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# ON THE EFFECTIVE MODULAE IN MICROLOCAL MODELS OF ELASTIC PERIODIC COMPOSITES

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## Introduction

In the microlocal mechanics of periodic composites the constitutive relations are assumed to include an additional term being the linear function of microlocal parameters related to the strain tensor by some algebraic equations. The idea was first advanced in [2] by an application of nonstandard analysis [1, 8] and then explored in [3, 4, 5, 6, 7]. The aim of the paper is to obtain effective modulae for a few special cases of the linear elastic composites with periodic structures including sheet reinforced and fiber reinforced composites. Results of the paper hold for composites with periodic structures consisting of two different linear elastic materials. The first material is interpreted as a reinforcement material being more rigid than the second material which is treated as a matrix. At the same time the area of the cross section across the matrix is much bigger than that across the reinforcement. We start the considerations from the formulation of the fundamental system of equations of the microlocal mechanic for periodic structures.

## List of symbols

Throughout the paper the symbol $\mathbf{A} \otimes \mathbf{B}$ stands for the tensor product of tensors $\mathbf{A}$ and $\mathbf{B}$. The double contraction of any two tensors $\mathbf{A}$, $\mathbf{B}$ is denoted by $\mathbf{AB}$ provided that $\mathbf{A}$ and $\mathbf{B}$ are the fourth-order tensors (otherwise the symbol $\mathbf{AB}$ denotes the simple product of $\mathbf{A}$ and $\mathbf{B}$) and by $\mathbf{A}[\mathbf{B}]$ if $\mathbf{A}$ is the fourth-order tensor and the $\mathbf{B}$ is of the second-order. For any function $R_+ \ni \varepsilon \to f(\varepsilon) \in R^m$, $m$ being the positive integer, and for an arbitrary real number $\gamma \in R_+$ the symbol $f(\varepsilon) \in o\,(\varepsilon^\gamma)$ stands for the condition that the function $\varepsilon \to \varepsilon^{-\gamma} f(\varepsilon)$ is bounded in all 0-neibourhoods. Subscripts $a$ and $b$ run over the set $\{1, 2, \ldots, n\}$, subscripts $\alpha$ and $\beta$ run over the set $\{1, 2, \ldots, \nu\}$, where positive integers $n$, $\nu$ will be specified below. Subscripts $i, j, k, l$ run over the set $\{1, 2, 3\}$. Moreover the summation convention is assumed to holds for every pair of the repeted small subscripts.

## 1. Basic equations

Let $\Delta \equiv (0, Y^1) \times (0, Y^2) \times \ldots \times (0, Y^\nu)$, for the given $\nu$-tuple $(Y^\alpha)_{\alpha=1}^{\nu}$, of positive real numbers, be the basic unit of some periodic linear-elastic composite in a certain undeformed state related to the known Cartesian orthogonal coordinate system. It is mean that (we restrict our considerations to ases $\nu = 1$ or $\nu = 2$):

(i) if $\nu = 1$ then we deal with a periodic linear-elastic sheet-reinforced composite, cf. Fig. 1.

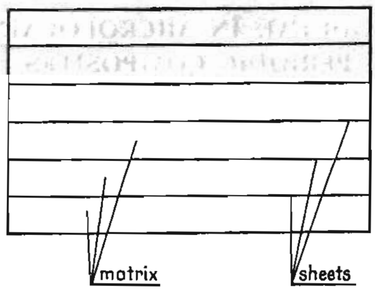


Fig. 1.

