STRENGTH OF A METAL SEVEN-LAYER RECTANGULAR PLATE WITH TRAPEZOIDAL CORRUGATED CORES

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1. Introduction

The buckling and bending problems were solved. Magnucki et al. (2016) formulated two analytical models of a seven-layer steel beam with a transverse sinusoidal corrugated main core and two sandwich facings with steel foam cores, and solved the problem of bending and buckling. Cheon and Kim (2015) suggested an equivalent plate model to analyze the mechanical behaviour of corrugated-core sandwich panels under tensile and bending loads. Mantari and Granados (2015) presented a static analysis of functionally graded plates. In the paper, a simply supported square sandwich plate was subjected to a bi-sinusoidal load. Vaidya et al. (2015) investigated the response of sandwich steel beams with corrugated cores to quasi-static loading by employing experimental and computational approaches. A parametric study was also carried out on large-scale structural size beams of a few meters in length.

The subject of this study is a metal seven-layer rectangular plate with a trapezoidal corrugated main core and two trapezoidal corrugated cores of facings. The plate is simply supported and loaded with a uniformly distributed pressure.

2. Mathematical modelling of a seven-layer plate

2.1. Displacements and strains

A seven-layer rectangular plate with the trapezoidal corrugated main core, two inner flat sheets, two trapezoidal corrugated cores of the facings and two outer flat sheets is shown in Fig. 1. The plate is simply supported on all its edges and subjected to a uniform pressure \( p_0 \).

![Fig. 1. Scheme of the seven-layer rectangular plate](image)

The direction of the core facings corrugations is orthogonal to the one of the main core corrugation. Trapezoidal corrugations of the main core and facings cores are shown in Fig. 2.

![Fig. 2. Scheme of the corrugations of the (a) main core and (b) faces cores](image)
Taking into account the layered structures of the plate, it is easy to notice that the straight line normal to the middle plane of the plate before bending does not remain straight and normal after bending. The hypothesis is assumed that the straight line – normal after bending – takes a shape of a broken line (Fig. 3). The problem of the hypothesis for multi-layer structures was described, e.g. by Carrera (2003) and Magnucki et al. (2016).

![Diagram of deformation of the normal to the middle plane of the plate](image)

**Fig. 3. Deformation of the normal to the middle plane of the plate**

The displacements with consideration of the hypothesis are as follows:

1) outer flat sheets
   - the upper sheet for \(-0.5 + 2x_1 + x_2 \leq \zeta \leq -(0.5 + x_1 + x_2)\)
     \[
     u(x, y, z) = -t_{c1} \left[ \zeta \frac{\partial w}{\partial x} + \psi(x, y) \right] \quad v(x, y, z) = -t_{c1} \left[ \zeta \frac{\partial w}{\partial y} + x_2 \phi(x, y) \right] \quad (2.1)
     \]
   - the lower sheet for \(0.5 + x_1 + x_2 \leq \zeta \leq 0.5 + 2x_1 + x_2\)
     \[
     u(x, y, z) = -t_{c1} \left[ \zeta \frac{\partial w}{\partial x} - \psi(x, y) \right] \quad v(x, y, z) = -t_{c1} \left[ \zeta \frac{\partial w}{\partial y} - x_2 \phi(x, y) \right] \quad (2.2)
     \]

2) trapezoidal corrugated cores of the facings
   - the upper core for \(-(0.5 + x_1 + x_2) \leq \zeta \leq -(0.5 + x_1)\)
     \[
     u(x, y, z) = -t_{c1} \left[ \zeta \frac{\partial w}{\partial x} + \psi(x, y) \right] \quad v(x, y, z) = -t_{c1} \left[ \zeta \frac{\partial w}{\partial y} - \left[ \zeta + \left( \frac{1}{2} + x_1 \right) \right] \phi(x, y) \right] \quad (2.3)
     \]
   - the lower core for \(0.5 + x_1 \leq \zeta \leq 0.5 + x_1 + x_2\)
     \[
     u(x, y, z) = -t_{c1} \left[ \zeta \frac{\partial w}{\partial x} - \psi(x, y) \right] \quad v(x, y, z) = -t_{c1} \left[ \zeta \frac{\partial w}{\partial y} - \left[ \zeta - \left( \frac{1}{2} + x_1 \right) \right] \phi(x, y) \right] \quad (2.4)
     \]
3) inner flat sheets
   - the upper sheet for \(-0.5 + x_1 \leq \zeta \leq -0.5\)
     \[
     u(x, y, z) = -t_{c1}\left[\zeta \frac{\partial w}{\partial x} + \psi(x, y)\right] \quad v(x, y, z) = -t_{c1}\zeta \frac{\partial w}{\partial y}
     \]  
     (2.5)
   - the lower sheet for \(0.5 \leq \zeta \leq 0.5 + x_1\)
     \[
     u(x, y, z) = -t_{c1}\left[\zeta \frac{\partial w}{\partial x} - \psi(x, y)\right] \quad v(x, y, z) = -t_{c1}\zeta \frac{\partial w}{\partial y}
     \]  
     (2.6)
4) main corrugated core for \(-0.5 \leq \zeta \leq 0.5\)
     \[
     u(x, y, z) = -t_{c1}\zeta \left[\frac{\partial w}{\partial x} - 2\psi(x, y)\right] \quad v(x, y, z) = -t_{c1}\zeta \frac{\partial w}{\partial y}
     \]  
     (2.7)

