MECHANIKA
TEORETYCZNA
I STOSOWANA
4, 25, (1987)

# COMPLEX POTENTIALS IN TWO-DIMENSIONAL PROBLEMS OF PERIODICALLY LAYERED ELASTIC COMPOSITES

ANDRZEJ KACZYŃSKI

*Instytut Matematyki, Politechnika Warszawska*

STANISŁAW J. MATYSIAK

*Instytut Mechaniki, Uniwersytet Warszawski*

## 1. Introduction

The study of the behavior of stresses in periodically layered elastic composites is of importance in many enginnering and geophysical applications. The problems of laminated materials have been treated by various methods (see for references, cf. [1]). One of the approaches is the linear theory of elasticity with microlocal parameters given by Woźniak [1 - 4]. This homogenized model of microperiodic multilayered bodies describes the micro-local effects, i.e. the effects due to the microperiodic structure of the body.

The aim of this paper is to adopt the complex variable method for two-dimensional problems of the periodically layered elastic composites. As a basis of the consideration we take into account the linear theory of elasticity with microlocal parameters [1 - 4]. The complex potentials are introduced for the reduction of the two-dimensional static problems of the layered periodic composites to the boundary values problems of analytical functions. The complex variable method is well-known, cf. [5] and it was applied fruitfully in the linear elasticity of anisotropic bodies, cf. [6].

In Section 2, based on the results of papers [1 - 4], the fundamental equations of the homogenized models of the periodic layered linear-elastic composited are presented for the two-dimensional static problems. The complex potentials for these equations are introduced in Section 3. In Section 4 a special example describing the stress distribution in the periodic two-layered half-space is considered. The solution of the problem is obtained for arbitrary given loads on the boundary.

## 2. Statement of the problem and governing equations

We consider a periodic laminated body in which every lamina is composed of two homogeneous isotropic linear-elastic layers. Let $\lambda_1, \mu_1$ and $\lambda_2, \mu_2$ be Lamé constants of the subsequent layers, $(x_1, x_2, x_3)$ comprise the Cartesian coordinate system such that the axis $x_1$ is normal to the layering. Let $l_1, l_2$ be the thicknesses of the layers, and $\delta$ be the thickness of the fundamental layer, so $\delta = l_1 + l_2$. The scheme of the open middle cross-section $B_0 \subset R^2$ of the considered body is given on Fig. 1. We assume the perfect bonding between the layers.

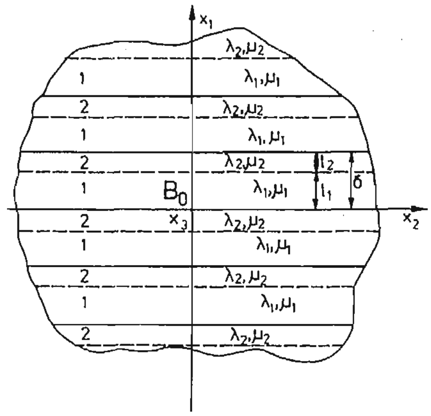


Fig. 1.

We confine attention to the two-dimensional static problems in which the displacement vector $\mathbf{u}$ is given in the form:

$$\mathbf{u}(x_1, x_2) \equiv \left(u_1(x_1, x_2), u_2(x_1, x_2), 0\right).$$

To determine the stress and strain distribution in the laminated body we take into consideration the homogenized model of linear elasticity with microlocal parameters given in [1 - 4], in which the components of displacement vector is assumed as follows:

$$u_\alpha(x_1, x_2) = w_\alpha(x_1, x_2) + h(x_1) q_\alpha(x_1, x_2), \ \alpha = 1, 2, \tag{2.1}$$

where $h: R \to R$ is the known a priori continuous $\delta$-periodic function, called the shape function [1 - 4], given by:

$$h(x_1) = \begin{cases} x_1 - \dfrac{l_1}{2} & \text{for} \quad x_1 \in \langle 0, l_1 \rangle \\ -\dfrac{\eta}{1-\eta} x_1 - \dfrac{l_1}{2} + \dfrac{l_1}{1-\eta} & \text{for} \quad x_1 \in (l_1, \delta >, \end{cases} \tag{2.2}$$

and $(\forall x_1 \in R)$, $h(x_1) = h(x_1 + \delta)$, where:

$$\eta \equiv \frac{l_1}{\delta} \in (0; 1). \tag{2.3}$$

The shape function h( $\cdot$ ) satisfies the conditions

$$\int\limits_{x_1 - \delta/2}^{x_1 + \delta/2} h(t)\, dt = 0, \quad \forall x_1 \in R, \ |h(x_1)| < \delta. \tag{2.4}$$