(ii) if $\nu = 2$ then we deal with a periodic linear-elastic fiber-reinforced composite, cf. Fig. 2.
Assume that there is given the finite decomposition $\bar{\Delta} = \cup\{\overline{\Delta(A)} | A = 1, \ldots, M\}$, $\Delta(A) \cap \Delta(B) = \emptyset$ for $A \neq B$, $\Delta(A) = \mathrm{int}\Delta(A)$ for $A = 1, \ldots, M$, $M$ being the given positive integer. Morover for every $A = 1, \ldots, M$, it is assumed that $\Delta(A)$ is occupied by the linear — elastic material in the reference state. By $\mathbf{H}(A) = (\mathbf{H}(A)^{ijkl})$ and $\varrho(A)$ we denote the tensor of elastic modulae and the mass density related to this material. Define:

$$\eta(A) \equiv \mathrm{mes}\Delta(A)/\mathrm{mes}\Delta,$$

$$\varrho \equiv \sum_{A=1}^{M} \eta(A)\varrho(A).$$

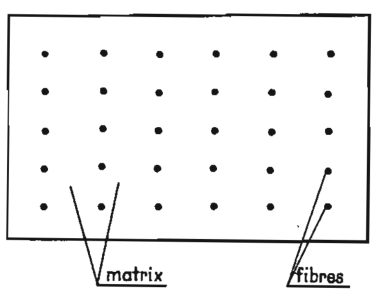


Fig. 2.

We shall introduce the decomposition of the basic unit on finite elements, [2]: $\bar{\Delta} = \cup\{\bar{\Delta}_E | E = 0, 1, \ldots, N\}$, $\Delta_E \cap \Delta_F = \emptyset$ for $E \neq F$, $E, F = 0, 1, \ldots, N$, $\Delta_E = \mathrm{int}\Delta_E$, $E = 0, 1, \ldots, N$, where $N \geqslant M+1$ is a given positive integer. We suppose that:

$$(\forall E = 0, 1, \ldots, N)(\exists A = 1, 2, \ldots, M)[\Delta_E \subset \Delta(A)],$$

and define:

$$\eta_E \equiv \operatorname{mes}\Delta_E/\operatorname{mes}\Delta .$$

We shall also introduce the system $l^1, \dots, l^n \colon R^\nu \to R^n$ of the shape functions, [2]. They are $\Delta$ — periodic continuous functions such that every $l^a$, $a = 1, \dots, n$, is linear in every set $\Delta_E$, $E = 0, 1, \dots, N$. Hence we denote:

$$\mathbf{\Lambda}^a_E = (\mathbf{\Lambda}^a_{E\alpha}) \equiv \left((l^a|_{\Delta_E})_{,\alpha}\right),\ \alpha \leqslant \nu,$$

where $\mathbf{\Lambda}^a_{E\alpha}$ are the known constants.

In microlocal mechanics of periodic composites, [2], it is assumed that the displacement field of the composite in some nonstandard model of analysis, [8], has the representation of the form:

$$u(X,t) = {}^*w(X,t) + \omega^{-1}{}^*l^a(\omega X){}^*q_a(X,t),\ (X,t) \in {}^*\Omega \times {}^*(0,t_f), \tag{1.1}$$

where:

— $\omega$ is an infinite hyperreal number,

— $t_f \in R_+$ is an arbitrary but fixed time instant,

— $\Omega$ is the region occupied by the composite in the reference configuration,

— $\boldsymbol{w} = \boldsymbol{w}(\cdot, t) \colon \Omega \to R^3$ and $\boldsymbol{q}_a = \boldsymbol{q}_a(\cdot, t) \colon \Omega \to R^3$ are certain basic unknowns termed the macrodisplacement field and the microlocal parameter fields, respectively. We shall also denote by $\boldsymbol{a} = \boldsymbol{a}(\cdot, t) \colon \Omega \to R^3$, $\boldsymbol{b} = \boldsymbol{b}(\cdot, t) \colon \Omega \to R^3$ the acceleration field and the body force field, respectively, and by $\mathbf{E}(\boldsymbol{w}) \colon \Omega \to R^6$ the macro-strain tensor field defined by:

$$\mathbf{E}(\boldsymbol{w}) = 0.5(\nabla\boldsymbol{w} + \nabla\boldsymbol{w}^T).$$

