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LINEARIZED EQUATIONS OF STABILITY OF ELASTIC-PLASTIC CONICAL SHELL INCLUDING THE EFFECTS OF PASSIVE PROCESSES

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1. Introduction

Buckling loads of elastic-plastic shells can be determined by means of two approaches. In the first one, called the constant load approach, it is assumed that the external load does not change in post-buckling state this is accompanied by arising local unloading regions (passive processes). In the second one, the so called SHANLEY approach is assumed [2, 3], i.e. that the load increases in the post-buckling state, and the passive processes develop only as a result of post-critical deflections. In paper [4] the SHANLEY approach has been used for calculating bifurcation loads of conical shells. The presented procedure account for the stability analysis of elastic-plastic shells basing on the two fundamental plasticity theories, i.e.: the incremental (plastic flow) theory, and the total strain (deformation) theory. It is also possible to use the results of paper [4] for analyzing elastic shells. The problem is quite complicated when including the effects of unloading. This leads to nonlinear differential equations; although geometrical linearity is assumed. It is the purpose of this paper to linearize these equations for a simply supported conical shell, with the assumption of a two-parametrical external load and a linear stress-deformation material hardening relation.

2. Stability equations and physical relations

The basic stability equations for a conical shell, according to linear shell theory, are as follows [4]:

$$F_{,xx}\sin\beta+\delta M_{x,xx}x\cos\beta+2\delta M_{x,x}\cos\beta+\frac{1}{x\cos\beta}\delta M_{\varphi,\varphi\varphi}-$$

$$-\delta M_{\varphi,x}\cos\beta+\frac{2}{x}\delta M_{x\varphi,\varphi}+2\delta M_{x\varphi,x\varphi}+\left(\frac{1}{x\cos\beta}w_{,\varphi\varphi}+w_{,x}\cos\beta\right)N_{20}+$$

$$+x\cos\beta w_{,xx}N_{10}+2\left(w_{,x\varphi}-\frac{1}{x}w_{,\varphi}\right)T_{10}=0, \tag{2.1}$$

$$w_{,xx}\sin\beta-\delta\gamma_{12,x\varphi}-\frac{1}{x}\delta\gamma_{12,\varphi}+\delta\varepsilon_{2,xx}x\cos\beta+\frac{1}{x\cos\beta}\delta\varepsilon_{1,\varphi\varphi}+$$

$$+2\delta\varepsilon_{2,x}\cos\beta-\delta\varepsilon_{1,x}\cos\beta=0,$$

where $\delta M_{\alpha\beta}$ are the additional buckling moments per unit length, $N_{\alpha\beta}$ are the membrane forces, and w is the normal deflection. Eq. $(2.1)_1$ is the equilibrium equation with introduced force function F, and eq. $(2.1)_2$ is the strain compatibility equation.

According to the constant load concept the local unloading regions appear at the moment of buckling; so the three main zones are distinguished (see I, II, III in Fig. 2). In the first zone, a part of the shell that was deformed into the plastic state before buckling,

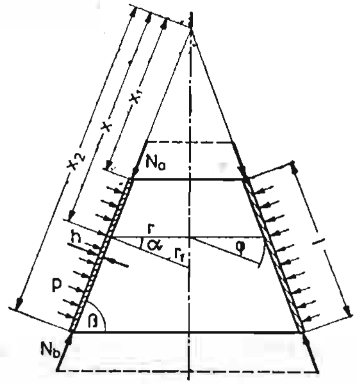

Fig. 1

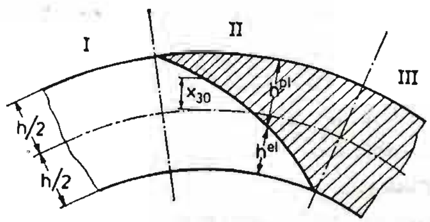

Fig. 2

returns to the elastic state; it is governed by physical relations of generalized Hooke's law. The second zone (II) is so distinguished that before buckling material is deformed plastically, but in post buckling the state a part of the material returns into the elastic state and the rest remains plastic. So, active and passive processes develop here. In the third zone (III) the plastic deformations hold for the pre-and post-buckling states; the unloading does not take place here. The physical relations in the first and in the third zones are evident, i.e. the generalized Hooke's law and apropriate plasticity relations, respectively.