The functions $w_\alpha(\cdot)$, $q_\alpha(\cdot)$, $\alpha = 1,2$ are unknown functions of class $C^2(\bar{B}_0)$, the functions $w_\alpha(\cdot)$ are interpreted as the components of „macro"-displacement vector and the functions

$q_\alpha(\cdot)$ are called the microlocal parameters. The vector $h \cdot q$ represents the "micro-displacement vector connected with the microperiodic structure of the body.

According to the results given in [1 - 4], the governing equations of the homogenized model of the microperiodic two-layered composites under consideration in the two-dimensional static case take the following form:

$$
\begin{aligned}
&(\tilde{\lambda}+\tilde{\mu})(w_{1,11}+w_{2,21})+\tilde{\mu}(w_{1,11}+w_{1,22})+\\
&\qquad([\lambda]+2[\mu])q_{1,1}+[\mu]q_{2,2}=0,\\
&(\tilde{\lambda}+\tilde{\mu})(w_{1,12}+w_{2,22})+\tilde{\mu}(w_{2,11}+w_{2,22})+\\
&\qquad[\lambda]q_{1,2}+[\mu]q_{2,1}=0,\\
&(\hat{\lambda}+2\hat{\mu})q_1+[\lambda](w_{1,1}+w_{2,2})+2[\mu]w_{1,1}=0,\\
&\qquad\hat{\mu}q_2+[\mu](w_{2,1}+w_{1,2})=0,
\end{aligned}
\tag{2.5}
$$

where

$$
\begin{aligned}
&\tilde{\lambda}=\eta\lambda_1+(1-\eta)\lambda_2, \quad [\lambda]=\eta(\lambda_1-\lambda_2),\\
&\tilde{\mu}=\eta\mu_1+(1-\eta)\mu_2, \quad [\mu]=\eta(\mu_1-\mu_2),\\
&\hat{\lambda}=\eta\left(\lambda_1+\frac{\eta}{1-\eta}\lambda_2\right), \quad \hat{\mu}=\eta\left(\mu_1+\frac{\eta}{1-\eta}\mu_2\right),
\end{aligned}
\tag{2.6}
$$

and the comma denotes partial derivatives with respect to the coordinates $x_\alpha$, $\alpha = 1,2$.

Solving Eqs. $(2.5)_{3-4}$ we obtain

$$
\begin{aligned}
q_1&=-\frac{[\lambda]}{\hat{\lambda}+2\hat{\mu}}(w_{1,1}+w_{2,2})-\frac{2[\mu]}{\hat{\lambda}+2\hat{\mu}}w_{1,1},\\
q_2&=-\frac{[\mu]}{\hat{\mu}}(w_{2,1}+w_{1,2}).
\end{aligned}
\tag{2.7}
$$

Next, substitution of the microlocal parameters $q_\alpha(\cdot)$ given by Eqs. (2.7) into Eqs. $(2.5)_{1-2}$ yields

$$
\begin{aligned}
A_1w_{1,11}+(B+C)w_{2,12}+Cw_{1,22}&=0,\\
A_2w_{2,22}+(B+C)w_{1,12}+Cw_{2,11}&=0,
\end{aligned}
\tag{2.8}
$$

where:

$$
\begin{aligned}
A_1&=\frac{(\lambda_1+2\mu_1)(\lambda_2+2\mu_2)}{(1-\eta)(\lambda_1+2\mu_1)+\eta(\lambda_2+2\mu_2)}>0,\\
B&=\frac{(1-\eta)\lambda_2(\lambda_1+2\mu_1)+\eta\lambda_1(\lambda_2+2\mu_2)}{(1-\eta)(\lambda_1+2\mu_1)+\eta(\lambda_2+2\mu_2)}>0,\\
C&=\frac{\mu_1\mu_2}{(1-\eta)\mu_1+\eta\mu_2}>0,\\
A_2&=A_1+\frac{4\eta(1-\eta)(\mu_1-\mu_2)(\lambda_1-\lambda_2+\mu_1-\mu_2)}{(1-\eta)(\lambda_1+2\mu_1)+\eta(\lambda_2+2\mu_2)}>0.
\end{aligned}
\tag{2.9}
$$

The stresses in the subsequent layers may be obtained from the Hooke's law taking into account the displacement (2.1), the shape function $h(\cdot)$ given by (2.2) and equations (2.7). The components of stress tensor $\sigma_{\alpha\beta}^{(j)}$, where index $j$ runs over 1, 2 and is related to the layers of the first kind (with material constants $\lambda_1, \mu_1$) and the second kind (with material constants $\lambda_2, \mu_2$), can be expressed by:

$$\begin{aligned}\sigma_{11}^{(J)} &= A_1 w_{1,1}+Bw_{2,2},\\ \sigma_{12}^{(J)} &= C(w_{1,2}+w_{2,1}),\\ \sigma_{22}^{(J)} &= D^{(J)}w_{1,1}+E^{(J)}w_{2,2},\end{aligned} \tag{2.10}$$

where:

$$D^{(J)} = \frac{\lambda_J}{\lambda_J+2\mu_J}A_1, \quad E^{(J)} = \frac{4\mu_J(\lambda_J+\mu_J)}{\lambda_J+2\mu_J}+\frac{\lambda_J}{\lambda_J+2\mu_J}B. \tag{2.11}$$

Within the framework of given above homogenized model we can formulate boundary value problems for the equations (2.8) in terms of the "macro-displacements $w_\alpha(\cdot)$ (by using Eqs. (2.10)).

## 3. Complex representation of the solution for the equations of homogenized model

We consider now two-dimensional problems of the "hypothetical " elastic orthotropic body which is described by strain-stress relations as follows

$$\begin{aligned}\sigma_{11} &= A_1\varepsilon_{11}+B\varepsilon_{22},\\ \sigma_{12} &= 2C\varepsilon_{12},\\ \sigma_{22} &= B\varepsilon_{11}+A_2\varepsilon_{22},\end{aligned} \tag{3.1}$$

where:

$$\varepsilon_{11} = w_{1,1}, \quad \varepsilon_{22} = w_{2,2}, \quad \varepsilon_{12} = 0.5(w_{1,2}+w_{2,1}), \tag{3.2}$$

and the constants $A_1, A_2, B, C$ are defined by (2.9).

The equilibrium equations

$$\sigma_{11,1}+\sigma_{12,2} = 0, \quad \sigma_{21,1}+\sigma_{22,2} = 0, \tag{3.3}$$

expressed in terms of $w_\alpha$, (by using (3.1), (3.2)) $\alpha = 1,2$ take the same form as equations (2.8). So, we can apply the well-known general solution of these equations given in, cf. [6], for the case of orthotropic elastic body.

If we express the components of stress tensor by the stress function $U(\cdot) \in C^4(\bar{B}_0)$.

$$\sigma_{11} = U_{,22}, \quad \sigma_{22} = U_{,11}, \quad \sigma_{12} = -U_{,12}, \tag{3.4}$$

then the equations (3.3) are satisfied identitely. Now solving equations (3.1) we have

$$\begin{aligned}\varepsilon_{11} &= \frac{A_2}{A_1A_2-B^2}\sigma_{11}-\frac{B}{A_1A_2-B^2}\sigma_{22},\\ \varepsilon_{22} &= -\frac{B}{A_1A_2-B^2}\sigma_{11}+\frac{A_1}{A_1A_2-B^2}\sigma_{22},\\ \varepsilon_{12} &= \frac{1}{2C}\sigma_{12}.\end{aligned} \tag{3.5}$$