It can be proved, [2], that the equations of motion:

$$\begin{aligned} \operatorname{div}\tilde{\mathbf{T}} + \varrho(\boldsymbol{b} - \boldsymbol{a}) &= \mathbf{0}, \\ \boldsymbol{S}^a &= \mathbf{0}, \end{aligned} \tag{1.2}$$

and the constitutive equations:

$$\begin{aligned} \tilde{\mathbf{T}} &= \sum_{E=0}^{N} \eta_E \mathbf{H}_E[\mathbf{E}(\boldsymbol{w}) + \mathbf{\Lambda}^b_E \otimes \boldsymbol{q}_b], \\ \boldsymbol{S}^a &= \sum_{E=0}^{N} \eta_E \mathbf{H}_E[\mathbf{E}(\boldsymbol{w}) + \mathbf{\Lambda}^b_E \otimes \boldsymbol{q}_b]\mathbf{\Lambda}^a_F, \end{aligned} \tag{1.3}$$

hold in $\Omega$ and for every $t \in (0, t_f)$; here $\mathbf{H}_E$ stands for $\mathbf{H}(A)$ iff $\Delta_E \subset \Delta(A)$, $E = 0, 1, \dots$ $\dots, N$, $A = 1, \dots, M$.

## 2. Formulation of the problem

The results below correspond to the class of periodic composites consisting of two linear -elastic materials: the matrix material and the reinforcement material. Hence $M = 2$. Let $\eta_M \equiv \eta(1)$, $\eta_R \equiv \eta(2)$, $\mathbf{H}_M \equiv \mathbf{H}(1)$, $\mathbf{H}_R \equiv \mathbf{H}(2)$, $\varrho_M \equiv \varrho(1)$, $\varrho_R \equiv \varrho(2)$. We are taking into account only composites for which $\varepsilon \equiv \sqrt{\eta_R}$ is a small parameter and where $\overline{\mathbf{H}}_R \equiv \eta_R \mathbf{H}_R$ as well as $\mathbf{H}_M$ are independent on $\varepsilon$. Hence also $\eta_M = 1-\eta_R \in o(1)$.

From now on we have to keep in mind that the terms in Eqs. (1.3) depend on the small parameter $\varepsilon$. The problem we are going to deal with is to determine the tensor $\mathbf{H}_{eff} = (H_{eff}^{ijkl})$, which is assumed to be independent on $\varepsilon$, such that the condition:

$$\mathbf{T} - \mathbf{H}_{eff}[\mathbf{E}(w)] \in o(\varepsilon), \tag{2.1}$$

is equivallent to the system $(1.2)_2$, (1.3) in the following sense:

(i) for any pair $(w, \mathbf{T})$ satisfying (2.1) there exists a system of microlocal parameters $\mathbf{q} = (q_{ai})$ making a triplet $(w, \mathbf{T}, \mathbf{q})$ to be a solution of $(1.2)_2$, (1.3),

(ii) if a triplet $(w, \mathbf{T}, \mathbf{q})$ is a solution of $(1.2)_2$, (1.3) then a pair $(w, \mathbf{T})$ satisfies (2.1). The tensor $\mathbf{H}_{eff}$ will be termed the effective modulae tensor.

## 3. Results

To determine the effective modulae tensor for the composite under consideration we are to restrict the system $l^1, \ldots, l^n$ of shape functions by taking into account only the situations that satisfy certain conditions which will be specified below. In order to do that we shall introduce the following denotations:

$$\mathbf{P} = (P^{\alpha jkl}) : P^{\alpha jkl} \equiv \begin{cases} \delta_{\alpha k}\delta_{jl}, & \text{if} \quad \alpha = j, \\ 0.5\sqrt{2}\,(\delta_{\alpha k}\delta_{jl} + \delta_{\alpha l}\delta_{jk}), & \text{if} \quad \alpha < j, \end{cases}$$

$$\mathbf{L}_E = (L_E^{jka}) : \quad \mathbf{L}_E[q] \equiv \mathbf{P}[\Lambda_E^b \otimes q_b] \quad \text{for} \quad q \equiv (q_{ak}).$$

Now we are to formulate the following **basic proposition**.

