MECHANIKA TEORETYCZNA
I STOSOWANA
Journal of Theoretical
and Applied Mechanics
3, 30, 1992

ON THE INFLUENCE OF DAMPING ON THE CRITICAL SPEED OF SPRING-MASS SYSTEM MOVING ALONG A TIMOSHENKO BEAM ON AN ELASTIC FOUNDATION

ROMAN BOGACZ

Institute of Fundamental Technological Research, Warsaw
and Cracow University of Technology

SLAWOMIR NOWAKOWSKI

Institute of Fundamental Technological Research, Warsaw

The paper is devoted to an analysis of behaviour of the densly distributed chain of spring-mass system moving with constant speed along on infinite Timoshenko beam on a visco-elastic foundation. The critical velocities of relative motion were determined on the basis of the kinetic stability criteria. It was proved that the instability regions are esentialy influenced also by a very small intensity of energy dissipation.

1. Introduction

The problem od self-excited vibrations of continuous systems under moving inertial loads or self-excitation of systems with travelling waves has been the object of intensive research in many fields of physics and engineering e.g. flutter phenomena in aeroelastic structures, vibrations of pipes containing fluid or the wheel-rail behaviour of high-speed trains (cf [1]).

The majority of previous studies devoted to the analysis of critical speeds and stability regions were restricted to the determination of parameter resonances in ordinary differential equations. In many cases the solutions were approximated by standing waves, which can lead to significant differences in the estimation of critical velocities (cf e.g. [2]). Bogacz et al. [3] gives an alternative formulation of the problem which yields a solution in the form of travelling waves. Considerations of the similar but elastic problem, presented Bogacz et al. [4] are interesting from the theoretical point of view only. For each physicaly real dynamic system an energy dissipation occurs. As follows from futher consideration that the stability regions

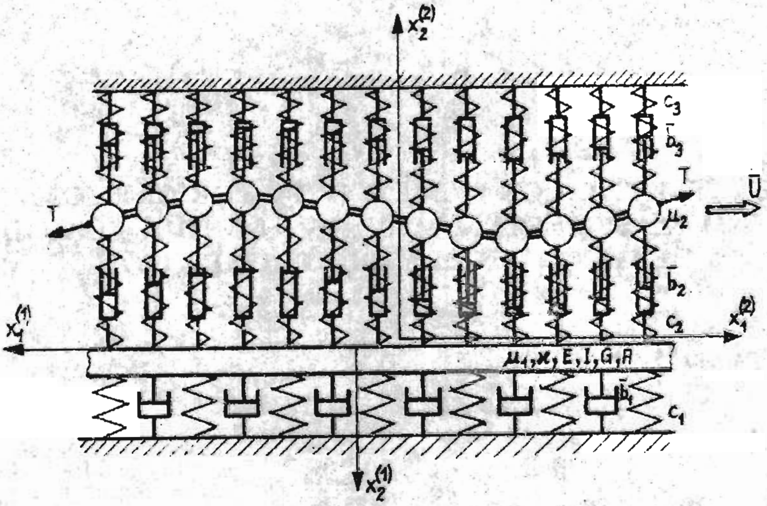


Fig. 1.

for systems with travelling waves are particularly sensitive on viscous damping. Let us consider the stability of the system shown in Fig.1., consisting of a chain of oscillators moving along a Timoshenko beam on a viscoelastic foundation. The effect of self-excitation follows from the interaction between the vibrations of the moving chain of oscillators and the travelling waves generated in the beam. This simple mechanical model besides its own practical significance can also give an insight into more complex and qualitatively new effects appearing in aeroelasticity and other fields of physics and engineering.

The aim of the present paper is to determine the critical parameters and the stability regions of the steady-state motion of the system (cf Fig.1), as a function of the system parameters, especially the speed of the oscillator chain. The stability problem will be studied under the following assumptions

- the beam as well as the chain of oscillators are of infinite length
- the speed of the oscillator chain relative to the beam is constant
- the trajectories of the undisturbed oscillators motion are straight
- the friction between oscillators and beam is neglected
- the oscillator chain consists of densly and uniformly distributed discrete one-degree-of-freedom spring-mass system
- the interaction between the beem oscillators and the surrounding is assumed to be viscoelastic

- the gravitational effects are neglected
- the mass of viscoelastic elements is neglegted

2. Equations of motion

The equations of the beam motion regarding the effects of shear destortion and rotatory inertia are given as follows

$$
\begin{gathered}
EI\frac{\partial^2\psi}{\partial\bar{x}_1^{(1)2}} + \kappa AG\Big(\frac{\partial w_1}{\partial\bar{x}_1^{(1)}} - \psi\Big) - \mu_1 I\frac{\partial^2\psi}{\partial t^2} = 0 \\
\kappa AG\Big(\frac{\partial^2 w_1}{\partial\bar{x}_1^{(1)2}} - \frac{\partial\psi}{\partial\bar{x}_1^{(1)}}\Big) - \mu_1 A\frac{\partial^2 w_1}{\partial t^2} - \bar{b}_1\frac{\partial w_1}{\partial t} - c_1 w_1 = -p_1(\bar{x}_1^{(1)},t)
\end{gathered}
\tag{2.1}
$$

where

ψ	–	rotation angle of the cross section
w_1	–	beam displacement in $\bar{x}_2^{(1)}$ – direction
A	–	area of the cross section
E	–	Young modulus
G	–	shear modulus
I	–	moment of inertia of the cross–section area
μ_1	–	mass per unit length of the beam
κ	–	coefficient describing the effective shear area
b_1	–	foundation coefficient of viscosity per unit length
c_1	–	foundation spring coefficient per unit length
p_1	–	distributed load per unit length in $\bar{x}_2^{(1)}$ – direction