Utilizing the strain compatibility equation

$$\varepsilon_{11,22}+\varepsilon_{22,11} = 2\varepsilon_{12,12}, \tag{3.6}$$

and equations (3.4), (3.5) we obtain the following equation for unknown function $U$:

$$A_1CU_{,1111}+(A_1A_2-2BC-B^2)U_{,1122}+A_2CU_{,2222} = 0. \tag{3.7}$$

If we introduce the differential operators:

$$D_k \equiv \frac{\partial}{\partial x_1}-s_k\frac{\partial}{\partial x_2}, \quad k = 1,2,3,4, \tag{3.8}$$

where $s_k$, $k = 1, 2, 3, 4$ are the roots of the characteristic equation

$$A_1Cs^4+(A_1A_2-2BC-B^2)s^2+A_2C = 0, \tag{3.9}$$

the equation (3.7) can be rewritten in the form

$$D_1D_2D_3D_4U = 0. \tag{3.10}$$

The solutions of characteristic equation (3.9) depend on material constants of the layers, and we can mark out two cases:

3.1. Case 1, $[\mu] \neq 0$. We assume now that $\mu_1 \neq \mu_2$. The equation (3.9) has four different pure imaginary complex roots $\pm ik_1$, $\pm ik_2$, where

$$\begin{aligned} k_1 &= \left(\frac{A_1A_2-2BC-B^2-\sqrt{\delta_1}}{2A_1C}\right)^{1/2}, \\ k_2 &= \left(\frac{A_1A_2-2BC-B^2+\sqrt{\delta_1}}{2A_1C}\right)^{1/2}, \\ \delta_1 &= (A_1A_2-2BC-B^2)^2-4A_1A_2C^2 > 0. \end{aligned} \tag{3.11}$$

By integration of the equation (3.10) we obtain the following general solution in the form

$$\begin{aligned} U(x_1, x_2) &= 2\mathrm{Re}[U_1(x_2+ik_1x_1)+U_2(x_2+ik_2x_1)] \\ &= 2\mathrm{Re}[U_1(z_1)+U_2(z_2)], \end{aligned} \tag{3.12}$$

where $U_1(z_1)$ and $U_2(z_2)$ are arbitrary holomorphic functions of the complex variable $z_1 = x_2+ik_1x_1$ and $z_2 = x_2+ik_2x_1$.

Introducing the complex potentials

$$\begin{aligned} \varphi(z_1) &\equiv \frac{dU_1}{dz_1}, \\ \psi(z_1) &\equiv \frac{dU_2}{dz_2}, \end{aligned} \tag{3.13}$$

and utilizing Eqs. (3.4), (3.1) and (2.10) we obtain the complex representation for the stress components $\sigma_{11}^{(j)}$ and $\sigma_{12}^{(j)}$:

$$\begin{aligned} \sigma_{11}^{(j)} &= \sigma_{11} = 2\mathrm{Re}[\varphi'(z_1)+\psi'(z_2)], \\ \sigma_{12}^{(j)} &= \sigma_{12} = -2\mathrm{Re}\,i[k_1\varphi'(z_1)+k_2\psi'(z_2)], \quad j = 1,2. \end{aligned} \tag{3.14}$$

To obtain the complex representation for the stress components $\sigma_{22}^{(j)}$ defined by $(2.10)_3$ we must determine the functions $w_{1,1}$ and $w_{2,2}$ by solving the following equations (we use Eqs. $(3.1)_{1,3}$, (3.2) and (3.14)):

$$\begin{aligned} A_1w_{1,1}+Bw_{2,2} &= 2\mathrm{Re}[\varphi'(z_1)+\psi'(z_2)], \\ Bw_{1,1}+A_2w_{2,2} &= -2\mathrm{Re}[k_1^2\varphi'(z_1)+k_2^2\psi'(z_2)]. \end{aligned} \tag{3.15}$$