Assume that:

1° the set $\Delta_R$ coincides with the fixed finite element $\Delta_0$, i.e., $\Delta_R = \Delta_0$,

2° $\mathbf{L}_0$ is a linear epimorphism matrix,

3° $\mathbf{L}_0 \in o(1)$, $\sum_{E=1}^{N} \eta_E \mathbf{L}_E \in o(\varepsilon^2)$, $\sum_{E=1}^{N} \eta_E \mathbf{L}_E^T \mathbf{H}_M \mathbf{L}_E \in o(\varepsilon^2)$.

Then:

$$\mathbf{H}_{eff} = \mathbf{H}_M + \overline{\mathbf{H}}_R - \overline{\mathbf{H}}_R \mathbf{P}^T (\mathbf{P}\overline{\mathbf{H}}_R \mathbf{P}^T)^{-1} \mathbf{P}\overline{\mathbf{H}}_R. \tag{2.2}$$

Proof. By means of 1° the constitutive relations (1.3) can be rewritten to the form:

$$\mathbf{T} = (\eta_M \mathbf{H}_M + \overline{\mathbf{H}}_R)[\mathbf{E}(w)] + \overline{\mathbf{H}}_R \mathbf{P}^T \mathbf{L}_0[\mathbf{q}] + \sum_{E=1}^{N} \eta_M \mathbf{H}_M \mathbf{P}^T \mathbf{L}_E[\mathbf{q}],$$

$$\mathbf{S} = (S^a) = \mathbf{L}_0^T \mathbf{P}\overline{\mathbf{H}}_R \mathbf{P}^T \mathbf{L}_0[\mathbf{q}] + \sum_{E=1}^{N} \eta_E \mathbf{L}_E^T \mathbf{P}\mathbf{H}_M \mathbf{P}^T \mathbf{L}_E[\mathbf{q}] + \tag{3.1}$$

$$+ \mathbf{L}_0^T \mathbf{P}\overline{\mathbf{H}}_R[\mathbf{E}(w)] + \sum_{E=1}^{N} \eta_E \mathbf{L}_E^T \mathbf{P}\mathbf{H}_M[\mathbf{E}(w)].$$

By means of 2° for any $E = 1, \dots, N$ there exist the unique matrix $\mathbf{L}_E$ such that:

$$\mathbf{L}_E = \tilde{\mathbf{L}}_E \mathbf{L}_0, \tag{3.2}$$

and:

$$\mathbf{L}_0^T \text{ is a linear monomorphism matrix.} \tag{3.3}$$

Since the matrix $\mathbf{P}\overline{\mathbf{H}}_R\mathbf{P}^T$ is invertible then, by means of (3.3), $\mathbf{L}_0^T\mathbf{P}\overline{\mathbf{H}}_R\mathbf{P}^T$ is also. By means of 3° we have:

$$(\mathbf{L}_0\mathbf{P}\mathbf{H}_R\mathbf{P}^T)^{-1} - (\mathbf{L}_0\mathbf{P}\overline{\mathbf{H}}_R\mathbf{P}^T + \sum_{E=1}^{N} \eta_E \mathbf{L}_E^T \mathbf{P}\mathbf{H}_M\mathbf{P}^T\tilde{\mathbf{L}}_E)^{-1} \in o(\varepsilon). \tag{3.4}$$

By virtue of (3.2), (3.4) from $(3.1)_2$, $(1.2)_2$ we obtain:

$$\mathbf{L}_0(\mathbf{q}) = -(\mathbf{L}_0^T\mathbf{P}\overline{\mathbf{H}}_R\mathbf{P}^T)^{-1}\mathbf{L}_0^T\mathbf{P}\overline{\mathbf{H}}_R[\mathbf{E}(w)] + o(\varepsilon),$$

and hence by virtue of $(3.1)_1$:

$$\mathbf{T} = (\mathbf{H}_M + \overline{\mathbf{H}}_R - \overline{\mathbf{H}}_R\mathbf{P}^T(\mathbf{P}\overline{\mathbf{H}}_R\mathbf{P}^T)^{-1}\mathbf{P}\overline{\mathbf{H}}_R)[\mathbf{E}(w)] + o(\varepsilon).$$

Now taking into account Eq. (2.1) we conclude that (2.2) holds, which ends the proof.

