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NUMERICAL CALCULATIONS OF NONLINEAR EQUILIBRIUM PATHS FOR SPHERICAL CANTILEVER SHELLS

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1. Introduction

In the paper some results of the nonlinear static analysis for the axisymmetric elastic, thin spherical cantilever shell has been presented. It is assumed that the shell is subjected to the conservative load characterized by a single scalar parameter λ. The static analysis has been performed by means of the finite element method. In the algorithm the equations of the geometrically nonlinear SANDERS-KOITER shell theory are taken into eccount. Each point on the equilibrium path has been determined from the solution of the system of nonlinear equilibrium equations by NEWTON-RAPHSON method with the possibility of a change of control parameter [1].

2. Fundamental relations and equations

We outline below some relations and equations of a proposed algorithm. More details one can find in the unpublished paper [2].

We assume that the generating line of the rotational shell midsurface is given by equation $r = r(z)$, $z \in [z_1, z_{N+1}]$. Division into the finite elements is performed by the sequence of values $z_1, \ldots, z_{N+1}$. Thus we obtain the finite element as a cone with curvilinear generatrix. In order to obtain fundamental relations for an element the local parametrization of generatrix $s = s(z)$ is introduced. For a case of axisymmetric load the displacement field of a shell midsurface is a function of one variable s only, $\tilde{u} = \tilde{u}(s)$. Tangential and normal components of this field we denote by u and w, respectively. Defining the nodal displacement vector of on element by

$$(2.1) \qquad \{q_e\}^T = [u_i, u'_i, w_i, \beta_{si}; u_{i+1}, u'_{i+1}, w_{i+1}, \beta_{si+1}]^T, \quad (i = 1, 2, \ldots, N),$$

we assume the approximation of components u and w of the displacement field for i-th element in the form of the 3-rd order polynomial with respect to s.

The potential energy of the shell is given by the sum

$$P(\tilde{u}) = U(\tilde{u}) + V(\tilde{u}) = \sum_N U^e + \sum_N V^e, \tag{2.2}$$

where

$$U^e = \pi \int_0^{L_i} \left\{ \frac{Eh}{1-\nu^2} (\varepsilon_s^2 + \varepsilon_\Theta^2 + 2\nu\varepsilon_s\varepsilon_\Theta) + \frac{Eh^3}{12(1-\nu^2)} (\varkappa_s^2 + \varkappa_\Theta^2 + 2\nu\varkappa_s\varkappa_\Theta) \right\} r\,ds, \tag{2.3}$$

is a strain energy of an element and

$$V^e = -2\pi \int_0^{L_i} (p_w w + p_u u) r\,ds = -2\pi\lambda \int_0^{L_i} (pw + qu) r\,ds, \tag{2.4}$$

is a potential energy of a conservative load with components: normal $p_w = \lambda p$ and tangential $p_u = \lambda q$.

The strain — displacement relations may be written in the form

$$\begin{aligned} &e_s = \varepsilon_s + \frac{1}{2}\beta_s^2, \quad e_\Theta = \varepsilon_\Theta, \quad \varkappa_s = -\beta_s', \quad \varkappa_\Theta = -\frac{\sin\Phi}{r}\beta_s, \\ &\varepsilon_s = u' - \Phi' w, \quad \varepsilon_\Theta = \frac{1}{r}(u\sin\Phi + w\cos\Phi), \quad \beta_s = w' + \Phi' u, \quad ()' = \frac{d}{ds}(\,), \end{aligned} \tag{2.5}$$

where Φ is the angle between axis of revolution and tangential to shell's meridian.

The strain energy (2.3) after substituting relations (2.5) can be expressed by $U^e = U_L^e + U_{NL}^e$ where term U_{NL}^e contains displacements and their derivatives to the power not exceeding two.

The equilibrium equations of a shell can be obtained from the stationarity condition of potential energy $\delta P(\tilde{u}, \lambda; \delta\tilde{u}) = 0$, where $\delta\tilde{u}$ is an arbitrary kinematically admissible variation of a displacement field. Hence

$$[K]\{q\} = \lambda\{Q\} - \{Q^*(\{q\})\}, \tag{2.6}$$

where: [K] is a classical stiffness matrix, $\{q\}$ is a vector of axisymmetric nodal displacements, $\{Q\}$ is a vector of unit loads, $\{Q^*\} = \left\{\frac{\partial U_{NL}}{\partial q}\right\}$ is a vector of "pseudo-forces".