Assuming the Kirchhoff-Love hypotheses the additional forces and moments during buckling in the shell are:

$$\delta N_{\alpha\beta} = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \delta\sigma_{\alpha\beta} dx_3, \qquad \delta M_{\alpha\beta} = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \delta\sigma_{\alpha\beta} x_3 dx_3. \tag{2.2}$$

When calculating the forces and the moments in the second zone each of the integrals (2.2) should be devided into two, i.e.: $\left\langle -\frac{h}{2}, x_{30} \right\rangle$, and $\left\langle x_{30}, +\frac{h}{2} \right\rangle$; x_{30} is a coordinate of active and passive processes boundary. We have for example

$$\delta N_1 - \frac{1}{2}\delta N_2 = \frac{Eh}{2}\int_{-1}^{\bar{x}_{30}} (\delta\varepsilon_1 - \delta\bar{\varkappa}_1\bar{x}_3)\,d\bar{x}_3 + \frac{h}{2}(E_s - E_t)\bar{\sigma}_x^0\varkappa \times$$
$$\times \int_{\bar{x}_{30}}^{+1} (\bar{x}_3 - \bar{x}_{30})\,d\bar{x}_3 + \frac{E_s h}{2}\int_{\bar{x}_{30}}^{+1} (\delta\varepsilon_1 - \delta\bar{\varkappa}_1\bar{x}_3)\,d\bar{x}_3, \tag{2.3}$$

where $\bar{x}_3 = 2x_3/h$ — dimensionless variable, and E_s, E_t are secant and tangent modules respectively. When appropriate calculations are made, for the total strain (deformation) theory one obtains:

$$\begin{aligned}
\delta N_1 &= B_1\left(\delta\varepsilon_1 + \frac{1}{2}\delta\varepsilon_2\right) + D_1\left(\delta\varkappa_1 + \frac{1}{2}\delta\varkappa_2\right) + B_2\bar{\sigma}_x\delta\varkappa,\\
\delta N_2 &= B_1\left(\delta\varepsilon_2 + \frac{1}{2}\delta\varepsilon_1\right) + D_1\left(\delta\varkappa_2 + \frac{1}{2}\delta\varkappa_1\right) + B_2\bar{\sigma}_\varphi\delta\varkappa,\\
\delta N_{12} &= B_1\delta\gamma_{12} + \frac{1}{2}D_1\delta\varkappa_{12} + B_2\bar{\tau}_{x\varphi}\delta\varkappa,\\
\delta M_1 &= \delta M_x = -D_1\left(\delta\varepsilon_1 + \frac{1}{2}\delta\varepsilon_2\right) - C_2\left(\delta\varkappa_1 + \frac{1}{2}\delta\varkappa_2\right) + C_3\bar{\sigma}_x\delta\varkappa,\\
\delta M_2 &= \delta M_\varphi = -D_1\left(\delta\varepsilon_2 + \frac{1}{2}\delta\varepsilon_1\right) - C_2\left(\delta\varkappa_2 + \frac{1}{2}\delta\varkappa_1\right) + C_3\bar{\sigma}_\varphi\delta\varkappa,\\
\delta M_{12} &= \delta M_{x\varphi} = -D_1\delta\gamma_{12} - \frac{1}{2}C_2\delta\varkappa_{12} + C_3\bar{\tau}_{x\varphi}\delta\varkappa,
\end{aligned} \tag{2.4}$$

where B_1, B_2, D_1, C_1, and C_2 are the stiffnesses of the shell, given by the formulas:

$$\begin{aligned}
B_1 &= \frac{2}{3}Eh[2 - f_\omega(1 - \bar{x}_{30})], \qquad B_2 = \frac{1}{8}Eh^2(f_k - f_\omega)(1 - \bar{x}_{30})^2,\\
D_1 &= \frac{1}{6}Eh^2 f_\omega(1 - \bar{x}_{30}^2), \qquad C_2 = \frac{1}{18}Eh^3[2 - f_\omega(1 - \bar{x}_{30}^3)],\\
C_3 &= \frac{1}{48}Eh^3(f_k - f_\omega)(1 + \bar{x}_{30}^2)(2 + \bar{x}_{30}), \qquad \bar{x}_{30} = \frac{2x_{30}}{h} < 1.
\end{aligned} \tag{2.5}$$