## 4. Example: sheet reinforced composites

We shall examine the case of $\nu = 1$, $n = 2$. By the previous comments we deal with the sheet reinforced composite, cf. Fig. 1. We suppose that $\Delta_R$ is the interval, $\Delta_R = (\alpha_1, \alpha_2)$, $0 < \alpha_1, \alpha_2 < Y$, $Y \equiv Y_1$, similituded to $\Delta = (0, Y)$ with respect to its common origin. As the shape function we take the $\Delta$ — periodic function $l: R \to R$, $l \equiv l^1$, the restriction of which is given by, cf. Fig. 3:

$$l^a|_\Delta(X) = \begin{cases} X\sqrt{\alpha Y}/(Y-\alpha) & \text{if} \quad X \in (0, 0.5(Y-\alpha)), \\ (0.5Y - X)\sqrt{\alpha Y}/\alpha & \text{if} \quad X \in \langle 0.5(Y-\alpha), 0.5(Y+\alpha)\rangle, \\ (X-Y)\sqrt{\alpha Y}/(Y-\alpha) & \text{if} \quad X \in (0.5(Y+\alpha), Y), \end{cases} \tag{4.1}$$

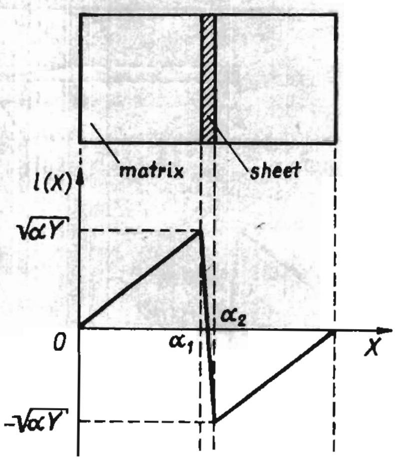


Fig. 3.

where $\alpha \equiv \alpha_2 - \alpha_1$. In this case we have $\varepsilon = \sqrt{\alpha/Y}$ and:

$$
\begin{gathered}
L^{jk} = H_R^{1jk1}, \\
\Big(\sum_{E=1}^{N} \eta_E \mathbf{L}_E^T \mathbf{P} \mathbf{H}_M\Big)^{ijk} = \varepsilon^2 H_M^{1ijk}. \\
\Big(\sum_{E=1}^{N} \eta_E \mathbf{L}_E^T \mathbf{P} \mathbf{H}_M \mathbf{P}^T \mathbf{L}_E\Big)^{jk} = \varepsilon/(1-\varepsilon^2)\, H_M^{1jk1}.
\end{gathered}
\tag{4.2}
$$

By means of (4.2) for the sheet reinforced composite under consideration there are satisfied conditions 1°, 2° and 3° and hence the tensor $\mathbf{H}_{eff}$ is given by the formula (2.2). If we deal with isotropic materials with the Lamé constants $\lambda_R$, $\mu_R$ and $\lambda_M$, $\mu_M$, respectively, then the formula (2.2) leads to the following values of the effective modulae, in which $\overline{\lambda}_R \equiv \eta_R \lambda_R$ and $\overline{\mu}_R \equiv \eta_R \mu_R$:

$$
H_{eff}^{ijkl} = \begin{cases}
\lambda_M + 2\mu_M + 4\overline{\mu}_R(\overline{\lambda}_R + \overline{\mu}_R)/(\overline{\lambda}_R + 2\overline{\mu}_R) & \text{if} \quad i = j = k = l \neq 1 \\
\lambda_M + 2\overline{\mu}_R \overline{\lambda}_R/(\overline{\lambda}_R + 2\overline{\mu}_R) \quad \text{if} \quad (i,k) = (j,l) \in \{(2,3),(3,2)\} & \\
\qquad\qquad \text{or if} \quad (i,k) = (l,j) \in \{(2,3),(3,2)\} & \\
H_M^{ijkl} \text{ if otherwise.} &
\end{cases}
$$