Assuming that the oscillator chain is moving in $\bar{x}_2^{(1)}$ – direction, we are looking for a steady–state solution in form of the travelling waves. At first, let us consider the case, in which the pressure acting on the beam is given by

$$
p_1(x_1^{(1)},t) = pe^{ik_1(\bar{x}_1^{(1)}-\bar{v}_1 t)} \tag{2.2}
$$

Then, the displacement of the beam takes a similar form

$$
w_1(x_1^{(1)},t) = w_{10}e^{ik_1(\bar{x}_1^{(1)}-\bar{v}_1 t)} \tag{2.3}
$$

where

k_1 – wave number
$\bar{v}_1$ – wave velocity in $\bar{x}_1^{(1)}$ – direction
p, w_{10} – amplitudes of pressure and beam displacement, respectively

We introduce now a moving system of dimensionless coordinates (x_1^*, x_2^*) related to the fixed system $(\bar{x}_1^{(1)}, \bar{x}_2^{(1)})$ by

$$x_1^* = \frac{\bar{x}_1^{(1)} - \bar{v}_1 t}{i_0} \qquad x_2^* = \frac{\bar{x}_2^{(1)}}{i_0} \tag{2.4}$$

where $i_0 = \sqrt{I/A}$ is the radius of gyration of the cross section. Then, the system of Eqs (2.1) takes the following dimensionless form in the coordinates (x_1^*, x_2^*) for steady-state

$$\begin{aligned} &(V_E^2 - v_1^2)\psi^{\rm II} + V_G^2(W_1^{\rm I} - \psi) = 0 \\ &(V_G^2 - v_1^2)W_1^{\rm II} - 2v_1 b_1 W_1^{\rm I} - W_1 - V_G^2\psi^{\rm I} + P{\rm e}^{ikx_1^*} = 0 \end{aligned} \tag{2.5}$$

where $(\ )^{\rm I}$ denotes the derivatives with respect to x_1^*, some abbreviations have also been introduced

$$\begin{aligned} &k = i_0 k_1 && W_1 = \frac{w_1}{i_0} && v_1 = \frac{\bar{v}_1}{v^*} \\ &V_G = \frac{V_G^*}{v^*} && V_E = \frac{V_E^*}{v^*} && b_1 = \frac{\bar{b}_1}{2\sqrt{A\mu_1 c_1}} \\ &P = p\sqrt{\frac{A}{Ic_1^2}} && V_G^* = \sqrt{\frac{\kappa G}{\mu_1}} && V_E^* = \sqrt{\frac{E}{\mu_1}} \\ &v^* = i_0\sqrt{\frac{c_1}{\mu_1 A}} \end{aligned} \tag{2.6}$$

It can easily be seen, that the system of Eqs (2.5) is equivalent to one of the following 4th-order equations

$$\begin{aligned} &(v_1^2 - V_G^2)(v_1^2 - V_E^2)\psi^{\rm IV} + 2v_1 b_1(v_1^2 - V_E^2)\psi^{\rm III} + \Big[v_1^2(V_G^2 + 1) - V_E^2\Big]\psi^{\rm II} + \\ &+2v_1 b_1 V_G \psi^{\rm I} + V_G^2\psi = PV_G^2 {\rm i}k{\rm e}^{ikx_1^*} \end{aligned} \tag{2.7}$$

$$\begin{aligned} &(v_1^2 - V_G^2)(v_1^2 - V_E^2)W_1^{\rm IV} + 2v_1 b_1(v_1^2 - V_E^2)W_1^{\rm III} + \Big[v_1^2(V_G^2 + 1) - V_E^2\Big]W_1^{\rm II} + \\ &+2v_1 b_1 V_G W_1^{\rm I} + V_G^2 W_1 = P\Big[V_G^2 - (v_1^2 - V_E^2)k^2\Big]{\rm e}^{ikx_1^*} \end{aligned} \tag{2.8}$$

In the next part of the study we are looking for both the relation between the beam displacement and the pressure acting on the beam and the similar relation for the oscillator chain. Limiting our consideration to small displacements only, we assume that the equation of motion of the chain of oscillators is linear

$$\mu_2\frac{\partial^2 w_2}{\partial t^2} - T\frac{\partial^2 w_2}{\partial \bar{x}^2} + \bar{b}_3\frac{\partial w_2}{\partial t} + \bar{b}_2\Big(\frac{\partial w_2}{\partial t} - \frac{\partial w_0}{\partial t}\Big) + c_3 w_2 + c_2(w_2 - w_0) = 0 \qquad (2.9)$$

where

w_2	–	oscillator displacement in $\bar{x}_2^{(2)}$ – direction
w_0	–	displacement of the contact points between the oscillator springs and the beam
μ_2	–	mass per unit length of the oscillator chain
$\bar{b}_2$, $\bar{b}_3$	–	relative and absolute viscosity damping coefficients per unit length, respectively
c_2, c_3	–	spring constants per unit length of the oscillator chain
T	–	tension of the oscillator chain

Describing the displacements of the oscillators in terms of dimensionless coordinates $(x_1^{(2)}, x_2^{(2)})$, where $x_\gamma^{(2)} = \bar{x}_\gamma^{(2)}/i_0$, $\gamma = 1,2$, it follows

$$\begin{aligned} w_0 &= i_0 W_0 e^{-ik(x_1^{(2)} - v_2 t)} \qquad && W_0 = \text{const.} \\ w_2 &= i_0 W_2 e^{-ik(x_1^{(2)} - v_2 t)} \qquad && W_2 = \text{const.} \end{aligned} \qquad (2.10)$$

Utilizing Eqs (2.6), the pressure acting on the oscillators takes the form,

$$b_2\Big(\frac{\partial w_2}{\partial t} - \frac{\partial w_0}{\partial t}\Big) + c_2(w_2 - w_0) = -P i_0 c_1 e^{-ik(x_1^{(2)} - v_2 t)} \qquad (2.11)$$

The system of Eqs (2.1) and (2.9) together with conditions (2.10) and (2.11) completely describe our problem. The next part of the paper will be devoted to the solution of the problem.