The quantities in (2.5) are as follows:

$$f_\omega = 1 - \frac{E_s}{E}, \quad f_k = 1 - \frac{E_t}{E}, \quad \bar{\sigma}_x = \sigma_x\sigma_i^{-1}, \quad \bar{\sigma}_\varphi = \sigma_\varphi\sigma_i^{-1},$$
$$\bar{\tau}_{x\varphi} = \tau_{x\varphi}\sigma_i^{-1}, \quad \delta\varkappa = \bar{\sigma}_x\delta\varkappa_1 + \bar{\sigma}_\varphi\delta\varkappa_2 + 2\bar{\tau}_{x\varphi}\delta\varkappa_{12}. \tag{2.6}$$

If before buckling the plastic deformations are small with comparison to elastic deformations one may put $f_\omega = 0$, then Eqs. (2.5) are reduced to the form:

$$B_1 = \frac{4}{3}Eh, \quad B_2 = \frac{1}{8}Eh^2 f_k(1 - \bar{x}_{30})^2, \quad D_1 = 0, \quad C_2 = D = \frac{Eh^3}{9},$$
$$C_3 = \frac{1}{48}Eh^3 f_k(1 + \bar{x}_{30})^2(2 - \bar{x}_{30}). \tag{2.7}$$

The physical relations (2.4) are coupled and nonlinear, because the position parameter $\bar{x}_{30}$, denoting the boundary between elastic and plastic region, is a variable and it depends on the unknown functions [2]:

$$
\begin{aligned}
\bar{x}_{30} &= 1-2\zeta, \qquad \zeta = \frac{1-\bar{x}_{30}}{2} = \frac{h^{pl}}{h} = \left[1-\sqrt{(1-f_k)(1+\phi)}\right] f_k^{-1}, \\
\phi &= \frac{f_k}{(1-f_k)Eh\delta\bar{\varkappa}}\left[\left(\bar{\sigma}_x-\frac{1}{2}\bar{\sigma}_\varphi\right)\delta N_1+\left(\bar{\sigma}_\varphi-\frac{1}{2}\bar{\sigma}_x\right)\delta N_2+3\bar{\tau}_{x\varphi}\delta N_{12}\right].
\end{aligned}
\tag{2.8}
$$

From the first three equations of (2.2) deformations $\delta\varepsilon_{\alpha\beta}$ may be expressed in terms of the force function F. Substituting $\delta M_{\alpha\beta}$ (expressing the curvatures $\delta\varkappa_{\alpha\beta}$ by the deflection w) and $\delta\varepsilon_{\alpha\beta}$ from (2.2) into stability equation (2.1) we obtain a set of two nonlinear differential equations for the deflection w and the force function F, to analyse the stability of an elastic-plastic conical shell under small deflections including effects of passive processes:

$$
\begin{aligned}
&\frac{2}{D}F_{,xx}\sin\beta+\alpha_1 w_{,xxxx}+\alpha_2 w_{,xxx}+\alpha_3 w_{,xx}+\alpha_4 w_{,x}+\alpha_5 w_{,xx\varphi\varphi}+ \\
&\quad+\alpha_7 w_{,x\varphi\varphi}+\alpha_9 w_{,\varphi\varphi}+\alpha_{10} w_{,\varphi\varphi\varphi\varphi}+\frac{2}{D}\left(\frac{1}{x\cos\beta}w_{,\varphi\varphi}+\right. \\
&\quad\left.+\cos\beta w_{,x}\right)\frac{px}{\mathrm{tg}\beta}+\frac{2}{D}\cos\beta w_{,xx}\left[\frac{p}{2\mathrm{tg}\beta}(x_1^2-x^2)-N_a x_1\right]+ \\
&\quad+\alpha_{11}F_{,xxxx}+\alpha_{12}F_{,xxx}+\alpha_{13}F_{,xx}+\alpha_{14}F_{,x}+\alpha_{15}F_{,xx\varphi\varphi}+\alpha_{17}F_{,x\varphi\varphi}+ \\
&\quad+\alpha_{19}F_{,\varphi\varphi}+\alpha_{20}F_{,\varphi\varphi\varphi\varphi} = 0, \\
&Ehw_{,xx}\sin\beta+\beta_1 F_{,xxxx}+\beta_2 F_{,xx\varphi\varphi}+\beta_3 F_{,\varphi\varphi\varphi\varphi}+\beta_4 F_{,x\varphi\varphi}+ \\
&\quad+\beta_5 F_{,xxx}+\beta_6 F_{,\varphi\varphi}+\beta_7 F_{,xx}+\beta_8 F_{,x}+\beta_{11} w_{,xxxx}+\beta_{12} w_{,xx\varphi\varphi}+ \\
&\quad+\beta_{13} w_{,\varphi\varphi}+\beta_{14} w_{,x\varphi\varphi}+\beta_{15} w_{,xxx}+\beta_{16} w_{,\varphi\varphi} = 0.
\end{aligned}
\tag{2.9}
$$