The system of nonlinear equations (2.6) is solved by the use the following version of an iterative NEWTON-RAPHSON method:

q^0 — initial approximation,

$$\left[K + \frac{\partial Q^*}{\partial q}(q^m)\right] \Delta q^{m+1} = -\{Kq^m + Q^*(q^m) - \lambda Q\}, \quad \text{for } m = 0, 1, 2, \ldots, \tag{2.7}$$

where $\Delta q^{m+1} = q^{m+1} - q^m$.

We can distinguish two cases.

1° The matrix $\left[K + \frac{\partial Q^*}{\partial q}\right]$ is well — conditioned in the neighborhood of a solution $q = q(\lambda)$.

2° The matrix $\left[K+\dfrac{\partial Q^*}{\partial q}\right]$ is ill — conditioned.

In the first case we calculate displacements $q = q(\lambda)$ using iteration procedure (2.7). In the second case we assume that one component q_r of a vector q is given but λ is unknown. Then (2.7) can be written in the form

$$\left[\tilde{K}+\frac{\partial Q^*}{\partial q}(\tilde{q}^m)\right]\Delta\tilde{q}^{m+1} = -\{K\tilde{q}^m+Q^*(\tilde{q}^m)-q_r T\}, \tag{2.8}$$

where: $[\tilde{K}]$ — is a modified matrix $[K]$ obtained via replacement of r-th column by vector $\{Q\}$. $\{T\}$ — r-th column of the matrix $[K]$

$$\tilde{q}^m = [q_1^m, \ldots, q_{r-1}^m, \tilde{\lambda}^m, q_{r+1}^m, \ldots, q_N^m]^T.$$

To obtain Δq^{m+1}, $\Delta\tilde{q}^{m+1}$ we use methods presented in details [1]. Initial iterate $q^0(\lambda)$ for a given $q(\lambda_1)$ and $q(\lambda_2)$ is calculated from the formula

$$q^0(\lambda) = q(\lambda_2)+\frac{\lambda-\lambda_2}{\lambda_2-\lambda_1}\{q(\lambda_2)-q(\lambda_1)\}. \tag{2.9}$$

Using displacements $\{q\} = \{q(\lambda)\}$ we can calculate stress resultants in the usual manner.

3. Numerical examples

Using the forementioned algorithm and a computer program some numerical examples have been computed. Computer Odra 1204 was used. Calculations connected with the numerical integration of the stiffness matrix, aggregation of a matrix, boundary conditions etc. were based on methods presented in [3].

The shallow spherical cantilever shell subjected to the uniformely distributed load was considered (see Fig. 1).

In examples No 1 and No 2 the thickness of the shell is constant but heights H are different. In example No 3 we have linear thickness distribution. The shells are divided into ten finite elements. The displacement of the shell external node for the three mentioned

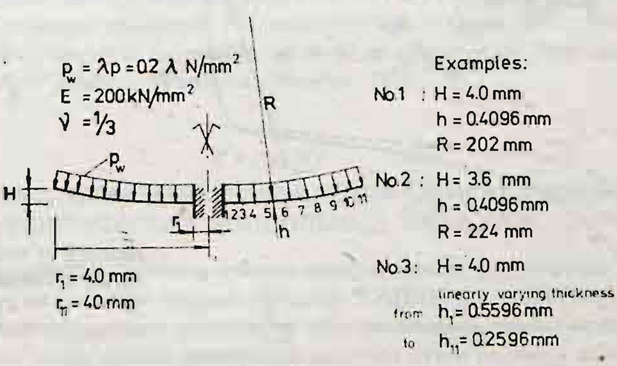

Fig. 1

examples is shown in Fig. 2. In the vicinity of the equilibrium path vertex the control parameter λ was replaced in (2.7) by component of displacement vector w_{11}, β_{11} and β_3. Calculations for particular values of control parameter ended, when