3. Solution of the problem

In the coordinate system (x_1^*, x_2^*) moving with the travelling wave, the beam displacement (2.3) takes the form

$$W_1 = W_{10} e^{ikx_1^*} \qquad W_{10} = \text{const.} \qquad (3.1)$$

Substituting Eqs (3.1) into Eqs (2.8) we obtain the relation

$$\begin{aligned} &(v_1^2 - V_G^2)(v_1^2 - V_E^2)k^4 + 2ik^3 v_1 b_1 (v_1^2 - V_E^2)W_{10} + \\ &- \Big[v_1^2(V_G^2 + 1) - V_E^2\Big]k^2 + V_G = \frac{P}{W_{10}}\Big[V_G^2 - (v_1^2 - V_E^2)k^2\Big] \end{aligned} \qquad (3.2)$$

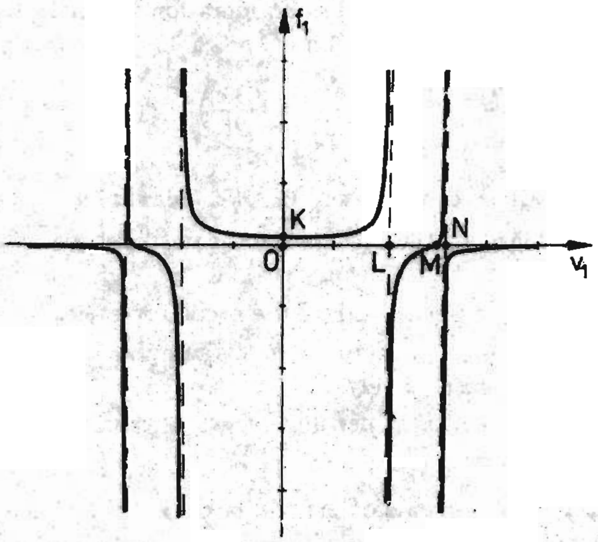


Fig. 2.

Hence, the ratio between the amplitude of beam displacement and pressure can be given as a function of the wave velocity

$$f_1(v_1) = \frac{V_G^2 - (v_1^2 - V_E^2)k^2}{k^4C_1 - k^2C_2 + V_E^2 + 2ikb_1v_1\left[k^2(V_1^2 - V_E^2) - V_G^2\right]} \tag{3.3}$$

where

$$C_1 = (v_1^2 - V_G^2)(v_1^2 - V_E^2)$$

$$C_2 = v_1^2(V_G^2 + 1) - V_E^2$$

which for the case $b_1 = 0$ is shown in Fig.2. Some characteristic values of the wave velocities are given by the formulae

$$v_L \equiv \overline{0L} = \sqrt{\frac{1}{2}\Big(\beta - \sqrt{\beta^2 - 4\gamma}\,\Big)}$$

$$v_M \equiv \overline{0M} = k^{-1}\sqrt{V_G^2 + k^2V_E^2}$$

$$v_N \equiv \overline{0N} = \sqrt{\frac{1}{2}\Big(\beta - \sqrt{\beta^2 - 4\gamma}\,\Big)}$$

where

$$\beta = V_G^2 + V_E^2 + k^{-2}(V_G^2 + 1)$$

$$\gamma = V_G^2 V_E^2 + k^{-2} V_E^2 + k^{-4} V_G^2$$

The value of the function $f_1(v_1)$ for $v_1 = 0$ reads

$$v_K \equiv \overline{0K} = \frac{V_G^2 + k^2 V_E^2}{k^2 V_G^2 V_E^2 + k^2 V_E^2 + V_G^2}$$

Let us now consider the chain of oscillators assuming that the pressure between the beam and the oscillators is described relative to the coordinate system $(\bar{x}_1^{(2)}, \bar{x}_2^{(2)})$

$$p_2(\bar{x}_1^{(2)}, t) = p e^{ik_2(\bar{x}_1^{(2)} - \bar{v}_2 t)} \tag{3.4}$$

The relations between the coordinates $(\bar{x}_1^{(1)}, \bar{x}_2^{(1)})$ and $(\bar{x}_1^{(2)}, \bar{x}_2^{(2)})$ are

$$\bar{x}_1^{(1)} + \bar{x}_1^{(2)} = \bar{U}t \qquad \bar{x}_2^{(1)} + \bar{x}_2^{(2)} = 0 \tag{3.5}$$

From the boundary conditions describing the conformity of displacements and pressure we have

$$w_1 = -w_0 \qquad p_1 = p_2 \qquad k_2 = -k \tag{3.6}$$

$$\bar{v}_1 - \bar{v}_2 = \bar{U} \tag{3.7}$$

where w_0 is the desplacement of the contact points between the oscillator springs and the beam in $\bar{x}_2^{(2)}$ – direction.

Substituting Eqs (2.10) into Eq (2.9) and using Eqs (2.11), we obtain the relation between the amplitudes of oscillator displacements and the pressure similar to Eq (3.3)

$$f_2(v_2) = \frac{W_{10}}{P} = \xi \frac{\alpha_2^2 + \alpha_3^2 - v_2^2 - i(b_2 + b_3)v_2}{v_2^2(\alpha_2^2 + b_2 b_3) + \alpha_2^2\alpha_3^2 + i[b_2(v_2^2 - \alpha_3^2) - \alpha_2^2 b_3]v_2} \tag{3.8}$$

where

$$\alpha_2^2 = \xi \frac{c_2}{c_1 v^{*2}} \qquad \alpha_3^2 = \frac{c_3}{\mu_2 k^2}$$
$$b_i = \frac{\bar{b}_i}{\mu_2 k^2 v^*} \qquad (i = 2, 3) \tag{3.9}$$
$$\xi = \frac{\mu_1 A}{\mu_2 k^2} \qquad v_2^2 = \frac{\bar{v}_2^2}{V^{*2}} \qquad v^{*2} = i_0^2 \frac{c_1}{\mu_1 A}$$