Now we come to linearizing the above equations. In the formulae for ζ (2.8) under the square root there is the function Φ. For elastic deformations $\Phi = 0$, for pure plastic deformations $\Phi = -f_k$. It can be prooved that $|\Phi| < f_k < 1$. Substituting ζ from (2.8) to the eqs. (2.9) we expand the characteristic terms in series, with respect to powers of Φ.

$$
(f_k-f_\omega)\zeta^2(3-2\zeta) = \left[\frac{f_k-f_\omega}{f_k}\lambda_{10}+\frac{f_k-f_\omega}{f_k}\lambda_{10}^*\Phi\right]+\frac{(1-f_k)^{\frac{3}{2}}}{f_k^3}(f_k-f_\omega)\Phi^3+\ldots, \tag{2.10}
$$

$$
\begin{aligned}
(f_k-f_\omega)\frac{\zeta^2}{1-f_\omega\zeta} &= \frac{f_k-f_\omega}{f_k^2}\,\frac{2-f_k-2\sqrt{1-f_k}}{1-\dfrac{f_\omega}{f_k}+\dfrac{f_\omega}{f_k}\sqrt{1-f_k}}+\frac{f_k-f_\omega}{1-f_k}\,\frac{2f_k-f_\omega-f_\omega\sqrt{1-f_k}}{2(f_k-f_\omega+f_\omega(\sqrt{1-f_k})^2}\times \\
&\quad\times\left(1-f_k-\sqrt{1-f_k}\right)\Phi+\ldots,
\end{aligned}
$$

where

$$
\begin{aligned}
\lambda_{10} &= \left[-8+12f_k-3f_k^2+8(1-f_k)^{\frac{3}{2}}\right]f_k^{-2}, \\
\lambda_{10}^* &= (1-f_k)\left[3f_k-6+6\sqrt{1-f_k}\right]f_k^{-2}.
\end{aligned}
\tag{2.11}
$$

In eqs. (2.9) terms there are also which cannot be linearized. However, their influence is so small, when plastic deformations are smaller than the elastic ones. So the nonlinearity parameter $\bar{x}_{30}$ we put $\bar{x}_{30} = -1$ on one hand, or with the Iliushin hypothesis [2] assuming zero values of force variations in the shell middle surface $\delta N_1 = \delta N_2 = \delta N_{12} = 0$ we take a $\bar{x}_{30}$ value accordingly on the other hand. In such an approach we obtain two different values of buckling load, and the set of equations (2.9) is linear with variable coefficients.

3. Method of Solution

The basic functions, i.e. the deflection w, and the force function F are taken as:

$$w(x, \varphi) = w_0 \sin \frac{m\pi}{l}(x-x_1)\cos n\varphi, \quad F(x, \varphi) = F_0 \sin \frac{m\pi}{l}(x-x_1)\cos n\varphi, \tag{3.1}$$

where m, and n are parameters. The functions (3.1) satisfy kinematic boundary conditions for simply supported shell edges, but the static boundary conditions are satisfied in part only. The previous investigations show that it is insignificant for shells of medium and large lengths whether all of the boundary conditions are satisfied. The linearized set of equations (2.9) we integrate using the GALERKIN type procedure. When F_1, and F_2 are the left-hand side of the eqs. (2.9) one may put

$$\int_0^{2\pi}\int_{x_1}^{x_2} F_1(x, \varphi) w(x, \varphi)\, dx r d\varphi = 0, \quad \int_0^{2\pi}\int_{x_1}^{x_2} F_2(x, \varphi) F(x, \varphi)\, dx r d\varphi = 0. \tag{3.2}$$