where \(x_1 = t_s/t_{c1}, \ x_2 = t_{c2}/t_{c1}\) are dimensionless parameters, \(\zeta = z/t_{c1}\) – dimensionless coordinate, \(t_{c1}, t_{c2}, t_s\) – thicknesses of the main core, facing cores and flat sheets (Fig. 2), \(\psi(x, y) = u_1(x, y)/t_{c1}, \phi(x, y) = v_1(x, y)/t_{c2}\) – dimensionless functions of displacements, \(u_1(x, y), v_1(x, y)\) – displacements in the \(x\) and \(y\) directions, respectively, \(w(x, y)\) – deflection (Fig. 3) – deflections of each layer are equal and referenced to the middle plate layer, so \(w(x, y, z) \equiv w(x, y)\) and \(\varepsilon_z \equiv 0\).

Thus, the linear relations for strains are as follows:
1) outer flat sheets (upper/lower)
   \[
   \varepsilon_{xz} = \gamma_{yz} = 0 \quad \gamma_{(u/l)} = \frac{\partial u}{\partial y} - t_{c1} \left[2\zeta \frac{\partial w}{\partial y} + \psi(x, y)\right]
   \]  
   (2.8)
2) trapezoidal corrugated cores of the facings (upper/lower)
   \[
   \gamma_{xz} = \gamma_{yz} = 0 \quad \gamma_{(u/l)} = t_{c1} \left[2\zeta \frac{\partial w}{\partial y} - \psi(x, y)\right]
   \]  
   (2.9)
3) inner flat sheets (upper/lower)
   \[
   \varepsilon_{xz} = \gamma_{yz} = 0 \quad \gamma_{(u/l)} = -t_{c1} \left[2\zeta \frac{\partial w}{\partial y} + \psi(x, y)\right]
   \]  
   (2.10)
4) main corrugated core
   \[
   \varepsilon_x = -t_{c1}\zeta \left[\frac{\partial w}{\partial x} - 2\psi(x, y)\right] \quad \varepsilon_y = -t_{c1}\zeta \frac{\partial w}{\partial y}
   \]  
   (2.11)

The sign “+” refers to the upper facing \((u)\), and the sign “−” refers to the lower facing \((l)\).

Strains (2.8)-(2.11) make a basis for formulation of the elastic strain energy of the seven-layer plate.
2.2. Total potential energy of the plate

The elastic strain energy of the plate is a sum of the energy of the individual layers

\[ U_{\sigma}^{(\text{plate})} = U_{\sigma}^{(s-o)} + U_{\sigma}^{(c-2)} + U_{\sigma}^{(s-i)} + U_{\sigma}^{(c-1)} \] (2.12)

Consecutive components of the sum are as follows:

1) energy of the outer flat sheets

\[ U_{\sigma}^{(s-o)}(\xi, \tau) = \frac{t_{\text{c1}}}{2} \int \int_{0}^{a} \int_{0}^{b} \left\{ -\left(\frac{1}{2} + x_1 + x_2 \right) \int \left[ \Phi_{\sigma_x}^{(u,s-o)}(\zeta) \right] d\zeta + \left(\frac{1}{2} + 2x_1 + x_2 \right) \int \left[ \Phi_{\sigma_y}^{(l,s-o)}(\zeta) \right] d\zeta \right\} dx \, dy \] (2.13)

where

\[ \Phi_{\sigma_x}^{(u,s-o)} = \sigma_x^{(u,l)} \varepsilon_x^{(u/l)} + \tau_{xy}^{(u,l)} \gamma_{xy}^{(u/l)} \] (2.14)

stresses (Hooke's law)

\[ \sigma_x^{(u/l)} = \frac{E}{1 - \nu^2} (\varepsilon_x^{(u/l)} + \nu \varepsilon_y^{(u/l)}) \quad \tau_{xy}^{(u/l)} = \frac{E}{2(1 + \nu)} \gamma_{xy}^{(u/l)} \] (2.15)

and strains – expressions (2.8).

Integration of expression (2.13) with respect to the coordinate \( \zeta \) provides

\[ U_{\sigma}^{(s-o)} = \frac{E t_{\text{c1}}}{1 - \nu^2} \int \int_{0}^{a} \int_{0}^{b} \left( C_2^{(s-o)} f_{22}^{(s-o)} - c_1^{(s-o)} f_{12}^{(s-o)} + x_1 f_{11}^{(s-o)} \right) dx \, dy \] (2.16)

where

\[ c_2^{(s-o)} = \frac{1}{12} (28x_1^2 + 18x_1(1 + 2x_2) + 3(1 + 2x_2)^2)x_1 \quad c_1^{(s-o)} = (1 + 3x_1 + 2x_2)x_1 \]

\[ f_{22}^{(s-o)} = \frac{\partial^2 w}{\partial x^2} + 2\nu \frac{\partial^2 w}{\partial x \partial y} + \left( \frac{\partial^2 w}{\partial y^2} \right)^2 + 2(1 - \nu) \left( \frac{\partial \psi}{\partial x} \right)^2 \]

\[ f_{12}^{(s-o)} = \frac{\partial^2 w}{\partial x \partial y} + \nu \frac{\partial \psi}{\partial y} + x_2 \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \frac{\partial \phi}{\partial y} + (1 - \nu) \left( \frac{\partial \psi}{\partial x} \right)^2 \]

\[ f_{11}^{(s-o)} = \frac{\partial^2 w}{\partial y^2} + \frac{1 - \nu}{2} \left( \frac{\partial \psi}{\partial y} \right)^2 + x_2 \left[ \frac{\partial^2 \psi}{\partial x \partial y} + (1 - \nu) \left( \frac{\partial \phi}{\partial y} \right)^2 \right] \]