From the conformity conditions (3.6) we obtain

$$f_1(v_1) - f_2(v_2) = \Phi(v_1, v_2) = 0 \tag{3.10}$$

Hence, the Eq (3.10) of complex form can be rewritten as the equivalent system of equations with real coefficients which together with Eqs (3.7) constitutes the characteristic equation of our problem. Eqs (3.10) and (3.7) take then the following form

$$\begin{aligned}&\Big[\xi e(v_1) + a(v_1)(\alpha_2^2 + b_2 b_3)\Big] v_2^2 - \xi d(v_1)(b_2 + b_3) v_2 + \\ &+ \xi e(v_1)(\alpha_2^2 + \alpha_3^2) - a(v_1)\alpha_2^2\alpha_3^2 = 0\end{aligned} \tag{3.11}$$

$$\begin{aligned}&a(v_1) b_2 v_2^3 - \xi d(v_1) v_2^2 - \Big[a(v_1)\alpha_2^2 b_3 - \xi e(v_1)(b_2 + b_3) + \\ &+ a(v_1) b_2 \alpha_3^2\Big] v_2 - d(v_1)\xi(\alpha_2^2 + \alpha_3^2) = 0\end{aligned} \tag{3.12}$$

$$v_1 - v_2 = U \qquad\qquad U = \frac{\bar{U}}{v^*} \tag{3.13}$$

where

$$\begin{aligned}a(v_1) &= V_G^2 - k^2(v_1^2 - V_E^2) \\ e(v_1) &= k^2(v_1^2 - V_G^2)(v_1^2 - V_E^2) - k^2\Big[v_1^2(V_G^2 + 1) - V_E^2\Big] + V_G^2 \\ d(v_1) &= 2kb_1 v_1 \Big[V_G^2 - k^2(v_1^2 - V_E^2)\Big]\end{aligned} \tag{3.14}$$

Eqs (3.11) in the elastic case are illustrated in Fig.3 in the v_1, v_2 – plane of wave velocities. Some characteristic values of the wave velocities are given by the formulae

$$\begin{aligned}v_A &\equiv 0A = \sqrt{\frac{\alpha_2^2\gamma}{\gamma^*}} & v_D &\equiv 0D = \sqrt{\frac{1}{2}\Big(\beta - \sqrt{\beta^2 - 4\gamma}\,\Big)} \\ v_B &\equiv 0B = \sqrt{\frac{1}{2}\Big(\beta - \sqrt{\beta^2 - 4\gamma}\,\Big)} & v_F &\equiv 0F = \alpha_2 \\ v_C &\equiv 0C = \sqrt{\frac{1}{2}\Big(\beta^* - \sqrt{\beta^{*2} - 4\gamma}\,\Big)} & v_E &\equiv 0E = \sqrt{\frac{1}{2}\Big(\beta^* - \sqrt{\beta^2 - 4\gamma}\,\Big)}\end{aligned} \tag{3.15}$$

where

$$\begin{aligned}\beta &= V_G^2 + V_E^2 + k^{-2}(V_G^2 + 1) & \beta^* &= V_G^2 + V_E^2 + k^{-2}(V_G^2 + \eta + 1) \\ \eta &= \frac{c_1}{c_2} & \gamma &= V_G^2 V_E^2 + k^{-2} V_E^2 + k^{-4} V_G^2 \\ \gamma^* &= V_G^2 V_E^2 + (\eta + 1)k^{-2} V_E^2 + (\eta + 1)k^{-4} V_G^2\end{aligned}$$

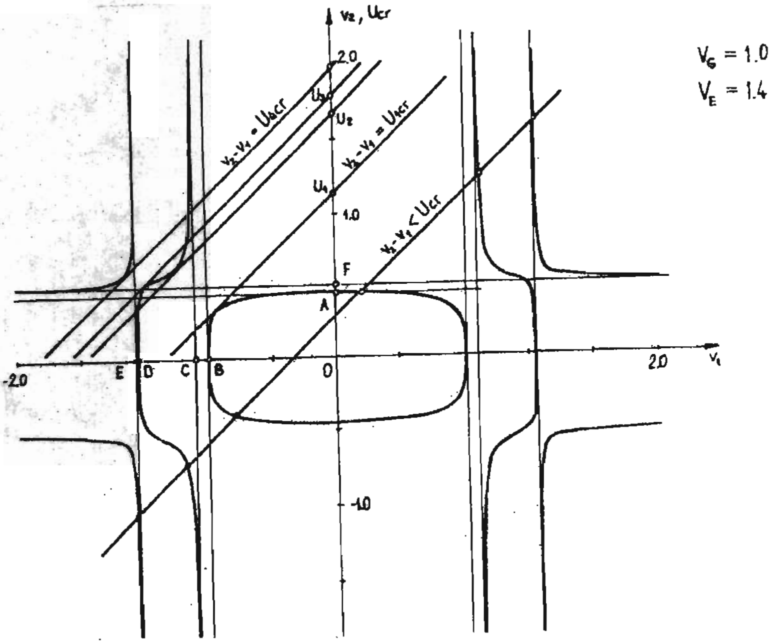

v_2, U_{cr}
2.0
1.0
-1.0
-2.0
2.0
v_1
$V_G = 1.0$
$V_E = 1.4$
$v_2 - v_1 = U_{3cr}$
$v_2 - v_1 = U_{2cr}$
$v_2 - v_1 < U_{cr}$
U_3
U_2
U_1
F
A
E
D
C
B
O

Fig. 3.

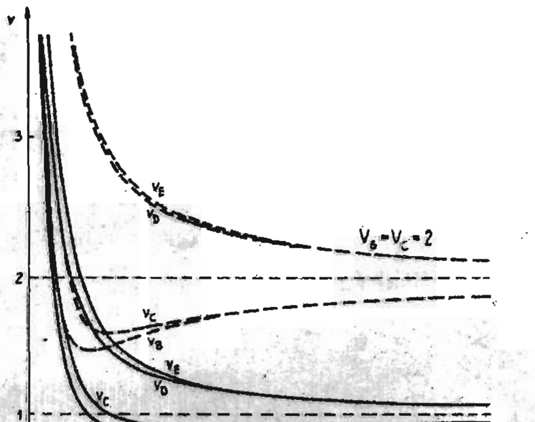

v
3
2
1
0
2
4
6
8
k
V_E
V_D
$V_G = V_C = 2$
V_C
V_B
V_E
V_D
V_C
V_B
$V_G = V_E = 1$

Fig. 4.