In the plastic range it is not possible to integrate analytically the equations, since not all of the calculated functions have an explicit form; a numerical procedure must be used. If appropriate transformations are made, a set of two algebraic equations is obtained. The resulting set of two equations is linear with respect to the vector of unknowns $U = U(w_0, F_0)$.

Using the static stability criterion, i.e. that the determinant of the above mentioned set of equations must be equal to zero, we obtain

$$\begin{aligned} \tilde{L}_{mn} = \frac{2Eh}{D}\bar{\tilde{C}}^2 - (\tilde{A}_1 + n^2\tilde{A}_2 + n^4\tilde{A}_3)(\tilde{B}_1 + n^2\tilde{B}_2 + n^4\tilde{B}_3) + \\ + (\tilde{A}_{10} + n^2\tilde{A}_{20} + n^4\tilde{A}_{30})(\tilde{B}_{10} + n^2\tilde{B}_{20} + n^4\tilde{B}_{30}) - \\ - \tilde{C}[Eh(\tilde{A}_{10} + n^2\tilde{A}_{20} + n^4\tilde{A}_{30}) + \frac{2}{D}(\tilde{B}_{10} + n^2\tilde{B}_{20} + n^4\tilde{B}_{30})] = 0. \end{aligned} \tag{3.3}$$

The buckling criterion, eq. (3.3) is transcendental and quite complicated, and it cannot be solved exactly; a numerical procedure must be used. The critical load can be calculated as the smallest positive root of eq. (3.3); however, it is necessary to minimalize it with respect to parameters m and n. The integrals $\tilde{A}_i$, $\tilde{B}_i$, $\tilde{A}_{i0}$, $\tilde{B}_{i0}$ are calculated numerically, where for example

$$\tilde{A}_1 = A_0 + \tilde{A}_1 \cos\beta, \tag{3.4}$$

$$A_0 = \frac{p(x_2-x_1)\cos\beta}{2D\mathrm{tg}\beta}\left\{\frac{1}{2}-\left(\frac{m\pi}{l}\right)^2\left[x_1^2\left(1-\frac{2\mathrm{tg}\beta}{\alpha_N}\right)-\frac{1}{3l}(x_2^3-x_1^3)\right]\right\}+$$
$$+\left(\frac{m\pi}{l}\right)^2\cos\beta e_{12}|_{x_1}^{x_2}, \qquad \alpha_N = \frac{px_1}{\alpha a},$$

$$\tilde{A}_1 = \begin{cases}\tilde{A}_1^{el}, & \text{for} \quad \sigma_i < \sigma_{pl}\\ \tilde{A}_1^{pl}, & \text{for} \quad \sigma_i \geqslant \sigma_{pl},\end{cases} \tag{3.4 [cont.]}$$

$$\tilde{A}_1^{el} = \int_{x_1}^{x_2} 2\left\{-\left[x\left(\frac{m\pi}{l}\right)^2+\frac{1}{x}\right]\sin^2\frac{m\pi}{l}(x-x_1)+\frac{l}{m\pi}\left[\left(\frac{m\pi}{l}\right)^2-\right.\right.$$
$$\left.\left.-\frac{1}{2x^2}\right]\sin\frac{2m\pi}{l}(x-x_1)\right\}dx,$$

$$\tilde{A}_1^{pl} = \int_{x_1}^{x_2}\left(\frac{m\pi}{l}\right)^2\left\{-\left(\frac{m\pi}{l}\right)^2 xe_{11}\sin^2\frac{m\pi}{l}(x-x_1)+\frac{m\pi}{2l}(e_{12}+e_{21})\sin\frac{2m\pi}{l}(x-x_1)-\right.$$
$$\left.-\frac{1}{x}e_{22}\cos^2\frac{m\pi}{l}(x-x_1)\right\}dx, \ldots$$

here σ_i is effective stress, σ_{pl} is plastic limit, e_{ij} are the shell stiffnesses (e_{ij} depend on the load).

