

## EXPERIMENTAL INVESTIGATIONS OF ELASTIC-PLASTIC STRAIN STATES ON VARIOUS STAGES OF MATERIAL PLASTIFYING

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In the paper, the possibility of application of various experimental methods to the analysis of elastic-plastic states at different levels of material plastifying is presented. For tests carried out on two-dimensional elements with different stress concentrators and loaded by tensile stresses, three experimental methods have been selected: Moiré method, method of photoelastic coating and the thermography method. On the basis of the tests results, the range of the applicability of chosen methods and their suitability to the elastic-plastic strain analysis at the development of material plastifying has been determined. The strain components distribution obtained from the Moiré method and the method of photoelastic coating has been compared. The possibility of increasing the accuracy of strain determination for the Moiré method by additional tests at the grid rotated by an angle of  $45^\circ$  with respect to the direction of tensile stress has been shown. The results of the investigations have been discussed.

*Keywords:* mechanics of solids, experimental methods, elastic-plastic states

### 1. Introduction and the experimental method selection

The analysis of elastic-plastic states finds many basic applications in engineering design, especially nowadays, since the economical trend towards building more lightweight and cheaper structures accepts partial material plastifying during exploitation.

Experimental studies on a great variety of non-linear problems are conducted in many research centers all over the world. These problems, like non-linear material characteristics, partial material plastifying, large deformation, material with imperfections, etc, always cause difficulties associated with the modelling of constructional materials. In such circumstances, the widely nowadays applied numerical methods as FEM still need final experimental verification. Especially the experimental methods which give information about the real object without any model simplification can be very useful as a good verification tool of theoretical and numerical design.

The experimental methods most commonly used to research work on elastic-plastic problems are: photoelasticity, especially the method of photoelastic coating (Pacey *et al.*, 2005; Foust *et al.*, 2011; Lamberson *et al.*, 2012; Diaz *et al.*, 2010), Moiré methods (Min *et al.*, 2006; Livieri and Nicoletto, 2003; Guo *et al.*, 2006), holographic interferometry (Lin, 2000; Balalov *et al.*, 2007), electronic speckle pattern interferometry method (ESPI) (Diaz *et al.*, 2001; Schajer *et al.*, 2005), digital image correlation method (DIC) (Vural *et al.*, 2011; Diaz *et al.*, 2004; Tarigopula *et al.*, 2008), strain gauge technique (Rasty *et al.*, 2007; Olmi, 2010), thermography (Pieczyska *et al.*, 2006; Connesson *et al.*, 2011).

The selection of the experimental method to study plastic “in-plane” deformation depends on several elements: the ability and accuracy of the method, the ease of its use in practice, the character of the obtained results and the possibility of their work out, etc.

The greatest potential taking into account a variety of research techniques and a diversity of the analyzed problems, create the photoelasticity and Moiré method.

To study elastic-plastic states, much more suitable is the method of photoelastic coating than a traditional photoelasticity which requires the use of optically active materials having characteristics corresponding to material characteristics of the tested element also in the non-linear range.

Similarly, among various Moiré techniques – the best applicability to study elastic-plastic states has the classical geometric Moiré method. The frequently nowadays applied interferential Moiré method has a very high sensitivity (density of the grid is here of several thousand lines/mm) and is used to the analysis of small areas. Its additional disadvantage is the necessity of coherent (laser) light application.

Holographic interferometry, which bases on phenomena occurring during the coherent light interference, can be used directly for measurement of displacement (or shape) of the structure. It shows good accuracy (strain measurement with an accuracy of  $0.1 \cdot 10^{-4}$ – $1 \cdot 10^{-4}$ ), but has a high mechanical sensitivity, and the result analysis is labor-consuming. It requires also laser light.