2) energy of the corrugated cores of the facings

\[ U_{\sigma}^{(c-2)} = \frac{1}{2} \int \int_{0}^{a} \int_{0}^{b} \left\{ \frac{1}{b_{\text{c2}}} \int_{A_{\text{T}}} \left[ \Phi_{\sigma_x}^{(u,l,c-2)} \right] dA_{\text{T}}^{(c-2)} + \frac{1}{b_{\text{c2}}} \int_{A_{\text{T}}} \left[ \Phi_{\sigma_y}^{(u,l,c-2)} \right] dA_{\text{T}}^{(c-2)} \right\} dx \, dy \] (2.17)

where

\[ \Phi_{\sigma_x}^{(u,l,c-2)} = \sigma_x^{(u,l)} \varepsilon_x^{(u/l)} + \sigma_y^{(u/l)} \varepsilon_y^{(u/l)} + \tau_{xy}^{(u/l)} \gamma_{xy}^{(u/l)} + \tau_{yz}^{(u/l)} \gamma_{yz}^{(u/l)} \] (2.18)

stresses

\[ \sigma_x^{(u/l)} = E \varepsilon_x^{(u/l)} \quad \sigma_y^{(u/l)} = E \varepsilon_y^{(c-2)} \quad \tau_{xy}^{(u/l)} = C_{xy}^{(c-2)} \gamma_{xy}^{(u/l)} \] (2.19)

and strains – expressions (2.9).
The area of one pitch of the trapezoidal corrugated cross section (Fig. 2)

\[ A^{(c-2)}_{tr} = 2l_{f}^{2}x_{02}(x_{f2}x_{b2} + \tilde{s}_{a2}) \]  

(2.20)

where \( x_{02} = t_{02}/t_{c-2} \), \( x_{f2} = b_{f2}/b_{02}, x_{b2} = b_{02}/t_{c-2} \) are dimensionless parameters, \( \tilde{s}_{a2} \) – dimensionless length of one pitch – trapezoid

\[ \tilde{s}_{a2} = \sqrt{(1 - x_{02})^2 + x_{b2}^2 \left( \frac{1}{2} - x_{f2} \right)^2} \]

Integration of expression (2.17) provides

\[ U^{(c-2)}_{e} = E\varepsilon_{c1}^3 \int_{0}^{a} \int_{0}^{b} \left[ \tilde{E}^{(c-2)}_{y} f_{22}^{(c-2)} + \tilde{f}_{12}^{(c-2)} + \tilde{G}_{xy}^{(c-2)} f_{11}^{(c-2)} + \tilde{G}_{yz}^{(c-2)} f_{10}^{(c-2)} \right] dx \, dy \]  

(2.21)

where

\[ f_{22}^{(c-2)} = \frac{a_{c2}^2 w^2}{2(3x_{f2}x_{b2} + \tilde{s}_{a2})(1 - x_{02})^2} \]

\[ f_{12}^{(c-2)} = 2C_{2y}^{(c-2)} \left( \frac{\partial^2 w}{\partial y^2} \right)^2 - C_{1y}^{(c-2)} \frac{\partial^2 w}{\partial y^2} \frac{\partial \phi}{\partial y} + C_{by}^{(c-2)} \frac{\partial \phi}{\partial y} \frac{\partial^2 w}{\partial y^2} \]

\[ f_{11}^{(c-2)} = 4C_{2y}^{(c-2)} \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 + C_{1y}^{(c-2)} \left( \frac{\partial^2 \phi}{\partial x} \right)^2 - \left( 2C_{1y}^{(c-2)} \frac{\partial \phi}{\partial y} + C_{1x}^{(c-2)} \frac{\partial \psi}{\partial y} \right) \frac{\partial^2 w}{\partial x \partial y} \]

\[ + x_2 \left( x_2 + \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y} \right) \frac{\partial \psi}{\partial y} \frac{\partial \psi}{\partial y} \]

\[ C_{2x}^{(c-2)} = \frac{1}{2} x_2 x_{02} \left[ \frac{1}{2} (1 - x_{02})^2 \left( x_{f2} + \frac{\tilde{s}_{a2}}{3 x_{b2}} \right) + (1 + 2x_1 + x_2) \left( x_{f2} + \frac{\tilde{s}_{a2}}{x_{b2}} \right) \right] \]

\[ C_{1x}^{(c-2)} = x_2 x_{02} (1 + 2x_1 + x_2) \left( x_{f2} + \frac{\tilde{s}_{a2}}{x_{b2}} \right) \]

\[ C_{0x}^{(c-2)} = 2x_2 x_{02} \left( x_{f2} + \frac{\tilde{s}_{a2}}{x_{b2}} \right) \]

\[ C_{by}^{(c-2)} = \frac{1}{3} x_2^3 \]

\[ C_{1y}^{(c-2)} = x_2 \left[ x_2^3 + x_1 (1 + x_2) + \frac{1}{4} (1 + 2x_2 + \frac{4}{3} x_2^3) \right] \]

\[ C_{1xy}^{(c-2)} = 2x_2 (1 + 2x_1 + x_2) \]

\[ \tilde{G}_{yz}^{(c-2)} = \frac{2}{(1 - \nu^2)x_{b2} f_{10} \tilde{s}_{a2}^3} \]

details in Lewinski et al. (2015)