4. Numerical Results and Conclusions

A research procedure elaborated by the Author [4] to find the buckling load from the buckling criterion, eq. (3.3), is used. In this procedure we evaluate the critical load numerically from the buckling criterion by searching for zero points of eq. (3.3) according to Newton's iteration technique; the integrals were evaluated by Simpson's rule. The buckling load is the lowest buckling load of many buckling loads for a specified range of m and n. The calculations were made on the computer ODRA 1305. Let us consider a circular conical shell loaded as in Fig. 1. In the presented series of investigations the following basic data have been assumed: $x_1 = 34.635$ cm, $x_2 = 77.635$ cm, $\beta = 20°$, $\alpha_N = px_1/N_a = 8$. We assume a linear stress hardening material with an isotropic strain hardening in which: $E = 2\cdot 10^5$ MPa, $E_t = 10000$ MPa, $\alpha_{pl} = 70$ MPa.

Fig 3 is a plot of curves representing the zero points p^* of the stability criterion (3.3), versus n, ($m = 1$), for different assumptions accepted in this paper. A minimum of each p^* curve is the buckling load. In Fig. 3 the present solutions are also compared with the author solutions [4] using the SHANLEY approach. Comparison of the results shows (see Fig. 3), that the inclusion of the effects of passive processes gives a higher critical load than the SHANLEY concept (the deformation theory in both cases is used); this was also stated previously in the analysis of plate stability [3]. The assumption of $\bar{x}_{30} = -1$ gives the results which are in better agreement with the SHANLEY concept, than using the ILIUSHIN hypothesis which says that the normal forces variations in the shell middle sturface vanish in the moment of buckling. When we use the simplified physical equations, eqs (2.7), i.e. $f_\omega = 0$. then the results are comparable with the ILIUSHIN hypothesis $\delta N_{\alpha\beta} = 0$.

We shall next obtain an elastoplastic solution of the cases in which a shell thickness parameter h is varied, with the rest of parameters taken constant, except of the angle β.

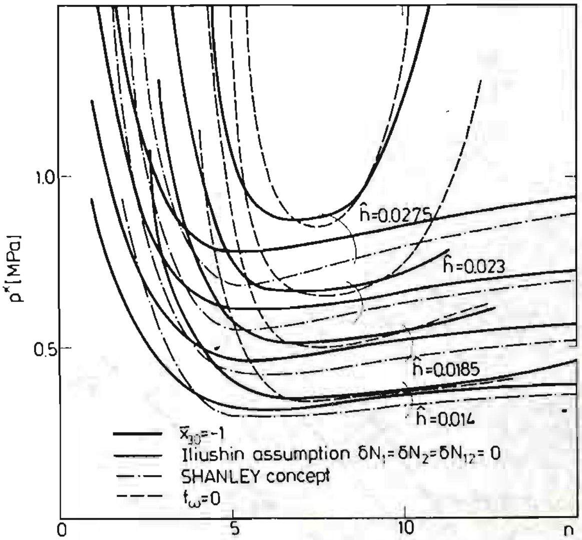

Fig. 3

Fig. 4 shows a plot of critical load as a function of shell thickness for different β using the simplified physical relations, $f_\omega = 0$ (2.7). It was ascertained, that a shell thickness increase is accompanied by the critical load increase; the curve shapes are approximately linear within the range of investigations. When angle β is increased, there is also an increase in the buckling load. The change of these two parameters did not affect the buckling form; $m = 1$, $n = 7$ (or 8).

Fig. 5 presents the results of calculations for different h and β, using non-simplified physical relations (2.5), where $f_\omega \neq 0$. For comparison Fig. 5 shows also the curves obtained on the basis of the Shanley concept for deformation theory (TD), and plastic flow theory (TPF), for $\beta = 20°$. Here one can see that when including the unloading and deformation theory of plasticity (as in this paper), the critical loads turn to be higher than when using the SHANLEY approach (unloading not included). However, the SHANLEY concept and incremental theory give critical loads (dotted line in Fig. 5) higher than in the case of deformation theory and the SHANLEY concept; but these are slightly different as to compare with critical loads obtained when including the unloading effects (see Fig. 5).