$$\max[\Delta q_i^{m+1} - \Delta q_i^m] < 10^{-6}$$

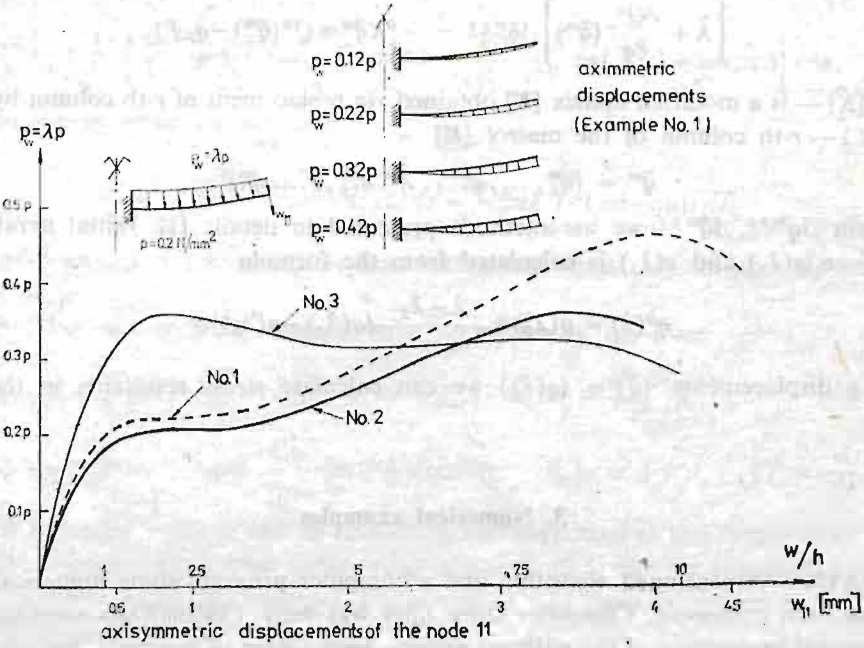

Fig. 2

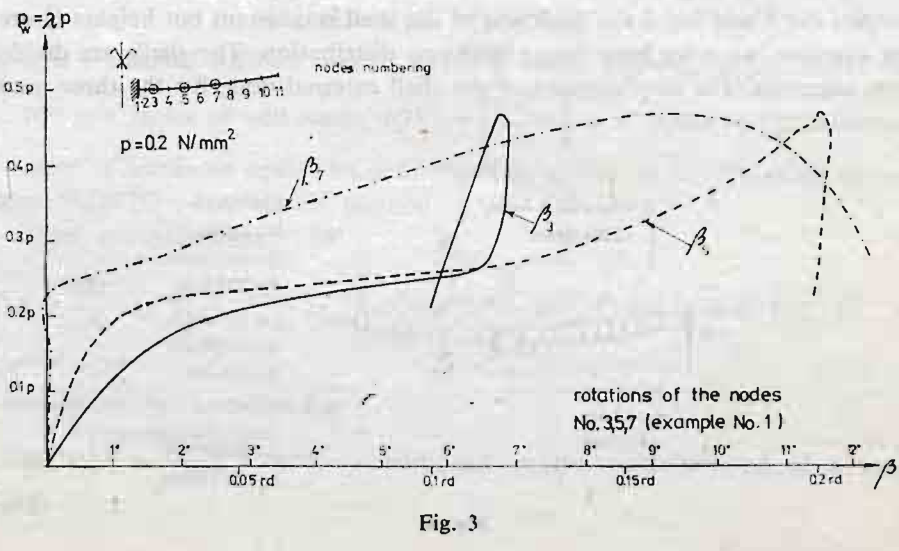

Fig. 3

where q_i^j — denotes the i-th component of displacement vector in j-th iteration step. For the error above defined each point on the path was calculated by means of 3 - 4 iteration (2.7). In Fig. 3 rotations β of nodes 3,5 and 7 are shown. The values of the stress resultants for example No 1 are shown in Fig. 4, where by dashed line on the plots of bending moments M_s the values from the example No 3 are marked.