Making use of coherent light requires also one of the modern experimental methods – electronic speckle pattern interferometry (ESPI). It has a lot of advantages – non-contact measurement, high sensitivity, resolution and accuracy; it gives surface images of displacement and strain components. However, ESPI method has also very serious limitations – high susceptibility on conditions in which experiment is performed, sensitivity to even slight movements. It is also not suitable for large deformations and requires the researcher to have high skills and a lot of experience.

The second modern experimental method, the digital image correlation method (DIC) also allows non-contact measurement and gives surface images of displacement and strain components. Compared to the ESPI method, it has a bigger measurement range, but lower resolution. The surface of the element needs special preparation and the results require a lot of calculations.

In contrast to modern experimental methods, one of the oldest commonly known but still most often used experimental technique is the strain gauge measurement. It enables one to get strain values with a very high accuracy ( $\sim 1 \cdot 10^{-6}$ ), but only in several points. That is why it is usually used in combination with other experimental methods, after determining the most loaded parts of a structure.

An auxiliary character has also usually the thermography method which is often used to detect material defects. Its accuracy is difficult to determine and depends mostly on temperature resolution of a thermovision camera and external conditions. However, the method allows one to observe thermal processes taking place in the material, what is a great advantage, especially concerning the elastic-plastic problems.

Taking into account advantages and disadvantages of various experimental methods and abilities of their application regarding to the elastic-plastic states analysis, the simplicity of their use in practice, equipment availability, etc., for further experimental testing, three methods have been selected: a method of photoelastic coating, Moiré method and the thermography method.

All the three methods give information about deformation of the whole tested area (not only at several points). They can be used to investigate real structure elements made of any material, in working conditions, even under heavy loading causing partial material plastifying. Their advantage is also excellent visualization of the process of progressive material plastifying. In particular, the methods of photoelastic coating and thermography allow direct observation of the process of formation and development of plastic zones, changes (expansion) of their boundaries and the direction of propagation.

The first two methods are optical methods, although each of them gives information about different physical quantities (method of photoelastic coating – strain, Moiré method – displacement). They are also comparable in terms of the level of accuracy (measurement or determination of strain with an accuracy  $\sim 1 \cdot 10^{-4}$ ). The third method – thermography is based on a quite

different physical phenomenon (emission of infrared radiation from the surface of the tested element), which allows, to a certain degree, verification of the results obtained from the first two methods.

The additional advantage of the photoelastic coating method is the possibility of direct strain measurement. The disadvantage is the dependence of the obtained results on the properties of the tested material (using the analytical method of strain separation).

The Moiré method has a purely geometrical character (measurement of the displacements is direct and does not depend on the properties of the tested material), determination of strain, however, requires differentiation of the obtained displacement values.

The photoelastic coating method and the Moiré method are methods very often used in worldwide research works, thus their choice seems to be the most evident and proper. Thermography, though less precise and giving results more qualitative than quantitative, can provide a useful and interesting complement to the first two methods, particularly for solving certain elastic-plastic problems.

## 2. Experimental testing

The experimental investigation of elastic-plastic states has been performed on two-dimensional models of structural elements weakened by different stress concentrators (holes) and subjected to tensile stresses. The elements of this type and loaded in such a way are often used in modern structures, particularly as different construction joints. The areas of the elements weakened by cut-outs are parts of structures which require special and accurate checking (Wung *et al.*, 2001; Olmi, 2010; Foust *et al.*, 2011). Shapes of stress concentrators have been designed on the basis of literature data and engineering practice (single central holes of various shapes and groups of circular holes of various configurations). The objects of discussion presented in the work are three of the models – Fig. 1. The first two models with one central hole have the same area of the most weakened cross-section in the  $x$  axis of symmetry (the effective cross-section) and differ only in shape of the hole. The third model is weakened by five circular holes cut symmetrically not only on the axis of symmetry  $x$ .