The obtained results were also the basis for plotting the diagram, Fig. 6, in coordinates p, N_a, that presents instability regions (ultimate load) of the shell for different coeffivients α_N. The points contained within an area limited by the coordinate axes and the curves refer to a stable condition, and for combination of p and N_a which corresponds to the position on the curve or the position outside the stability region, the shell is found to be in an unstable condition. It is seen that the curves for the SHANLEY approach and for ILIUSHIN concept of $\delta N_{\alpha\beta} = 0$, differ somewhat in form; the inclusion of the effects

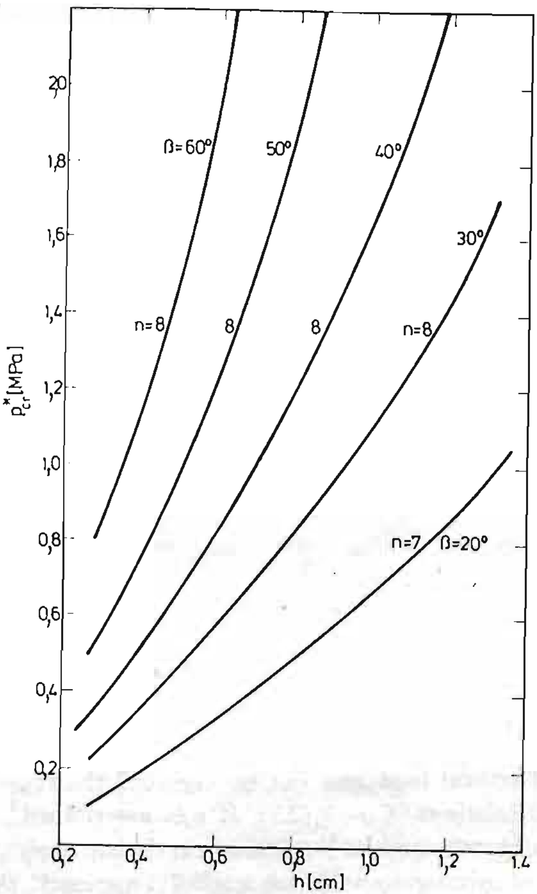

Fig. 4

of unloading gives higher critical loads, but the discrepancies are larger when the coefficient α_N is small.

It is worthnoting that the effects of passive processes on the inelastic buckling strength of conical shells subjected to axial compression and external pressure are significant for some cases, and these effects may be determined by the procedure given in this paper. The computer program developed in this research can also treat a linear elastic problem, because the terms resulting from plastic deformations are neglected automatically by conditional transfers in the program.

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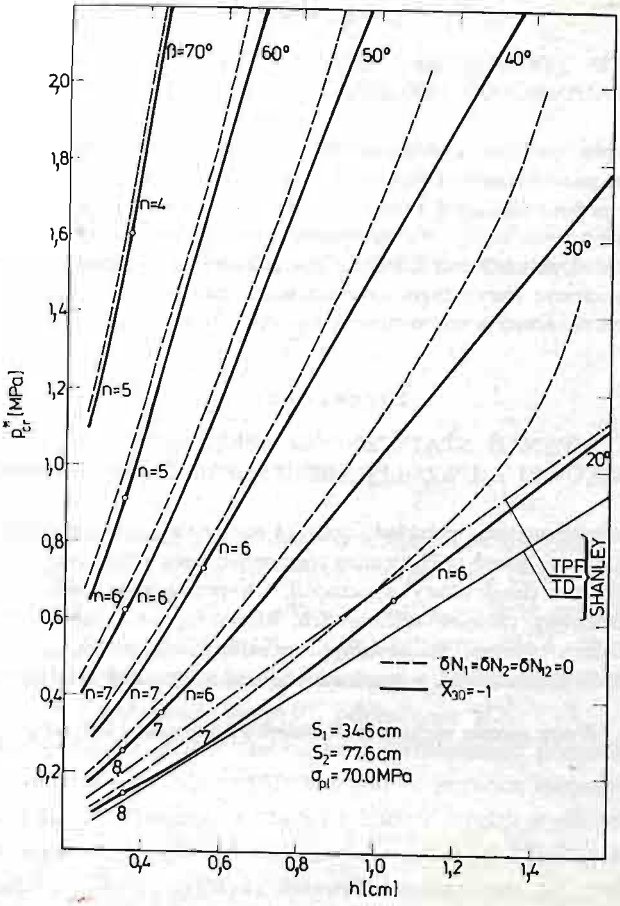
β=70°
60°
50°
40°
30°
20°
n=4
n=5
n=5
n=6
n=6
n=6
n=6
n=7
n=7
n=6
n=6
7
7
8
8
TPF
TD
SHANLEY
δN1=δN2=δN12=0
X̄30=-1
S1=34.6 cm
S2=77.6 cm
σpl=70.0 MPa
p*cr [MPa]
h [cm]
2,0
1,8
1,6
1,4
1,2
1,0
0,8
0,6
0,4
0,2
0,4
0,6
0,8
1,0
1,2
1,4