## 4. Example: fiber reinforced composites

We shall examine here the case of $\nu = 2$, $n = 2$. By the previous comments we deal with the fiber reinforced composite, cf. Fig. 3. We suppose that $\Delta_R$ is $\alpha_1 \times \alpha_2$ rectangul similituded to $\Delta$ with respect to its common origin. As the shape functions we take the pair $(l^1, l^2): R^2 \to R^2$ of $\Delta$ — periodic functions the restrictions of which to $\Delta$ is given by (no summation with respect to $a$), cf. Fig. 4:

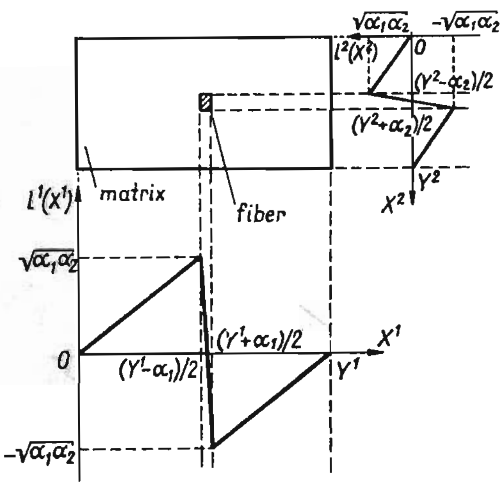


Fig. 4.

$$
l^a|_{\Delta}(X^1, X^2) = \begin{cases} \sqrt{\alpha_1\alpha_2}\; X^a/(Y^a-\alpha_a) & \text{if} \quad X^a \in (0, 0.5(Y^a-\alpha_a)), \\ (0.5Y^a - X^a)\sqrt{\alpha_1\alpha_2}/\alpha_a & \text{if} \quad X^a \in \langle 0.5(Y^a-\alpha_a), 0.5(Y^a+\alpha_a)\rangle, \\ \sqrt{\alpha_1\alpha_2}\,(X^a-Y^a)/(Y^a-\alpha_a) & \text{if} \quad X^a \in (0.5(Y^a+\alpha_a), Y^a), \end{cases} \quad (5.1)
$$

In this case we have $\varepsilon = \sqrt{\alpha_1\alpha_2/Y_1Y_2}$ and:

$$
(\mathbf{H}_R\mathbf{P}^T\mathbf{L}_0)^{ijka} = \sqrt{\alpha}^{(-1)^{a-1}} H_R^{ijka},
$$

$$
\Big(\sum_{E=1}^{N} \eta_E \mathbf{L}_E^T \mathbf{P}\mathbf{H}_M\Big)^{ijka} = -\sqrt{\alpha}^{(-1)^{a-1}} \varepsilon^2 H_M^{ijka}, \quad (5.2)
$$

$$
\Big(\sum_{E=1}^{N} (\eta_E \mathbf{L}_E^T \mathbf{P}\mathbf{H}_M \mathbf{P}^T\mathbf{L}_E\Big)^{ajkb} = \begin{cases} \varepsilon^2\big(\delta/(1-\varepsilon/\sqrt{\alpha})-\alpha\big) H_M^{1jk1} & \text{if} \quad a=b=1, \\ \Big(\big(\alpha\varepsilon(1-(\alpha/\delta)\,\varepsilon/\sqrt{\alpha})/\sqrt{\alpha}\big)^{-1} - \alpha^{-1}\Big) H_M^{2jk2} & \text{if} \quad a=b=2, \\ -\varepsilon^2 H_M^{1jk2} & \text{if} \quad (a,b)=(1,2) \text{ or if } (a,b)=(2,1), \end{cases}
$$