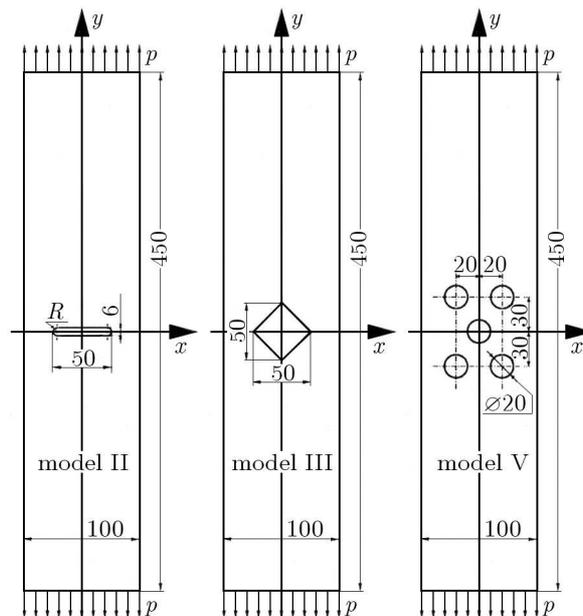


Fig. 1. Models of constructional elements

The models have been made of a duralumin sheet 3mm thick, from which stripes of 100 mm in width and 450 mm in length have been cut out. The length of the stripes was taken large enough to compensate potential non-uniformity of tensile stresses distribution applied at their ends.

The characteristic of the material (aluminum alloy EN-AW-2024) has been determined experimentally on the basis of a standard static uniaxial tensile test and it is shown in Fig. 2.

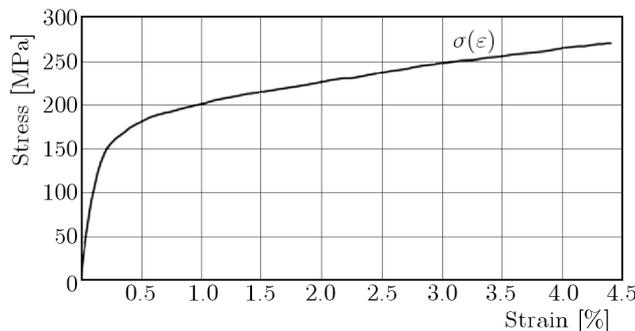


Fig. 2. Material characteristic

After mechanical working and special surface preparation, the models have been covered with: a cross-type grid of 20 lines/mm (for the Moiré method) with a 2 mm thick photoelastic coating of strain constant:  $f_\varepsilon = 1.114 \cdot 10^{-3}$  1/fringe order (for the photoelastic coating method) and with a layer of graphite (for the thermography method). Next, different holes have been cut out in the way which allowed avoiding creation of plastic strains as a result of machining.

The models have been loaded at their ends with uniformly distributed tensile stresses  $p$ . As the measure of the loading intensity, the ‘loading factor’  $s$  has been accepted. It has been calculated as the ratio of the average tensile stresses at the cross-section weakened by the hole on the axis of symmetry perpendicular to the stretching direction in relation to the offset yield strength  $R_{0.2} = 182$  MPa (taken from the material characteristic).

The loading of the models has been increased step by step within the over-elastic range of the material. At selected levels of loading, images of the Moiré pattern (for the displacement  $u(x, y)$  and  $v(x, y)$ ) and isochromatic pattern (for dark- and light-field polariscope) have been registered. For the thermography method, the loading of the models increased continuously and the temperature changes on the specimen surface have been recorded by a thermovision camera.

On the basis of the experimental data obtained from the Moiré method and the method of photoelastic coating, quantitative analysis of the elastic-plastic strain and stress around the stress concentrators has been made (Kozłowska, 2008, 2013). Due to low resolution of thermal images obtained from the infrared camera, the thermograms have given only qualitative information about deformation of the elements (Kozłowska, 2012).

### 3. Determination of the suitability of selected experimental methods to elastic-plastic analysis in dependence on the material plastifying level

The experiment has been performed within a wide range of the loading – from the moment of occurring first plastic deformations to the elements failure. But not at every level of loading all of the selected methods have been equally useful.