The dependence of the characteristic velocities v_B, v_C, v_D and v_E on the wave number k is shown in Fig.4.

Now let us consider the polynomial

$$M(k^2) = (v_1^2 - V_G^2)(v_1^2 - V_E^2)k^4 - \Big[v_1^2(V_G^2+1) - V_E^2\Big]k^2 + V_G \tag{3.16}$$

which is the denominator of the function $f_1(v_1)$. From $M(k^2) = 0$ we find the the roots k_1^2, k_2^2

$$\begin{aligned} k_1^2(v_1^2) &= \frac{1}{2(v_1^2 - V_G^2)(v_1^2 - V_E^2)}\Big[v_1^2(V_G^2+1) - V_E^2 - d_1(v_1^2)\Big] \\ k_2^2(v_1^2) &= \frac{1}{2(v_1^2 - V_G^2)(v_1^2 - V_E^2)}\Big[v_1^2(V_G^2+1) - V_E^2 + d_1(v_1^2)\Big] \end{aligned} \tag{3.17}$$

Here

$$d_1(v_1^2) = \sqrt{\Delta(v_1^2)} = \mathrm{i}\sqrt{-\Delta(v_1^2)}$$

$$\Delta(v_1^2) = (V_G^2 - 1)v_1^4 + 2\Big[2V_G^4 + V_E^2(V_G^2-1)\Big]v_1^2 + V_E^2(V_E^2 - 4V_G^2)$$

The polynomial $\Delta(v_1^2)$ has two roots, $v_{10}^2 \equiv v_0^2$ but for physical reasons we have to take only the nonnegative one $(v_0^2 \geq 0)$

$$v_0^2 = \begin{cases} \dfrac{0.5\sqrt{\Delta_1} - 2V_G^4 - V_E^2(V_G^2-1)}{(V_G^2-1)^2} & \text{if} \quad V_G \neq 1 \\[2ex] V_E^2(1 - 0.25V_E^2) & \text{if} \quad V_G = 1 \end{cases} \tag{3.18}$$

where

$$\Delta_1 = 16V_G^6\Big[V_G^2(V_E^2+1) - V_E^2\Big]$$

The velocity v_0 is the critical velocity of the travelling waves. It can be proved, that

$$\begin{aligned} k_j^2(v_0^2) &> 0 \qquad \text{if} \quad V_E^2 < V_G^2(V_G^2+1) \\ k_j^2(v_0^2) &< 0 \qquad \text{if} \quad V_E^2 > V_G^2(V_G^2+1) \end{aligned} \qquad j = 1,2 \tag{3.19}$$

but $v_0 = V_G$ if $V_E^2 = V_G^2(V_G^2+1)$.

We see from Fig.4 that the function $v_B = v_B(k)$ reaches a minimum only if $V_E^2 < V_G^2(V_G^2+1)$. The minimal value, $\min\limits_k v_B(k) = v_B(k_0) \equiv v_0$, is the first critical wave speed. The corresponding critical wave number k_0 is given by

$$k_0^2(v_0^2) = \frac{V_0^2(V_G^2+1) - V_E^2}{2(v_0^2 - V_G^2)(v_0^2 - V_E^2)} \tag{3.20}$$

It can be shown that in the case of $V_E^2 > V_G^2(V_G^2+1)$ (cf (3.19)), $v_B = v_B(k)$ is the monotonic decreases function from $v_B \to \infty$ for $k \to 0$ to $v_B \to V_G$ for $k \to \infty$, i.e. the behavior of $v_B = v_B(k)$ is similar to that of $v_D(k)$ and $v_E(k)$ shown in Fig.4 and, thus, k_0 tends to infinity. However, in the case of $V_E^2 < V_G^2(V_G^2+1)$, cf Fig.4, there exist finite minimum values v_0, where $v_0 < \min(V_E, V_G)$. It is interesting to note that depending on the system parameters the range of stability (U_2, U_3) changes and sometimes even vanishes [4]. From this consideration it follows that the critical speed v_0 of travelling waves in the system under consideration, neglecting viscosity, can be smaller than the minimum possible velocity of wave propagation in the elastic subsystem (which follows for $\alpha_2^2 \to 0$).

As can be seen from a comparison of the characteristic curves given in Fig.3 with the corresponding ones given by Bogacz et al. [5] for the limit case of a Bernoulli–Euler beam, the effects of shear deformation and rotary inertia are of the great influence on the critical velocities of travelling waves. They can change qualitatively the configuration of the stability regions. In general, the existence of four critical velocities, which constitute two ranges of unstable motion is possible (cf Fig.3). For some specific system parameters these regions can merge to one large instability region as shown in [4].

4. Stability analysis for the case with damping

In the present case, taking damping into regard the solution (2.3) may be unstable, stable or asymptotically stable, depending on the value of the velocity U. Stability of the steady-state solution (3.1) requires $\mathrm{Im}(kv_\nu) \le 0$ for $\nu = 1,2$ and in the case of instability $\mathrm{Im}(kv)_\nu > 0$, $\nu = 1$ or $\nu = 2$. Now we will analyze the stability behavior depending on the velocity U. First let us observe that the solution is stable if and only if there exist six complex roots $v_\nu^{(n)}$, $\nu = 1,2$, $n = 1,2,3,4,5,6$, of Eq (3.10) such that

$$m(kv_\nu^{(n)}) \le 0 \tag{4.1}$$

The plot of the Mikhaylov curve for this case is shown in Fig.5. If the inequality sign holds the asymptotic stability follows while in case where the equality sign holds the stability follows when the real roots are different.