where $\alpha \equiv \alpha_2/\alpha_1$, $\delta \equiv Y^2/Y^1$ are assumed to be independent on $\varepsilon$. By means of (5.2)for the fiber reinforced composite under consideration there are satisfied conditions 1°, 2° and 3° and hence the tensor $\mathbf{H}_{eff}$ is given by the formula (2.2). If we deal with the isotropic materials with the Lamé constants $\lambda_R$, $\mu_R$ and $\lambda_M$, $\mu_M$, respectively, then the formula (2.2) leads to the following values of the effective modulae:

$$
H_{eff}^{ijkl} = \begin{cases} \lambda_M + 2\mu_M + (3\bar{\lambda}_R + 2\bar{\mu}_R)\bar{\mu}_R/(\bar{\lambda}_R + 2\bar{\mu}_R) & \text{if } (i,j) = (k,l) = (3,3) \\ H_M^{ijkl} & \text{if} \quad \text{otherwise}, \end{cases}
$$

where $\bar{\lambda}_R \equiv \eta_R\lambda_R$ and $\bar{\mu}_R \equiv \eta_R\mu_R$.

## Final remarks

By the application of the microlocal theory of periodic composites the effective tensor of elastic modulae $\mathbf{H}_{eff}$ has been obtained in the form of the formula (2.2). The formula (2.2) holds in a few special cases. The cases of the sheet reinforced composite and of the fiber reinforced composite has been examined in Sects. 4, 5. If the tensor $\mathbf{H}_{eff}$ is determined then the problems for the linear elastic composite can be treated as the problems similar to those of the linear elasticity theory.

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Резюме

ЭФФЕКТИВНЫЕ МОДУЛИ В МИКРОЛОКАЛЬНЫХ МОДЕЛЯХ УПРУГИХ ПЕРИОДИЧНЫХ КОМПОЗИТОВ

В микролокальной механике периодичных композитов в определяющих соотношениях существует неклассическое слагаемое — линейная функция микролокальных параметров. Тензор деформации относится к микролокальным параметрам посредством известной системы алгебраических уравнений. Цель этой статьи — вычислить эффективные модули для некоторого класса линейно-упругих периодичных композитов на основании микролокальной механики. Итоги статьи касаются к композитом состоящимся из двух различных материалов. Первый, считаемый укрепляющим материалом, более жесткий другого, считаемого матрицей. Предполагается, что мера разреза матрицы в направлении поперечном относительно упрочнения несличимо малая в сравнении с той же мерой касающейся укрепляющего материала.

Streszczenie

EFEKTYWNE MODUŁY W MIKROLOKALNYCH MODELACH SPRĘŻYSTYCH PERIODYCZNYCH KOMPOZYTÓW

W mikrolokalnej mechanice kompozytów periodycznych relacje konstytutywne zawierają dodatkowy człon, który jest liniową funkcją parametrów mikrolokalnych. Parametry mikrolokalne są z kolei związane z tensorem odkształceń poprzez pewien układ równań algebraicznych. Celem pracy jest otrzymanie efektywnych modułów dla pewnej klasy liniowo sprężystych kompozytów. Punktem wyjścia jest tu mikrolokalna mechanika periodycznych kompozytów liniowo-sprężystych. Rezultaty pracy dotyczą kompozytów periodycznych złożonych z dwóch materiałów. Pierwszy z nich, interpretowany jako materiał zbrojeniowy, jest dużo sztywniejszy niż drugi traktowany jako matryca. Jednocześnie zakłada się, że miara przekroju matrycy w kierunku poprzecznym do zbrojenia jest dużo większa w stosunku do analogicznej miary dla materiału zbrojeniowego.

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