As proved by an experiment, photoelastic coating method allows analysis of the plastic strain starting from the beginning of their occurring in the material – level of about  $s \approx 0.5$  (tensile stress  $p \approx 45$  MPa) for the models with a single hole or  $s \approx 0.3$  (tensile stress  $p \approx 44$  MPa) for the model with five holes. At that loading level, the Moiré method is not very useful (to low

number of Moiré fringes). Exemplary images of isochromatic and Moiré patterns at the discussed loading level for the area around the stress concentrator of model III are shown in Fig. 3.

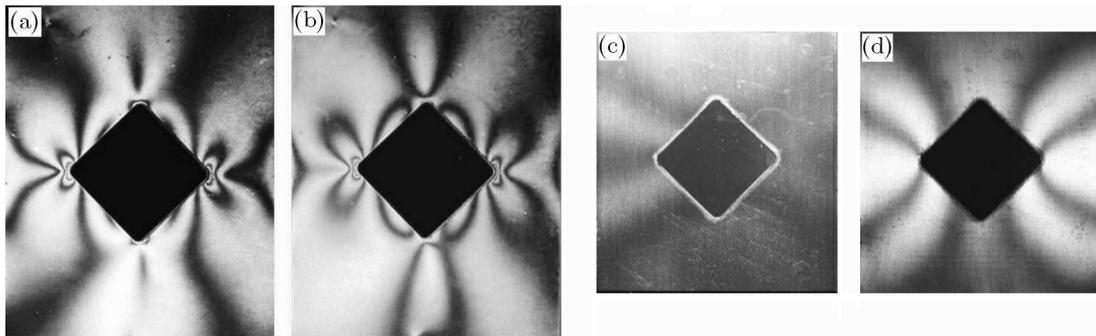


Fig. 3. Model III ( $s = 0.495$ ) – isochromatic pattern: (a) dark-field polariscope, (b) light-field polariscope; Moiré fringe pattern: (c)  $u(x, y)$  surface, (d)  $v(x, y)$  surface

For significant plastic deformation, for a loading level over  $s \approx 1$  (tensile stress  $p \approx 91$  MPa) for the models with a single hole or  $s \approx 0.8$  (tensile stress  $p \approx 116$  MPa) for the model with five holes, the method of photoelastic coating is no longer useful. It is so because the photoelastic coating can crack (Fig. 4), come off (Fig. 5) or (in the best case) isochromatic fringes in the most plastified areas become quite unreadable (Fig. 6). The Moiré method, however, at the same loading level or even higher, enables proper analysis of elastic-plastic states (Figs. 4-6).

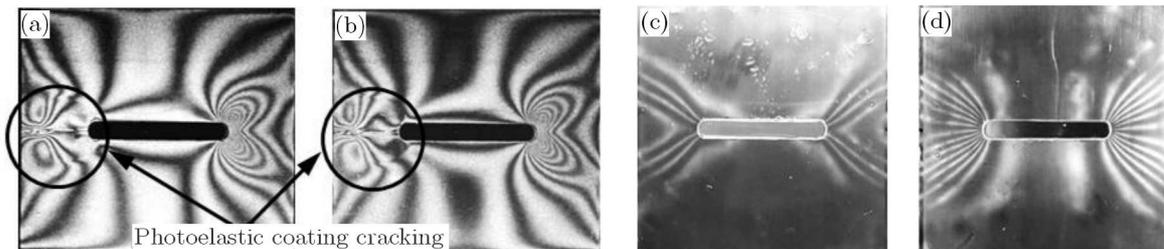


Fig. 4. Model II ( $s = 1.136$ ) – isochromatic pattern: (a) dark-field polariscope, (b) light-field polariscope; Moiré fringe pattern: (c)  $u(x, y)$  surface, (d)  $v(x, y)$  surface