3 energy of the inner flat sheets

\[ U^{(s-i)}_{e} = \frac{\nu \varepsilon_{c1}^3}{2} \int_{0}^{a} \int_{0}^{b} \left\{ \int_{-\frac{1}{2}}^{\frac{1}{2}+x_1} \left[ \tilde{\Phi}^{(u/s,i)}_{\sigma,\varepsilon} \right] d\zeta + \int_{\frac{1}{2}}^{\frac{1}{2}+x_1} \left[ \tilde{\Phi}^{(u/s,i)}_{\varepsilon,\tau} \right] d\zeta \right\} dx \, dy \]  

(2.22)

where

\[ \tilde{\Phi}^{(u/s,i)}_{\sigma,\varepsilon} = \sigma^{(u/l)}_{\sigma,\varepsilon} \varepsilon^{(u/l)}_{\sigma,\varepsilon} + \sigma^{(u/l)}_{\sigma,\varepsilon} \varepsilon^{(u/l)}_{\varepsilon,\tau} + \sigma^{(u/l)}_{\varepsilon,\tau} \varepsilon^{(u/l)}_{\varepsilon,\tau} + \sigma^{(u/l)}_{\varepsilon,\tau} \varepsilon^{(u/l)}_{\varepsilon,\tau} \]

(2.23)

stresses (Hooke’s law)

\[ \sigma^{(u/l)}_{\sigma,\varepsilon} = \frac{E}{1 - \nu^2} \left( \varepsilon^{(u/l)}_{\sigma,\varepsilon} + \nu \varepsilon^{(u/l)}_{\varepsilon,\tau} \right) \]

\[ \varepsilon^{(u/l)}_{\varepsilon,\tau} = \frac{E}{2(1 + \nu)} \varepsilon^{(u/l)}_{\varepsilon,\tau} \]

(2.24)

and strains – expressions (2.10).
Integration of expression (2.22) with respect to the coordinate $\zeta$ provides

$$U_{c}^{(s-i)} = \frac{E_{c1}^{3}}{1 - \nu^{2}} \int_{0}^{a} \int_{0}^{b} \left( C_{2}^{(s-i)} f_{22}^{(s-i)} - C_{1}^{(s-i)} f_{12}^{(s-i)} + x_{1} f_{11}^{(s-i)} \right) \, dx \, dy$$

where

$$C_{2}^{(s-i)} = \frac{1}{4} \left( 1 + 2 x_{1} + \frac{4}{3} t_{2}^{2} \right) x_{1}$$

$$C_{1}^{(s-i)} = (1 + x_{1}) x_{1}$$

$$f_{22}^{(s-i)} = \left( \frac{\partial^{2} w}{\partial x^{2}} + \nu \frac{\partial^{2} w}{\partial y^{2}} \right) \frac{\partial \psi}{\partial x} + (1 - \nu) \frac{\partial^{2} w}{\partial x \partial y} \frac{\partial \psi}{\partial y}$$

$$f_{12}^{(s-i)} = \left( \frac{\partial^{2} w}{\partial x^{2}} + \nu \frac{\partial^{2} w}{\partial y^{2}} \right) \frac{\partial \psi}{\partial x} + \frac{1}{2} \left( \frac{\partial \psi}{\partial y} \right)^{2}$$

4) energy of the main corrugated core

$$U_{c}^{(c-i)} = \frac{1}{2 b_{01}} \int_{0}^{a} \int_{0}^{b} \left\{ \int_{A_{Tr}} \Phi_{\sigma,\epsilon}^{(c-i)} \, dA_{Tr} \right\} \, dx \, dy$$

where

$$\Phi_{\sigma,\epsilon}^{(c-i)} = \sigma_{x} \epsilon_{x} + \sigma_{y} \epsilon_{y} + \tau_{xy} \gamma_{xy} + \tau_{xz} \gamma_{xz}$$

stresses

$$\sigma_{x} = E_{x}^{(c-i)} \epsilon_{x} \quad \sigma_{y} = E_{y}^{(c-i)} \epsilon_{y} \quad \tau_{xy} = G_{xy}^{(c-i)} \gamma_{xy} \quad \tau_{xz} = G_{xz}^{(c-i)} \gamma_{xz}$$

and strains – expressions (2.11).

The area of one pitch of the trapezoidal corrugated cross section (Fig. 2)

$$A_{Tr}^{(c-i)} = 2 t_{c1}^{2} x_{01}^{2} (x_{f1} x_{b1} + \tilde{s}_{a1})$$

where $x_{01} = t_{01} / t_{c1}$, $x_{f1} = b_{f1} / b_{01}$, $x_{b1} = b_{01} / t_{c1}$ are dimensionless parameters, $\tilde{s}_{a1}$ – dimensionless length of one pitch – trapezoid

$$\tilde{s}_{a1} = \sqrt{(1 - x_{01})^{2} + x_{b1}^{2} \left( \frac{1}{2} - x_{f1} \right)^{2}}$$