The regions S_I of U for which

$$S_I = \Big\{U : v_\nu,\ \mathrm{Im}(v_\nu) > 0,\ \Phi(v_\nu, U) = 0\Big\} \tag{4.2}$$

will be called instability regions. Since the analytical determination of the critical parameters is complicated, let us apply a geometrical approach. The critical values

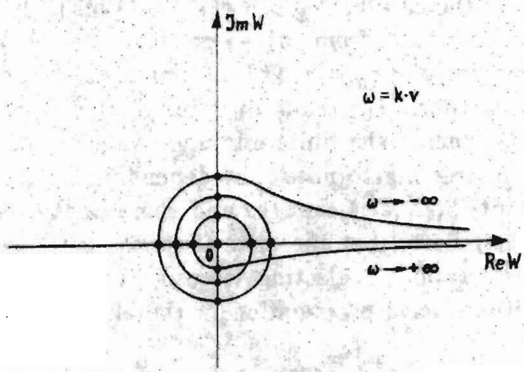


Fig. 5.

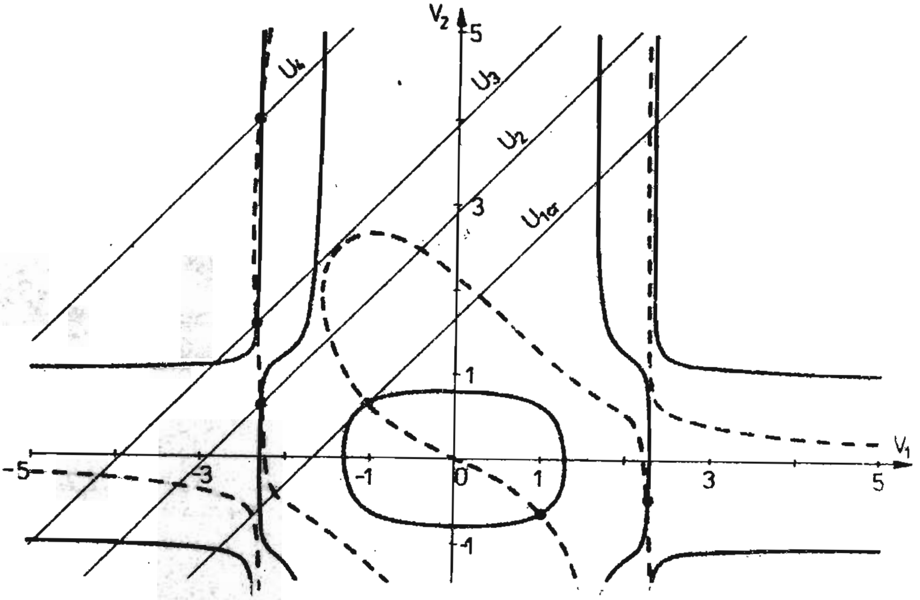
of U at the boundaries of the instability region S_I are determined in the (v_1, v_2) – plane by the straight lines $v_2 = v_1 - U$ passing through the intersection points of the curve $\mathrm{Re}\Phi = 0$ and $\mathrm{Im}\Phi = 0$ Eq (3.10).

In the region $S = \Big\{U : U \in [U_{1cr}, U_{2cr}]$ or $[U_{3cr}, U_{4cr}]\Big\}$, the solution (2.10) describes waves with amplitudes increasing in time. Beside this solution there exists also a trivial solution; thus, according to Lapunov's instability criterion, region S is the region of instability, $S = S_I$.

Now let us determine the instability regions for certain particular cases. For the numerical analysis following parameters of the system were taken as the constant

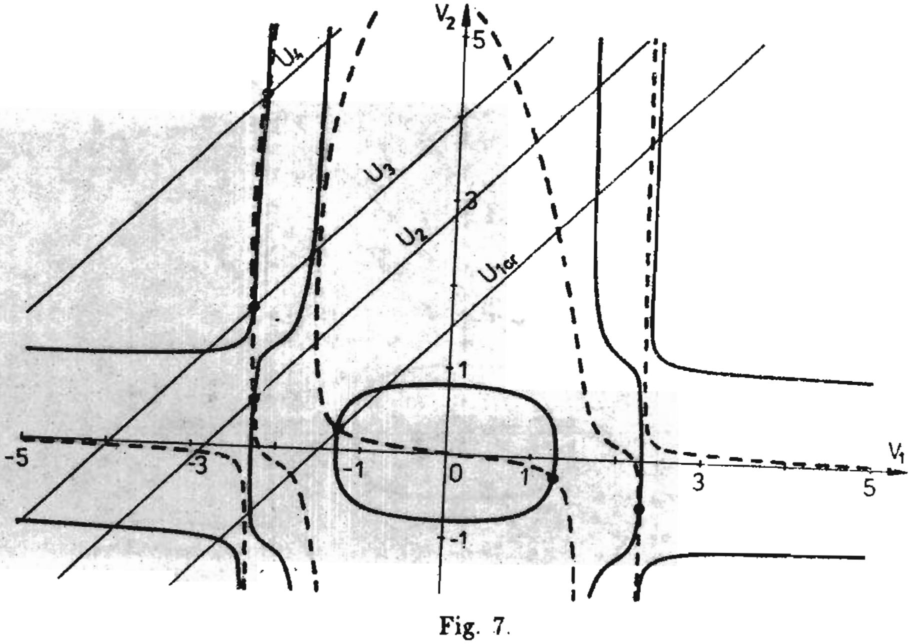
$$V_G = 1 \qquad V_E = 2 \qquad k = 1 \qquad \kappa = 1$$
$$\alpha_2^2 = 1 \qquad \alpha_3^2 = 0 \qquad \xi = 1$$

The values of dimensionless damping coefficients b_1, b_2 and b_3 were changed, and results are illustrated on graphs in Fig.6 ÷ 11. On the basis of the characteristic equations derived it can be found that the critical velocities depend on the products of the damping coefficient and their ratios. The characteristic results are illustrated in the (v_1, v_2) – plane. The curves representing the real part of the characteristic equations equations $\mathrm{Re}\Phi(v_1, v_2) = 0$ depend on the product $b_i b_j$ $(i, j = 1, 2, 3)$ and for the case $b_1(b_2 + b_3) \to 0$ they are identical with the elastic case. The imaginary part of the characteristic equations depends both on the products $b_i b_j$ and the ratio $b_i b_j^{-1}$. The curves representing the imaginary part $\mathrm{Im}\Phi(v_1, v_2) = 0$ for $b_i b_j^{-1} = 1$ are shown in Fig.6, and for the cases $(b_1 + b_2)b_3^{-1} \ll 1$ and $(b_2 + b_3)b_1^{-1} \to 0$ they are shown in Fig.7 and Fig.8, respectively. It is easy to see that in the case of $b_i b_j \to 0$, the critical value of the motion velocity is not greater than that in the elastic case, and sometimes the difference is very pronounced (Fig.8). It can be easily seen that increase of the value of the beam damping b_1