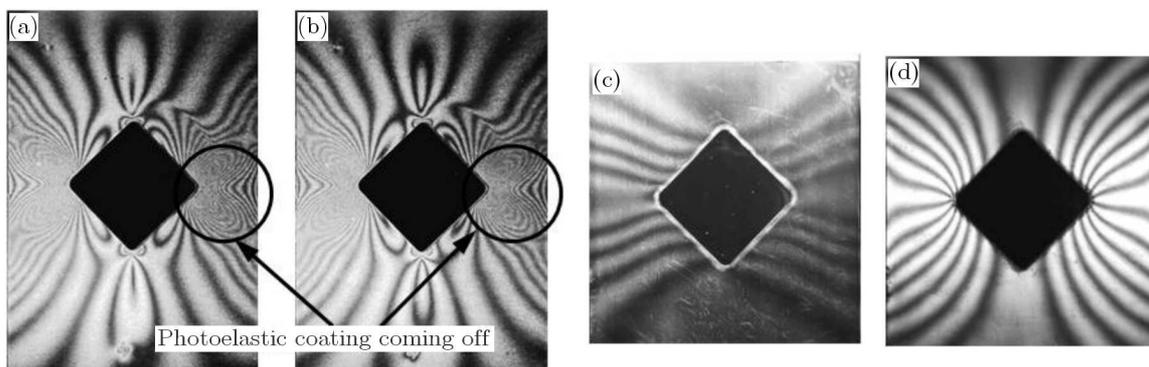


Fig. 5. Model III ( $s = 1.136$ ) – isochromatic pattern: (a) dark-field polariscope, (b) light-field polariscope; Moiré fringe pattern: (c)  $u(x, y)$  surface, (d)  $v(x, y)$  surface

The discussed examples show an approximate range of the application of the Moiré method and the method of photoelastic coating depending on the level of plastic deformation of the material. This range may be changed to some extent because the sensitivity of these two methods depends on the selection of the proper “measuring element”. Greater possibilities creates here

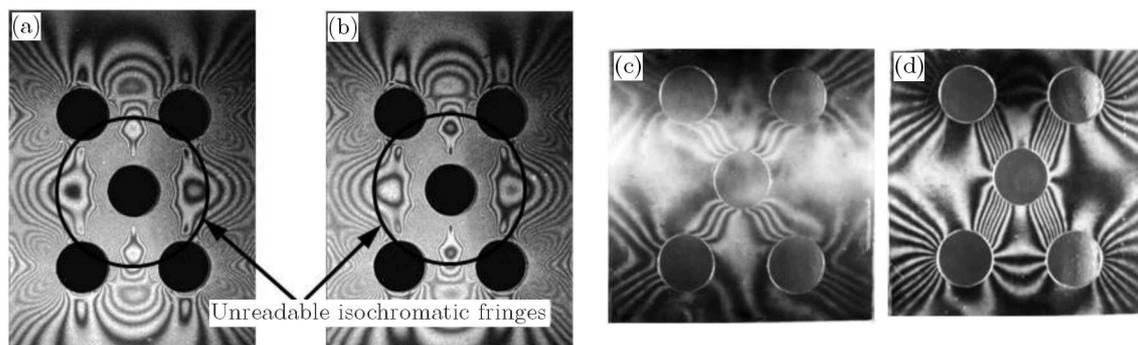


Fig. 6. Model V ( $s = 0.778$ ) – isochromatic pattern: (a) dark-field polariscope, (b) light-field polariscope; Moiré fringe pattern: (c)  $u(x, y)$  surface, (d)  $v(x, y)$  surface

the Moiré method by changing density of the grids – for the geometric Moiré method, the number of lines per millimeter may vary from a few to several tens (most commonly from 20 to 40).

The change in sensitivity of the method of photoelastic coating can be achieved by variation of thickness of an optically active layer, but in a much smaller range. Usually, the thickness of the applied photoelastic coating is 1 to 3 mm, although you can find one of 0.25 mm. A thicker layer causes stiffening of the element and introduces too much of measurement inaccuracy (averaging over the thickness). The upper limit of the capabilities of the photoelastic coating method is the layer cracking or its coming off the base at higher loading levels.