Integration of expression (2.30) provides

$$U_{c}^{(c-i)} = E_{c1}^{3} \int_{0}^{a} \int_{0}^{b} \frac{1}{24} \left( E_{x}^{(c-i)} f_{22}^{(c-i)} + \frac{1}{2} G_{xy}^{(c-i)} f_{12}^{(c-i)} + \frac{1}{6} G_{xz}^{(c-i)} f_{11}^{(c-i)} + 2 G_{xx}^{(c-i)} f_{10}^{(c-i)} \right) \, dx \, dy$$

where

$$f_{22}^{(c-i)} = \frac{\partial^{2} w}{\partial x^{2}} - 4 \frac{\partial^{2} w}{\partial x \partial y} + 4 \left( \frac{\partial \psi}{\partial x} \right)^{2}$$

$$f_{12}^{(c-i)} = \frac{\partial^{2} w}{\partial x^{2}} - 2 \frac{\partial^{2} w}{\partial x \partial y} \frac{\partial \psi}{\partial y} + \left( \frac{\partial \psi}{\partial y} \right)^{2}$$

$$f_{11}^{(c-i)} = \frac{\partial^{2} w}{\partial x \partial y}$$

$$f_{10}^{(c-i)} = \frac{\psi^{2}(x, y)}{t_{c1}^{2}}$$

$$\bar{E}_{x}^{(c-i)} = \frac{x_{b1} x_{01}^{3}}{2 (x_{f1} x_{b1} + \tilde{s}_{a1})}$$

$$\bar{G}_{xy}^{(c-i)} = \frac{x_{01}}{2 (1 + \nu)}$$

$$\bar{G}_{xz}^{(c-i)} = 2 x_{01} (1 - x_{01}) (3 x_{f1} x_{b1} + \tilde{s}_{a1})$$

$$\bar{G}_{xx}^{(c-i)} = \frac{1 - x_{01}}{4 (1 - \nu^{2}) x_{b1} f_{u} (\frac{x_{01}}{\tilde{s}_{a1}})^{3}}$$

detail in Lewinski et al. (2015).
The work of the load, a uniformly distributed pressure $p_0$, is in the following form

$$W = \int_0^a \int_0^b p_0 w(x, y) \, dx \, dy$$

(2.31)

The total potential energy is a sum of elastic strain energy (2.12) and work (2.31).

### 3. Equations of equilibrium and its solution

The principle of minimum total potential energy

$$\delta(U_{\text{plate}}^e - W) = 0$$

(3.1)

where $U_{\text{plate}}^e$ is the elastic strain energy of the plate (2.12) and $W$ is the work of the load (2.31).

The system of the equations of equilibrium – three partial differential equations derived based on principle (3.1) is in the following form

$$\mathbf{R}_w^{(s-o-1)} + \mathbf{R}_w^{(c-2)} + \mathbf{R}_w^{(c-1)} = \frac{p_0}{E\ell_c^2}$$

(3.2)

where

$$\mathbf{R}_w^{(s-o-1)} = \frac{1}{1 - \nu^2} \left\{ 2(C_2^{(s-o)} + C_2^{(s-i)}) \nabla^4 w - C_1^{(s-o)} \left[ \frac{\partial}{\partial x}(\nabla^2 \psi) + x_2 \frac{\partial}{\partial y}(\nabla^2 \phi) \right] \right\}$$

$$\mathbf{R}_w^{(c-2)} = 2\mathbf{R}_w^{(c-2)} - \mathbf{R}_w^{(c-2)} - \mathbf{R}_w^{(c-2)}$$

$$\mathbf{R}_w^{(c-2)} = C_i x_{2,c} \frac{\partial^4 w}{\partial x^4} + C_i^{(c-2)} \left( 4G_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + \tilde{E}_y(c-2) \frac{\partial^4 w}{\partial y^4} \right)$$

$$\mathbf{R}_w^{(c-2)} = \frac{\partial}{\partial x} \left( 2C_i x_{2,c} \frac{\partial^2 \psi}{\partial x^2} + C_i^{(c-2)} C_i^{(c-2)} \frac{\partial^2 \psi}{\partial y^2} \right)$$

$$\mathbf{R}_w^{(c-2)} = C_i^{(c-2)} \frac{\partial}{\partial x} \left( 2G_{xy}^{(c-2)} \frac{\partial^2 \phi}{\partial x^2} + \tilde{E}_y(c-2) \frac{\partial^2 \phi}{\partial y^2} \right)$$

$$\mathbf{R}_w^{(c-1)} = \frac{1}{12} \mathbf{R}_w^{(c-1)} - \frac{1}{6} \mathbf{R}_w^{(c-1)}$$

$$\mathbf{R}_w^{(c-1)} = \tilde{E}_x(c-1) \frac{\partial^2 \psi}{\partial x^2} + 2G_{xy}^{(c-2)} \frac{\partial^2 \psi}{\partial y^2} - 4 \tilde{G}_{xy}^{(c-2)} \frac{\partial^2 \psi}{\partial y^2}$$

and

$$\mathbf{R}_w^{(s-o-1)} + \mathbf{R}_w^{(c-2)} + \mathbf{R}_w^{(c-1)} = 0$$

(3.3)

where

$$\mathbf{R}_w^{(s-o-1)} = \frac{1}{1 - \nu^2} \left\{ \left( C_1^{(s-o)} + C_1^{(s-i)} \right) \frac{\partial}{\partial x}(\nabla^2 w) - 2x_1 \left[ 2 \frac{\partial^2 \psi}{\partial x^2} + (1 - \nu) \frac{\partial^2 \psi}{\partial y^2} \right] \right\}$$