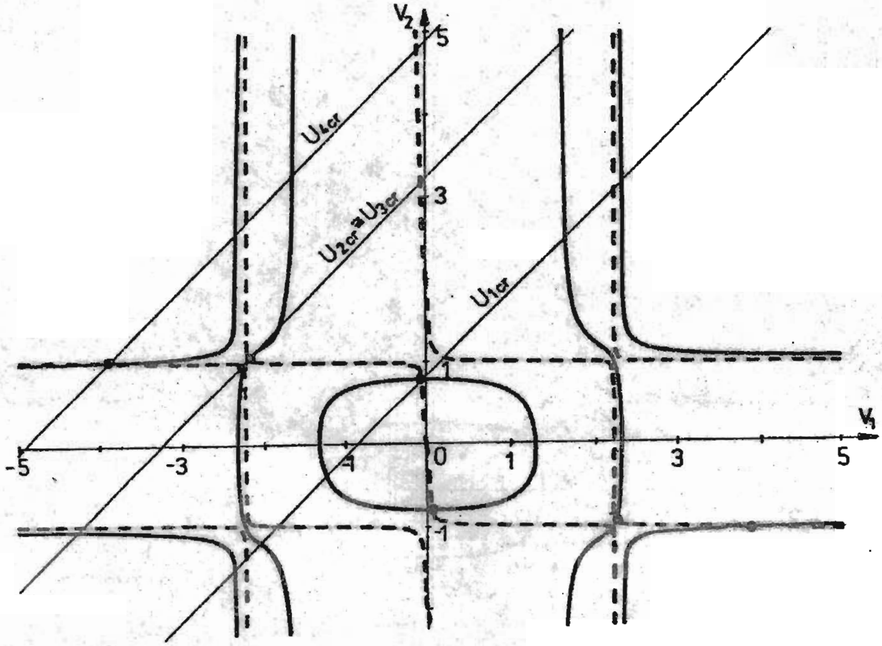
V_2
5
3
1
-1
V_1
-5
-3
-1
0
1
3
5
U_4
U_3
U_2
U_{1cr}

Fig. 6.



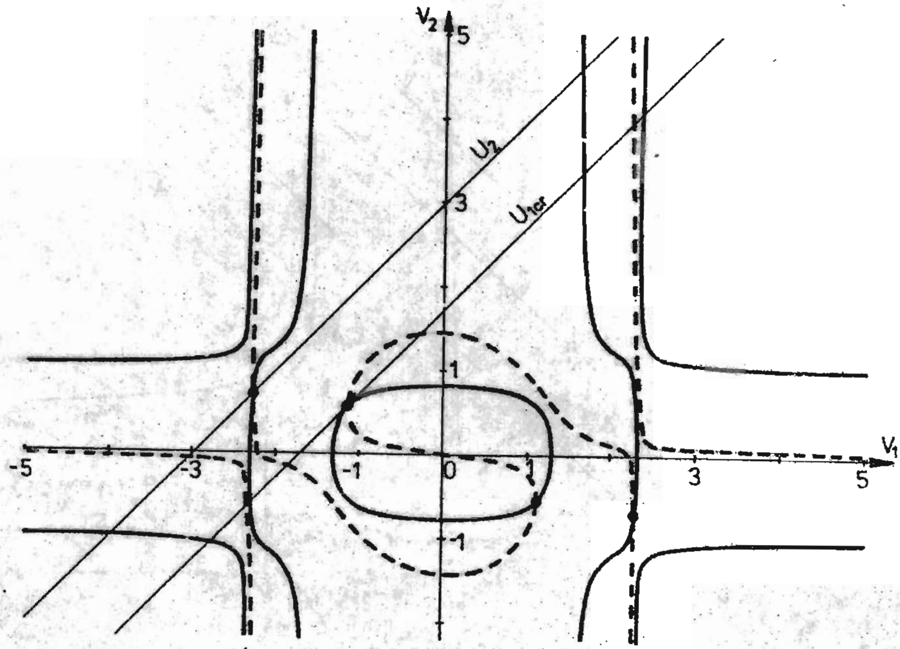
V_2
5
3
1
-1
V_1
-5
-3
-1
0
1
3
5
U_4
U_3
U_2
U_{1cr}

Fig. 7.



V2
5
3
1
-1
V1
-5
-3
-1
0
1
3
5
U4cr
U2cr=U3cr
U1cr

Fig. 8.



V2
5
3
1
-1
V1
-5
-3
-1
0
1
3
5
U2
U1cr

Fig. 9.