This disadvantage does not apply to the Moiré method, because even if the grid is imprinted to a photographic film and is affixed to the surface of the element, it forms a flexible thin layer, very strongly connected with the base. In the case of a grid applied directly to the surface of the element (e.g. etched), the problem of the grid coming off does not exist at all.

For the grids used in experimental testing (20 lines per millimeter), the range of measured strain was  $\sim 0.2\%$  to  $1.5\%$ , while for the photoelastic coating of 2 mm thick, the maximum determined plastic strain was up to  $\sim 0.9\%$ .

The thermal images obtained from infrared camera did not give sufficiently precise information about the temperature distribution on the surface of the tested element. The accuracy of temperature measurement by an infrared camera, however, depended mainly on its resolution, and that increases with the technical possibilities.

Even if the thermograms do not allow one to obtain the values of strain components, they show directly the full development of plastic zones, from the beginning of their creation to the failure of the element as e.g. in model III (Fig. 7)

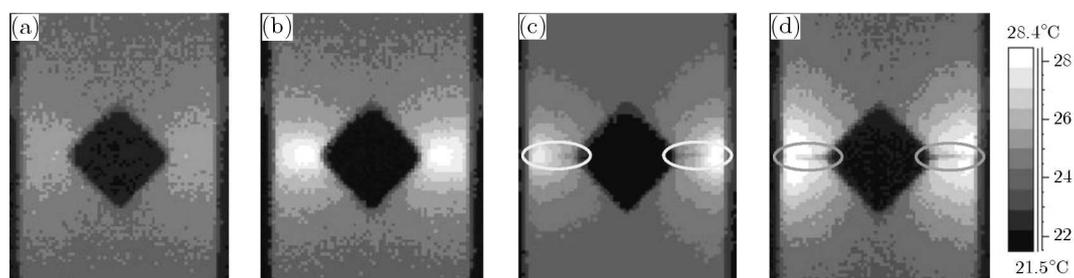


Fig. 7. Thermograms (model III) for loading levels: (a)  $s = 0.989$ , (b)  $s = 1.172$ , (c)  $s = 1.304$  (first cracks), (d)  $s > 1.355$  (element failure)

The method of thermography has also no limitations resulting from the properties of the layer covering model (graphite), as it is in the case of the photoelastic coating method (the optically active layer) or even in the Moiré method, when the grid is affixed to the surface of

the model. Thus, this method can be used, both as a preliminary tool to select an area of the element to be tested with a more accurate method (the formation of plastic zones) as well as a way to observe the mechanism of plastic material failure in the range already out of reach for other experimental methods.

#### 4. The accuracy of the determination of strain components by selected experimental methods

Although the Moiré method and the method of photoelastic coating have different ranges of the best suitability for quantitative analysis of the elastic-plastic strain and stress components, there is a certain range of loading level for which both methods can be properly used.

To compare the accuracy of the Moiré method and the method of photoelastic coating, the loading levels for which both methods give results freely allowing one to determine the strain components have been chosen. The analysis of model II and III has been carried out at the loading level  $s = 0.952$ , which corresponds to tensile stresses  $p = 87$  MPa. The average stress on the  $x$  axis was then  $\sigma_{av} = 173$  MPa (average strain  $-\varepsilon_{av} = 0.38\%$ ). For model V the loading level accepted for analysis, was  $s = 0.687$  (tensile stress  $p = 100$  MPa), for which the average stress on the axis of symmetry  $x$  – was  $\sigma_{av} = 125$  MPa (average strain  $-\varepsilon_{av} = 0.15\%$ ).

At the chosen loading levels, a quite significant plastification of the material already occurred around the stress concentrators, on the one hand large enough to enable measurement by the Moiré method, on the other hand, still allowing using the method of photoelastic coating.

The analysis of the elastic-plastic strain state for the models with one hole is shown on the example of model III, for which the images of isochromatic and Moiré patterns around the stress concentrator at the loading level  $s = 0.952$  are presented in Fig. 8.

