lambda} \right) E \tag{2.10}
\]

Assuming that the critical compressive stresses are proportional to the yield point, the foregoing formula can be generalized to the form valid for the different duraluminium materials:

\[
\sigma_{cr} = \left( 0.00181 + \frac{0.0218}{\lambda} \right) \frac{1.2827 \cdot R_{0.2}}{245} E \tag{2.11}
\]

\[
\sigma_{cr} = \left( 9.48 + \frac{114}{\lambda} \right) \cdot 10^{-6} E R_{0.2} \tag{2.12}
\]

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Naprężenia krytyczne warstwowych cylindrów w stanach niesprężystych.
(Badania doświadczalne)

Streszczenie


Przedstawiono stosunkowo prostą funkcję, która bardzo dobrze aproksymuje wyniki badań.

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