$$\mathbf{R}_w^{(c-2)} = \frac{\partial}{\partial x} \left( 2C_i x_{2,c} \frac{\partial^2 w}{\partial x^2} + C_i^{(c-2)} \tilde{G}_{xy}^{(c-2)} \frac{\partial^2 w}{\partial y^2} \right) - 2 \left( G_{0x}^{(c-2)} \frac{\partial^2 \psi}{\partial x^2} + x_2 G_{xy}^{(c-2)} \frac{\partial^2 \psi}{\partial y^2} \right)$$

$$\mathbf{R}_w^{(c-2)} = \frac{\partial^2 \phi}{\partial x \partial y}$$

$$\mathbf{R}_w^{(c-1)} = \frac{1}{6} \frac{\partial}{\partial x} \left( \tilde{E}_x(c-1) \frac{\partial^2 \psi}{\partial x^2} + 2G_{xy}^{(c-2)} \frac{\partial^2 \psi}{\partial y^2} \right) - \frac{1}{3} \left( \tilde{E}_x(c-1) \frac{\partial^2 \psi}{\partial x^2} + G_{xy}^{(c-2)} \frac{\partial^2 \psi}{\partial y^2} \right) - 4 \tilde{G}_{xx}^{(c-2)} \frac{\psi(x, y)}{\ell_c^2}$$
and
\[ R_{\phi}^{(s-o-i)} + R_{\phi}^{(c-2)} = 0 \] (3.4)
where
\[ R_{\phi}^{(s-o-i)} = \frac{1}{1 - \nu^2} \left\{ x_2 C_1^{(s-o)} \frac{\partial}{\partial y} (\nabla^2 w) - x_1 x_2^2 \left[ (1 - \nu) \frac{\partial^2 \phi}{\partial x^2} + 2 \frac{\partial^2 \phi}{\partial y^2} \right] - x_1 x_2 (1 + \nu) \frac{\partial^2 \psi}{\partial x \partial y} \right\} \]
\[ R_{\phi}^{(c-2)} = \frac{\partial}{\partial y} (2 \tilde{G}_{xy}^{(c-2)} \frac{\partial^2 w}{\partial x^2} + \tilde{E}_{y}^{(c-2)} \frac{\partial^2 w}{\partial y^2}) \]
\[ R_{\phi,\phi}^{(c-2)} = 2 C_{0y}^{(c-2)} \left( \tilde{G}_{xy}^{(c-2)} \frac{\partial^2 \phi}{\partial x^2} + \tilde{E}_{y}^{(c-2)} \frac{\partial^2 \phi}{\partial y^2} \right) - 2 x_2 \tilde{G}_{yz}^{(c-2)} \frac{\phi(x,y)}{t_{c1}^2} \]
\[ \nabla^2 w = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \]

Three equations of equilibrium (3.2), (3.3) and (3.4) with three unknown functions \( w(x,y) \), \( \psi(x,y) \) and \( \phi(x,y) \) are approximately solved assuming three unknown functions in the forms
\[ w(x,y) = w_a \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \]
\[ \psi(x,y) = \psi_a \cos \frac{\pi x}{a} \sin \frac{\pi y}{b} \]
\[ \phi(x,y) = \phi_a \sin \frac{\pi x}{a} \cos \frac{\pi y}{b} \] (3.5)

where \( w_a, \psi_a, \phi_a \) are parameters of the functions, \( a, b \) – sizes of the plate (Fig. 1).

Substituting these functions into equations (3.2), (3.3) and (3.4) and using the Galerkin method, three algebraic equations are obtained
\[ \alpha_{11} w_a - \alpha_{12} \frac{b}{a} \psi_a - \alpha_{13} \frac{b}{a} \phi_a = \frac{16 a^2 b^2 p_0}{\pi^3 t_{c1}^2} \]
\[ \alpha_{21} \frac{\pi}{a} w_a - \alpha_{22} \frac{\pi}{a} \psi_a - \alpha_{23} \phi_a = 0 \] (3.6)

where the dimensionless elements
\[ \alpha_{11} = \frac{a^{(1)}}{a^{(1)}} + \frac{a^{(1)}}{a^{(1)}} + \alpha_{11}^{(1)} \]
\[ \alpha_{23} = x_2 \left( \frac{x_1}{1 - \nu} + x_2 \tilde{G}_{xy}^{(c-2)} \right) \]
\[ \alpha_{12} = \frac{1}{1 - \nu^2} \left[ C_1^{(s-o)} + C_1^{(s-1)} \left( \frac{b}{a} + \frac{a}{b} \right) \right] + \frac{1}{6} \left[ \tilde{E}_{x}^{(c-1)} \left( \frac{b}{a} \right)^2 + \tilde{G}_{xy}^{(c-1)} \left( \frac{a}{b} \right)^2 \right] \]
\[ \alpha_{13} = x_2 \left( \frac{b}{a} + \frac{a}{b} \right) + C_1^{(s-2)} \left( \tilde{G}_{xy}^{(c-2)} + \tilde{E}_{y}^{(c-2)} \right) \frac{a}{b} \]
\[ \alpha_{21} = \alpha_{12} \]
\[ \alpha_{31} = \alpha_{13} \]
\[ \alpha_{32} = \alpha_{23} \]
\[ \alpha_{33} = \frac{x_1 x_2}{1 - \nu^2} \left[ \frac{b}{a} + \frac{a}{b} \right] + 2 C_{0y}^{(c-2)} \left( \tilde{G}_{xy}^{(c-2)} + \tilde{E}_{y}^{(c-2)} \right) \frac{a}{b} + \frac{2 x_2}{\pi^2} \tilde{G}_{yz}^{(c-2)} \frac{ab}{t_{c1}^2} \]
Solving equations (3.6) one obtains

\[
\begin{align*}
  w_a &= \frac{16 a^2 b^2 p_0}{E \pi^6 \alpha w t_c^3} \\
  \psi_a &= \frac{16 \alpha \psi a^2 b p_0}{E \pi^4 \alpha w t_c^3} \\
  \phi_a &= \frac{16 \alpha \phi a^2 b p_0}{E \pi^4 \alpha w t_c^3} \\
\end{align*}
\]  