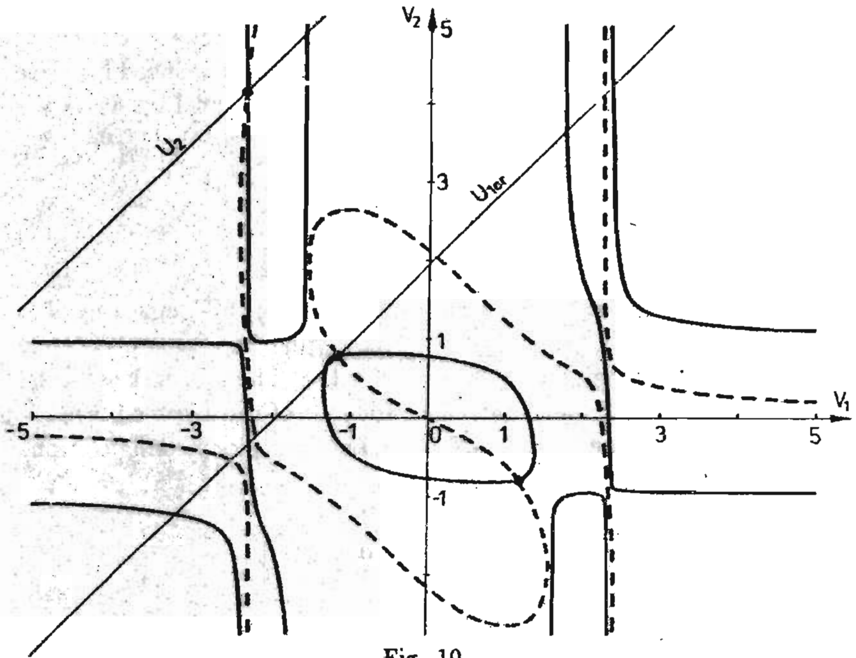


Fig. 10.

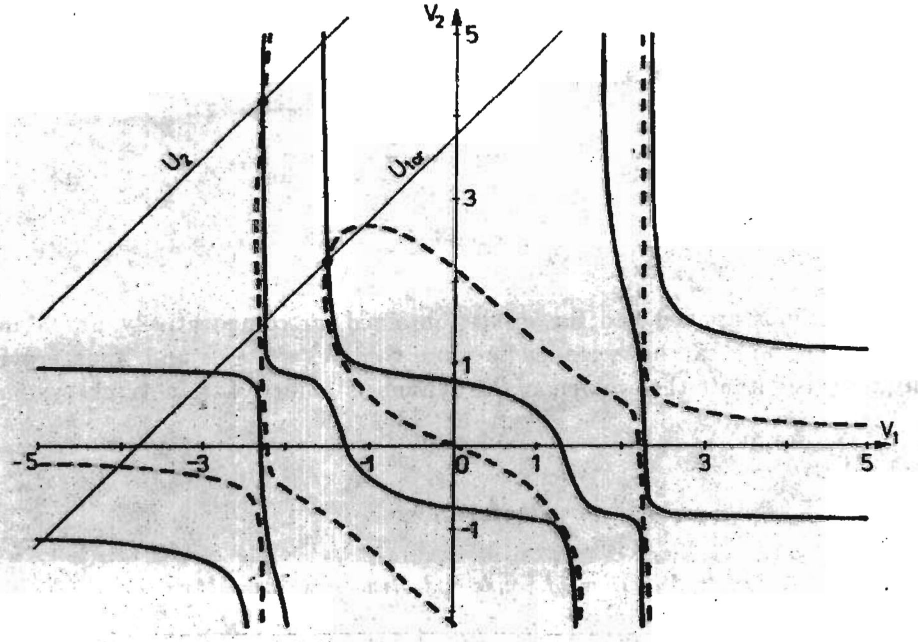


Fig. 11.

yield reduction of the critical velocity and destabilize the system. For the case of $b_1 = b_3 = 0.01$ and $b_2 = 0.1$ which is shown in Fig.9, the value of critical velocity of motion is similar as in the elastic case. The value of critical speed increase when the coefficients of damping are relatively large (cf Fig.10 and Fig.11).

5. Conclusions

As can be seen from the comperison of results given by Bogacz et al. [4] the regions of instability for the damping coefficent equal to zero are different from those in the case of damping tending to zero (Fig.12). The comparison with results given by Bogacz et al. [5] shows significant influence of the shear deformation and rotary inertia on the qualitative change of instability regions configuration.

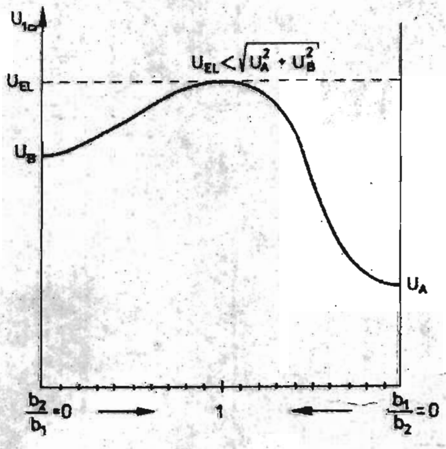


Fig. 12.

The methods applied and the results obtained for comparatively simple models with damping can be extended to more complex systems and, thus provide additional insight into the problem of the dynamic stability of train–track–systems.

References

1. FRYBA L., 1972, *Vibration of Solids and Structures under Moving Loads*, Academia, Prague

2. BENEDETTI G.A., 1971, *Dynamic instability of beam carrying moving masses*, J.Appl.Mech., 4, **38**

3. BOGACZ R., KRZYŻYŃSKI T., POPP K., 1986, *On wave solutions for beams on a viscoelastic foundation subjected to a travelling and oscillating force*, in: Refined Dynamical Theories of Beams, Plates and Shalls and their Applications, Proc.Euromech Colloquium 219, Kassel

4. BOGACZ R., NOWAKOWSKI S., POPP K., 1986, *On the stability of Timoshenko beam on an elastic foundation under a moving spring–mass system*, Acta Mechanica No. 58

5. BOGACZ R., DAMIELSKI J., POPP K., 1985, *On the influence of viscosity on stability of train–track–systems*, Arch.Mech., **37**, 6

O wpływie tłumienia na prędkość krytyczną układu usprężynowanych mas poruszającego się wzdłuż belki Timoszenki na sprężystym podłożu

Streszczenie

Pracę poświęcono analizie zachowania łańcucha gęsto rozłożonych oscylatorów poruszających się ze stałą prędkością wzdłuż nieograniczonej belki Timoszenki na lepkosprężystym podłożu. Określono prędkości krytyczne względnego ruchu stosując kinetyczne kryterium stabilności. Wykazano, że ukształtowanie obszarów niestabilności jest w istotny sposób zależne od energii dysypowanej układu nawet przy niewielkiej intensywności tłumienia.

Manuscript received February 9, 1992; accepted for print February 21, 1992