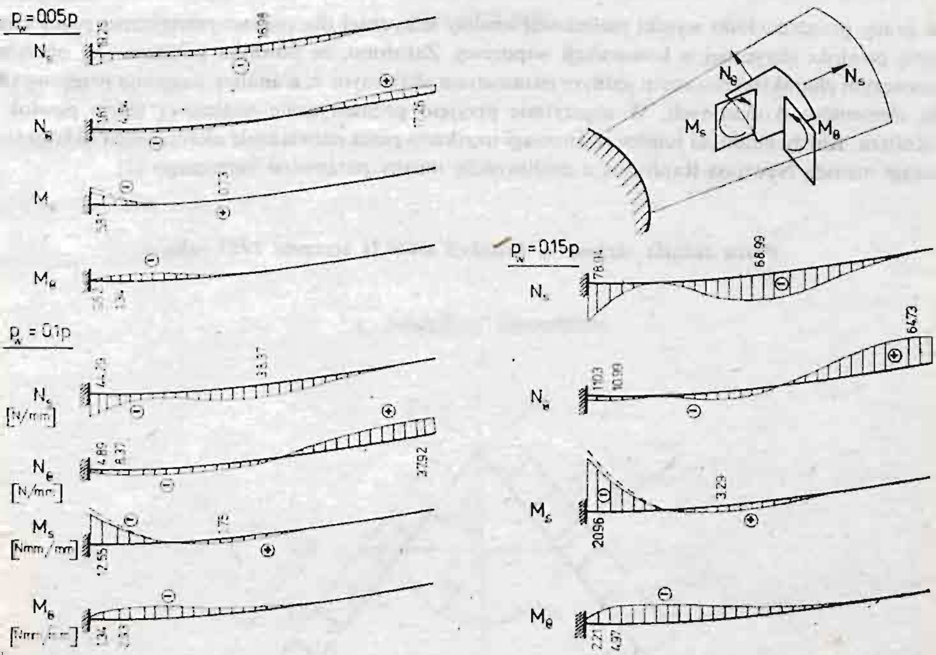

Fig. 4

References

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2. J. GOŁAŚ et al., *Numerical solutions of shells of revolution using linear and geometrically nonlinear theory of elastic and finite elements method,* Research supported by Polish Academy of Sciences under Grant 05.12, Department of Structural Mechanics, Polytechnic College in Opole, 1977 - 1980.
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Резюме

ЧИСЛЕННОЕ РЕШЕНИЕ ЗАДАЧИ НАХОЖДЕНИЯ КРИВОЙ РАВНОВЕСИЯ ДЛЯ СФЕРИЧЕСКИХ КОНСОЛЬНЫХ ОБОЛОЧЕК

Представлены результаты численного анализа статики тонкой сферической оболочки. Принимается геометрически нелинейную теорию оболочек САНДЕРСА-КОЙТЕРА. Нагрузка оболочки описывается одним скалярным параметром λ, а численный анализ приводится методом конечных элементов. Все точки кривой равновесия найдены путем решения нелинейной системы уравнений методом НЬЮТОНА-РАПСОНА.

В программе на ЭВМ (I) существует возможность замены параметра λ произвольной неизвестной системы уравнений.

Streszczenie

NUMERYCZNE OBLICZANIE KRZYWOLINIOWYCH ŚCIEŻEK RÓWNOWAGI DLA POWŁOK SFERYCZNYCH O KONSTRUKCJI WSPORCZEJ

W pracy przedstawiono wyniki nieliniowej analizy statycznej dla osiowosymetrycznej pracy cienkiej, sprężystej powłoki sferycznej o konstrukcji wsporczej. Założono, że powłoka poddana jest obciążeniem zachowawczym charakteryzowanym jednym parametrem skalarnym λ, a analizę statyczną przeprowadzono metodą elementów skończonych. W algorytmie przyjęto geometrycznie nieliniową teorię powłok Sandersa-Koitera. Każdy punkt na ścieżce równowagi uzyskano przez rozwiązanie nieliniowego układu równań równowagi metodą Newtona-Raphsona z możliwością zmiany parametru sterującego [1].

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