(3.7)

where

\[
\alpha_w = \alpha_{11} - (\alpha_{\psi} \alpha_{12} + \alpha_{\phi} \alpha_{13}) \\
\alpha_{\psi} = \frac{b \alpha_{21} \alpha_{33} - a \alpha_{31} \alpha_{23}}{a (\alpha_{22} \alpha_{33} - \alpha_{23}^2)} \\
\alpha_{\phi} = \frac{a \alpha_{31} \alpha_{22} - b \alpha_{21} \alpha_{32}}{b (\alpha_{22} \alpha_{33} - \alpha_{23}^2)}
\]

The stresses on the outer sheets and in the middle of the plate, for \( \zeta_o = \mp (0.5 + 2x_1 + x_2) \) and \( x = a/2, \ y = b/2 \) are

\[
\begin{align*}
  \sigma_x &= \frac{1}{1 - \nu^2} \left[ \left( \frac{a}{b} + \nu \frac{a}{b} \right) \zeta_o \pm (\alpha_{\psi} + \nu x_2 \alpha_{\phi}) \right] \frac{16 ab}{E \pi^4 \alpha w t_c^3} p_0 \\
  \sigma_y &= \frac{1}{1 - \nu^2} \left[ \left( \frac{a}{b} + \nu \frac{a}{b} \right) \zeta_o \pm (\nu \alpha_{\psi} + x_2 \alpha_{\phi}) \right] \frac{16 ab}{E \pi^4 \alpha w t_c^3} p_0 \\
\end{align*}
\]

(3.8)

and the equivalent stress (Huber-Mises-Hencky)

\[
\sigma_{eq} = \sqrt{f_{\sigma x}^2 - f_{\sigma x} f_{\sigma y} + f_{\sigma y}^2} \frac{16 ab}{E \pi^4 (1 - \nu^2) \alpha w t_c^3} p_0
\]

(3.9)

where

\[
\begin{align*}
  f_{\sigma x} &= \left( \frac{b}{a} + \nu \frac{a}{b} \right) \zeta_o \pm (\alpha_{\psi} + \nu x_2 \alpha_{\phi}) \\
  f_{\sigma y} &= \left( \frac{a}{b} + \nu \frac{a}{b} \right) \zeta_o \pm (\nu \alpha_{\psi} + x_2 \alpha_{\phi})
\end{align*}
\]

4. Finite element model of the seven-layer plate

A family of simply supported rectangular plates of dimensions 2024 mm x 2000 mm subjected to a uniform load of 0.01 MPa has been considered. The linear static analysis was carried out using the finite element software ABAQUS. A quarter of the rectangular plate was modeled. The linear S4R shell elements were placed at the mid-surface of the plate layers (Fig. 4).

Fig. 4. The meshing scheme of a simply supported plate

The mesh density study was carried out to refine the global mesh size to 4 mm. The mesh convergence plot for the maximum deflection in the middle of the top face sheet is presented in Fig. 5.
Perfect bonding between the cores and the flat sheets was assumed. The interaction between flanges of the cores and the flat sheets was provided with the use of the tie constraint. The flanges of the cores were slave surfaces and the flat sheets were master surfaces.

The boundary conditions were imposed only to edges of the flat sheets (master surfaces) – each edge was simply supported. The implementation of the symmetry and the simply supported boundary conditions on a quarter of the plate is schematically shown in Fig. 6.

5. Results of numerical calculations of deflection and stresses of the plate

The aim of these calculations was to verify the results obtained through the linear finite element analysis with those obtained through an analytical method. The maximum deflection and the equivalent stress of the family of seven-layer rectangular plates, using both analytical and finite element methods, was evaluated. The results of the parametric studies for changes of $b_{02}$ and $b_{01}$ are collected in Case 1 and Case 2, respectively.

**Case 1.** The study for constant area of the trapezoidal corrugation of the facing core $A_{Total}^{(c-2)} = nA_{Tyr}^{(c-2)}$, where $n$ is the number of the corrugations and $A_{Tyr}^{(c-2)}$ (2.20) is the area of one pitch of the trapezoidal corrugated cross section. The numerical calculations are carried out for the rectangular plate with the following sizes: $a = 2024$ mm, $b = 2000$ mm, $t_s = 0.8$ mm, $t_{c1} = 11.2$ mm, $t_{01} = 0.8$ mm, $b_{01} = 46$ mm, $b_{f1} = 10$ mm, $t_{c2} = 9.2$ mm, $b_{f2} = 8$ mm, $A_{Total}^{(c-2)} = 1811.83$ mm$^2$, $p_0 = 0.01$ MPa, and material constants...
\[ E = 2 \cdot 10^5 \text{ MPa}, \ \nu = 0.3. \] The results of the calculations are presented in Table 1. The values in the ABAQUS columns in Table 1 enclosed in parentheses are percentage differences with respect to the analytical ones (the absolute value of the relative deviation).