After simple calculations we have functions $w_{1,1}$, $w_{2,2}$ expressed by the complex potentials and next from $(2.10)_3$ we obtain

$$\sigma_{22}^{(J)} = 2\mathrm{Re}[c_1^{(J)}\varphi'(z_1)+c_2^{(J)}\psi'(z_2)], \tag{3.16}$$

where:

$$c_\alpha^{(J)} = \frac{(A_2+k_\alpha^2 B)D^{(J)}-(B+k_\alpha^2 A_1)E^{(J)}}{A_1 A_2-B^2}, \quad \alpha = 1,2. \tag{3.17}$$

Finally taking into account equations (3.5), (3.1) and (3.2) we arrive at the following expressions for the "macro"-displacements $w_\alpha$:

$$\begin{aligned} w_1 &= -2\mathrm{Re}[q_1\varphi(z_1)+q_2\psi(z_2)], \\ w_2 &= -2\mathrm{Re}[p_1\varphi(z_1)+p_2\psi(z_2)], \end{aligned} \tag{3.18}$$

where:

$$p_\alpha = \frac{A_1 k_\alpha^2+B}{A_1 A_2-B^2}, \quad q_\alpha = \mathrm{i}\frac{A_2+Bk_\alpha^2}{k_\alpha(A_1 A_2-B^2)}, \quad \alpha = 1,2. \tag{3.19}$$

The equations (3.18), (3.16) and (3.14) constitute the complex representation of the general solution for the homogenized model of microperiodic two-layered elastic composites presented in Section 2, in which the shear modulus $\mu_1$, $\mu_2$ satisfy $\mu_1 \neq \mu_2$. In this way two-dimensional problems of the composite bodies were reduced to the boundary value problems for holomorphic functions (complex potentials $\varphi(\cdot)$, $\psi(\cdot)$) of complex variables, which are well-known, cf. [5], [7].

3.2. Case 2, $[\mu] = 0$. We now assume that $\mu_1 = \mu_2$. Then, from Eqs. (2.9), (3.11) it follows that

$$\begin{gathered} \mu_1 = \mu_2 = C, \quad A_1 = A_2 = B+2C = \frac{(\lambda_1+2C)(\lambda_2+2C)}{(1-\eta)\lambda_1+\eta\lambda_2+2C}, \\ \delta_1 = 0, \quad k_1 = k_2 = 1. \end{gathered} \tag{3.20}$$

The equation (3.9) has two double roots $\pm\mathrm{i}$, what it means that equation (3.10) can be reduced to the biharmonic equation. The stress function $U(x_1, x_2)$ takes the Goursat's form

$$U(x_1, x_2) = \mathrm{Re}[\bar{z}U_1(z)+U_2(z)], \tag{3.21}$$

where $U_1(z)$, $U_2(z)$ are arbitrary holomorphic functions of the complex variable $z = x_2+\mathrm{i}x_1$. The stress function $U$ given by (3.21) is the same as in the classical theory of elasticity, cf. [5, 7].

The "macro"-displacements $w_\alpha$ and components of stress tensor $\sigma_{\alpha\beta}^{(J)}$, $\alpha, \beta = 1, 2$ can be written in terms of complex potentials $U_1(z)$, $U_2(z)$ and $\Phi(z) \equiv U_1'(z)$, $\Psi(z) \equiv U_2''(z)$ as follows:

$$\begin{aligned} 2C(w_2+\mathrm{i}w_1) &= \varkappa U_1(z)-z\overline{\Phi(z)}-\overline{U_2'(z)}, \\ \sigma_{11}^{(J)} &= \mathrm{Re}[2\Phi(z)+\bar{z}\Phi'(z)+\Psi(z)], \\ \sigma_{12}^{(J)} &= \mathrm{Im}[\bar{z}\Phi'(z)+\Psi(z)], \\ \sigma_{22}^{(J)} &= \mathrm{Re}[2a^{(J)}\Phi(z)-\bar{z}\Phi'(z)-\Psi(z)], \end{aligned} \tag{3.22}$$

where, (see equation (3.20))

$$a^{(J)} = 1+\frac{2C(\lambda_J+2C-A_1)}{(A_1-C)(\lambda_J+2C)}, \quad j = 1,2,$$
$$\varkappa = \frac{A_1+C}{A_1-C}. \tag{3.23}$$