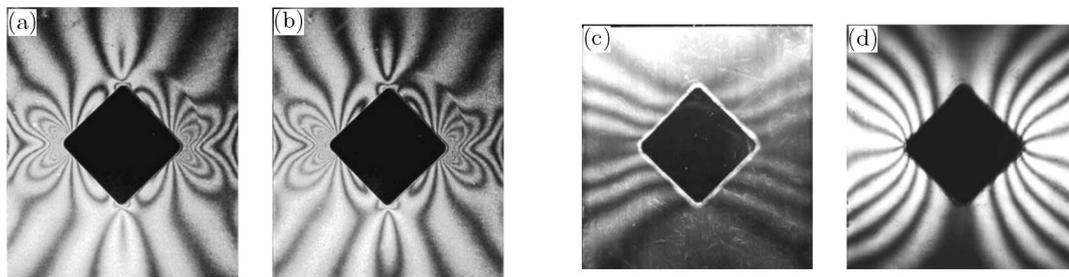


Fig. 8. Isochromatic pattern for model III –  $s = 0.952$ : (a) dark-field polariscope, (b) light-field polariscope; Moiré fringe pattern – (c)  $u(x, y)$  surface, (d)  $v(x, y)$  surface

To compare the results obtained from the Moiré method and the method of photoelastic coating, the strain components distribution on a horizontal axis of symmetry  $x$  perpendicular to the loading direction (segment  $AB$  – Fig. 1) have been assumed. In addition, on the same diagram it is also shown the distribution of strain  $\varepsilon_x$  and  $\varepsilon_y$  obtained from numerical (FEM) calculations (Fig. 9).

For model V (with five circular holes), the images of isochromatic and Moiré pattern around the stress concentrators at the loading level  $s = 0.687$  are shown in Fig. 10.

To compare the results obtained from the Moiré method and the method of photoelastic coating, the strain components distribution in the horizontal axis of symmetry  $x$  perpendicular to the loading direction (segment  $AB$  – Fig. 1) has been assumed (Fig. 11). For this model, FEM calculations have not been performed.

As follows from the presented diagrams, the strain components  $\varepsilon_x$  and  $\varepsilon_y$  distribution in the axis of symmetry  $x$  (segment  $AB$ ) for the chosen loading level obtained from the Moiré method and the method of photoelastic coating are approximate. The differences between the calculated

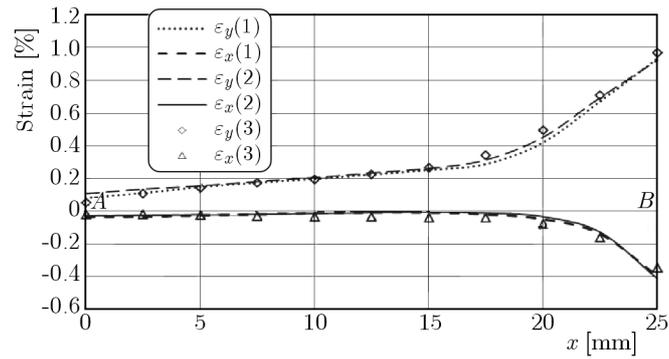


Fig. 9. Strain components distribution in the axis of symmetry  $x$  for model III –  $s = 0.952$ : (1) Moiré method, (2) method of photoelastic coating, (3) FEM calculations