Fig. 5

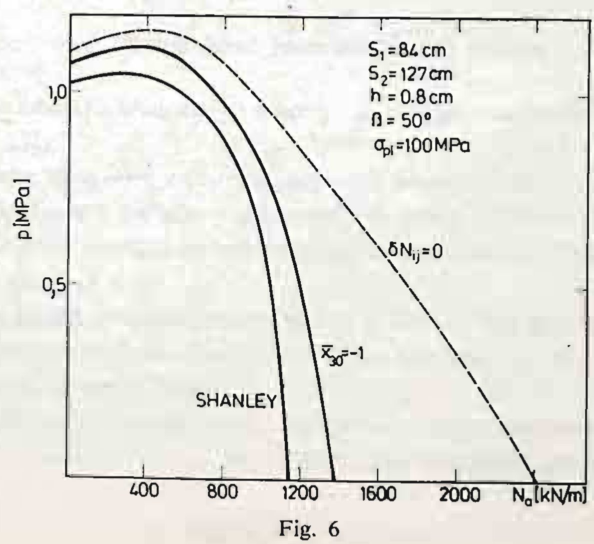
S1=84 cm
S2=127 cm
h = 0.8 cm
β = 50°
σpl=100 MPa
δNij=0
X̄30=-1
SHANLEY
p [MPa]
1,0
0,5
400
800
1200
1600
2000
Na [kN/m]

Fig. 6

Резюме

ЛИНЕАРИЗАЦИЯ УРАВНЕНИЙ УПРУГО ПЛАСТИЧЕСКОЙ УСТОЙЧИВОСТИ КОНИЧЕСКОЙ ОБОЛОЧКИ С УЧЕТОМ РАЗГРУЗКИ

В работе рассмотрен проблем упруго-пластической устойчивости оболочки в виде усеченного конуса под действием равномерного поперечного давления и осевого сжатия. Уравнения задачи построены на основе деформационной теории пластичности и теории пластического течения. Эти уравнения получены с учетом разгрузки материала, и их линеаризация сделана с помощью расположения в степенные ряды нелинейных членов. Линеаризованнуе уравнения решены методом Бубнова-Галеркина. Результаты могут быть использованы для определения критических нагрузок в упругих, упруго-пластических и чисто-пластических состояниях.

Streszczenie

LINEARYZACJA RÓWNAŃ STATECZNOŚCI SPRĘŻYSTO-PLASTYCZNEJ POWŁOKI STOŻKOWEJ Z UWZGLĘDNIENIEM PROCESÓW BIERNYCH

W pracy przedstawiono analizę i przykłady obliczeń numerycznych stateczności sprężysto-plastycznej powłoki stożkowej obciążonej bocznym ciśnieniem równomiernym i ściskającą siłą wzdłużną. Uwzględniono odciążenie materiału w chwili utraty stateczności, a wyprowadzone równania zlinearyzowano przez rozłożenie w szereg potęgowy członów nieliniowych. Równania rozwiązano metodą ortogonalizacyjną Galerkina. W przykładach obliczeń numerycznych przedstawiono porównanie wyników uzyskanych w oparciu o różne podejścia stosowane w teorii stateczności konstrukcji plastycznych.

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