Table 1. The deflection and the equivalent stresses of the plate for the first case \( A_{Total}^{(c-2)} = \text{const} \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( b_{02} ) [mm]</th>
<th>( t_{02} ) [mm]</th>
<th>Analytical ( w_a ) [mm]</th>
<th>ABAQUS ( w_a ) [mm]</th>
<th>Analytical ( \sigma_{eq} ) [MPa]</th>
<th>ABAQUS ( \sigma_{eq} ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>40.0</td>
<td>0.8</td>
<td>5.30</td>
<td>5.41 (2.1%)</td>
<td>59.4</td>
<td>60.01 (1.0%)</td>
</tr>
<tr>
<td>60</td>
<td>33.333</td>
<td>0.751</td>
<td>5.33</td>
<td>5.43 (1.9%)</td>
<td>59.7</td>
<td>60.86 (1.9%)</td>
</tr>
<tr>
<td>70</td>
<td>28.571</td>
<td>0.6968</td>
<td>5.38</td>
<td>5.48 (1.8%)</td>
<td>60.1</td>
<td>62.40 (3.8%)</td>
</tr>
<tr>
<td>80</td>
<td>25.0</td>
<td>0.6409</td>
<td>5.43</td>
<td>5.53 (1.7%)</td>
<td>60.5</td>
<td>63.44 (4.7%)</td>
</tr>
<tr>
<td>90</td>
<td>22.222</td>
<td>0.5867</td>
<td>5.51</td>
<td>5.61 (1.8%)</td>
<td>60.9</td>
<td>64.94 (6.4%)</td>
</tr>
<tr>
<td>100</td>
<td>20.0</td>
<td>0.5363</td>
<td>5.61</td>
<td>5.72 (1.9%)</td>
<td>61.2</td>
<td>66.29 (8.0%)</td>
</tr>
</tbody>
</table>

Case 2. The study for constant area of the trapezoidal corrugation of the main core \( A_{Total}^{(c-1)} = mA_{Tr}, \) where \( m \) is the number of the corrugations and \( A_{Tr}^{(c-1)} \) (2.29) is the area of one pitch of the trapezoidal corrugated cross section. The numerical calculations are carried out for the rectangular plate with the following sizes: \( a = 2024 \text{ mm}, \ b = 2000 \text{ mm}, \ t_s = 0.8 \text{ mm}, \ t_{c1} = 11.2 \text{ mm}, \ b_{f1} = 10 \text{ mm}, \ t_{c2} = 9.2 \text{ mm}, \ t_{02} = 0.8 \text{ mm}, \ b_{02} = 40 \text{ mm}, \ b_{f2} = 8 \text{ mm}, \ A_{Total}^{(c-1)} = 1876.03 \text{ mm}^2, \ p_0 = 0.01 \text{ MPa}, \ E = 2 \cdot 10^5 \text{ MPa}, \ \nu = 0.3. \) The results of the calculations are presented in Table 2. The values in the ABAQUS columns in Table 2 enclosed in parentheses are percentage differences with respect to the analytical ones (the absolute value of the relative deviation).

Table 2. The deflection and the equivalent stresses of the plate for the second case \( A_{Total}^{(c-1)} = \text{const} \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( b_{01} ) [mm]</th>
<th>( t_{01} ) [mm]</th>
<th>Analytical ( w_a ) [mm]</th>
<th>ABAQUS ( w_a ) [mm]</th>
<th>Analytical ( \sigma_{eq} ) [MPa]</th>
<th>ABAQUS ( \sigma_{eq} ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>44</td>
<td>46.0</td>
<td>0.8</td>
<td>5.30</td>
<td>5.41 (2.1%)</td>
<td>59.34</td>
<td>60.01 (1.1%)</td>
</tr>
<tr>
<td>54</td>
<td>37.481</td>
<td>0.7349</td>
<td>5.33</td>
<td>5.43 (1.8%)</td>
<td>59.44</td>
<td>60.07 (1.1%)</td>
</tr>
<tr>
<td>64</td>
<td>31.625</td>
<td>0.6652</td>
<td>5.41</td>
<td>5.49 (1.5%)</td>
<td>59.73</td>
<td>60.47 (1.2%)</td>
</tr>
<tr>
<td>74</td>
<td>27.351</td>
<td>0.5973</td>
<td>5.65</td>
<td>5.65 (0.0%)</td>
<td>60.55</td>
<td>61.33 (1.3%)</td>
</tr>
<tr>
<td>84</td>
<td>24.095</td>
<td>0.5245</td>
<td>6.48</td>
<td>5.98 (8.0%)</td>
<td>63.90</td>
<td>63.00 (1.4%)</td>
</tr>
</tbody>
</table>

6. Conclusions

As a conclusion, it can be said that the results obtained through the analytical and the numerical method are consistent with each other. It proves that the broken-line hypotheses assumed for deformation of the cross-section in \( x \) and \( y \) directions are sufficient for evaluating the plate deflection and equivalent stresses. It can also be seen that decreasing of the parameter \( b_{01} \) has much more influence on the increase of the maximum deflection of the plate. This effect is due to a significant change in the shear rigidity of the plate.

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References


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