3.3. **Remark.** Setting $[\lambda] = 0$, $[\mu] = 0$, i.e. $\lambda_1 = \lambda_2$ and $\mu_1 = \mu_2$ we obtain the case of homogeneous isotropic linear-elastic body. From equation (3.20) and (3.23) it follows now that:

$$a^{(J)} = 1, \quad \varkappa = \frac{\lambda_1+3\mu_1}{\lambda_1+\mu_1}. \tag{3.24}$$

Equations (3.22) together with (3.24) constitute the well-known complex representation of the general solution for the homogeneous isotropic linear-elastic body, [5, 7].

## 4. Example of application

We consider a two-layered microperiodic elastic half-space $x_1 \geqslant 0$, the scheme of which is given on Fig. 2. Let the half-space be loaded on the boundary $x_1 = 0$ by the force $(N(x_2), T(x_2), 0)$, where $N(x_2)$, $T(x_2)$ are known functions described normal and tangent

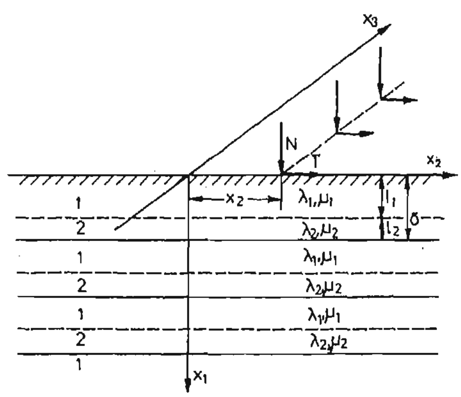


Fig. 2.

components of the force vector. The boundary conditions of equations (2.8) in this case can be written:

$$\begin{aligned} \sigma_{11}^{(1)}(0, x_2) &= A_1 w_{1,1}(0, x_2)+Bw_{2,2}(0, x_2) = -N(x_2),\\ \sigma_{12}^{(1)}(0, x_2) &= C(w_{1,2}(0, x_2)+w_{2,1}(0, x_2)) = -T(x_2). \end{aligned} \tag{4.1}$$

According to the results given in Section 3 we consider two cases:

**4.1. Case 1,** $[\mu] \neq 0$. Taking into account the complex representation (3.14), (3.16) and (3.18) and utilizing the solution of adequate problem for the ortotrophic linear-elastic half-space $x_1 \geqslant 0$, [6] (with the constitutire relations given by (3.1)) we arrive at the solution:

$$\varphi(z_1) = \frac{1}{k_1-k_2}\frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{k_2 N(\xi)-iT(\xi)}{\xi-z_1}d\xi,$$
$$\psi(z_2) = \frac{-1}{k_1-k_2}\frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{k_1 N(\xi)-iT(\xi)}{\xi-z_2}d\xi. \tag{4.2}$$

Knowing the complex potentials $\varphi$ and $\psi$ we can obtain easily the "macro"-displacements $w_\alpha$ (by using Eqs. (3.18) and (4.2)) and stresses $\sigma_{\alpha\beta}^{(J)}$ (by using Eqs. (3.14), (3.16) and (4.2)).

**4.2. Remark.** If we put in equations (4.2)

$$N(x_2) = \begin{cases} \dfrac{p}{2\varepsilon} & \text{for} \quad |x_2| \leqslant \varepsilon, \\ 0 & \text{for} \quad |x_2| < \varepsilon \end{cases} \qquad T(x_2) = 0 \quad \text{for} \quad x_2 \in R, \tag{4.3}$$

where $\varepsilon > 0$, and consider the limit case $\varepsilon \to 0$ we obtain the solution for the concentrated normal load $N(x_2) = P\delta(x_2)$ in the form:

$$\varphi(z_1) = \frac{C_1}{z_1}, \quad \psi(z_2) = \frac{C_2}{z_2}, \tag{4.4}$$

where constants $C_1$, $C_2$ are given by

$$C_1 = \frac{Pk_2}{2\pi i(k_2-k_1)}, \quad C_2 = -\frac{Pk_1}{2\pi i(k_2-k_1)}. \tag{4.5}$$

The obtained above complex potentials $\varphi$ and $\psi$ together with equations (3.14) and (3.16) give the stress distribution consistent with the results obtained in [8] by using the Fourier transform methods.

**4.3. Case 2,** $[\mu] = 0$. Taking into account the complex representation (3.22) and utilizing the solution of adequate problem for the homogeneous isotropic linear-elastic half-space $x_1 \geqslant 0$, [5] we arrive at the solution:

$$\Phi(z) = -\frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{N(\xi)-iT(\xi)}{\xi-z}d\xi,$$
$$\Psi(z) = -\frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{N(\xi)+iT(\xi)}{\xi-z}d\xi + \frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{N(\xi)-iT(\xi)}{(\xi-z)^2}\xi\, d\xi. \tag{4.6}$$

In the case of the concentrated force given by Eq. (4.3), where $\varepsilon \to 0$, we obtain

$$\Phi(z) = -\frac{P}{2\pi z}, \quad \Psi(z) = \frac{P}{2\pi z}. \tag{4.7}$$

## 5. Conclusions

The presented method of complex potentials for the homogenized model of microperiodic two-layered composites is very useful and effective in two-dimensional problems. The method may be applied to solve contact problems, crack problems in the laminated bodies. It is possible to developed and adopt it for the two-dimensional problems of multilayered microperiodic composites within the framework of the linear elasticity with microlocal parameters, [1 - 4].

### References

1. S. J. MATYSIAK, Cz. WOŹNIAK, *Micromorphic effects in a modelling of periodic multilayered elastic composites*, Int. J. Engng. Sci., 25 5, 1987, 549—559.
2. Cz. WOŹNIAK, *Nonstandard analysis and microlocal effects in the multilayered bodies*, Bull. Pol. Acad. Sci., Techn. Sci., 34, 7—8, 1986, 385—392.
3. Cz. WOŹNIAK, *Homogenized thermoelasticity with microlocal parameters*, Bull. Pol. Acad. Sci., Techn. Sci., 35, 3—5, 1987, 133—143.
4. Cz. WOŹNIAK, *On the linearized problems of thermoelasticity with microlocal parameters*, Bull. Pol. Acad. Sci., Techn. Sci., 35, 3—5, 1987, 143—153.
5. Н. И. Мусхелишвили, Некоторые основныо задачи математической теории упругостш, Изд. Академии Наук СССР, Мозква, 1949
6. С. Г. Лехницкий, *Теория упругости анизотропого тела*, Изд. Наука, Москва, 1977.
7. А. И. Каландия, *Математические методы двумерной упругости*, Изд. Наука, Москва, 1973.
8. A. KACZYŃSKI, S. J. MATYSIAK, *The influence of microlocal effects on singular stress concentrations in periodic two-layered elastic composites*, Bull. Pol. Acad. Sci., Techn. Sci., 35, 7—8, 1987, 371—382.

Р е з ю м е

КОМПЛЕКСНЫЕ ПОТЕНЦИАЛЫ В ДВУМЕРНЫХ ЗАДАЧАХ СЛОИСТЫХ СРЕД ПЕРИОДИЧЕСКОЙ СТРУКТУРЫ

В работе введено комплексные потенциалы для двумерных задач слоистых упругих сред периодической структуры рассмотренных в рамках модели Возняка линейной теории упругости с микролокальными параметрами.

S t r e s z c z e n i e

POTENCJAŁY ZESPOLONE W DWUWYMIAROWYCH ZAGADNIENIACH KOMPOZYTÓW WARSTWOWYCH O STRUKTURZE PERIODYCZNEJ

W ramach modelu Woźniaka liniowej teorii sprężystości z mikrolokalnymi parametrami wyprowadzono zespoloną reprezentację dla dwuwymiarowych zagadnień kompozytów warstwowych o strukturze periodycznej.

*Praca wpłynęła do Redakcji dnia 20 listopada 1986 roku.*