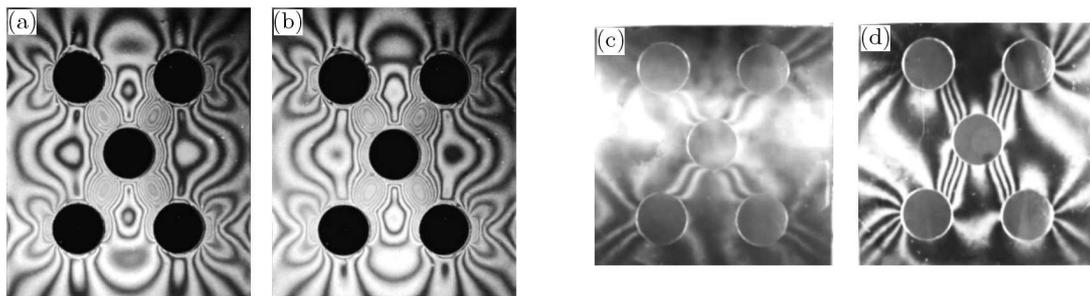


Fig. 10. Isochromatic pattern for model V –  $s = 0.687$ : (a) dark-field polariscope, (b) light-field polariscope; Moiré fringe pattern – (c)  $u(x, y)$  surface, (d)  $v(x, y)$  surface

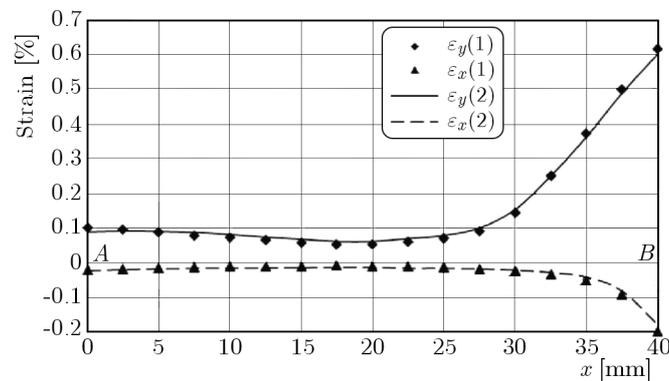


Fig. 11. Strain components distribution in the axis of symmetry  $x$  for model V –  $s = 0.687$ : (1) Moiré method, (2) method of photoelastic coating

values of strain components do not exceed a few percent (6% to 8%). A higher divergence occurs between the experimental results and numerical calculations, but even there it does not exceed 10% to 12%.

## 5. The influence of grid configuration on the accuracy of determination of strain components distribution

The strain components obtained on the basis of the Moiré fringes at the traditionally affixed grid (in accordance with the axes of symmetry of the model – the direction of tension) can be determined accurately not in every part of the tested model. Where the surfaces of deformation are not much diversified in the direction of the axis of the coordinate system (directions of differentiation), the derivatives can be calculated with a certain error.

As it has been said, the measurement sensitivity of the Moiré method may be increased by changing density of used grids. It is not always convenient, when the increasing of the accuracy is needed only in the part of the tested element. In such a case, an improvement of the measurement accuracy may be achieved by performing additional tests with grids rotated with respect to the direction of the basic grid by a certain angle.

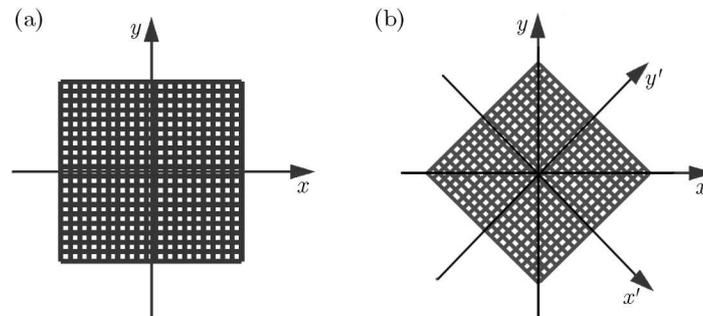


Fig. 12. The grid affixed according to the direction of tensile stresses (a), grid rotated by an angle of  $45^\circ$  (b)

In order to verify the possibility of increasing accuracy of determination of the elastic-plastic strain distribution around stress concentrators, additional tests have been performed at the grid rotated by an angle of  $45^\circ$  with respect to the direction of tensile stress (coordinate system  $x-y$ ) – Fig. 12.

An exemplary comparison of the strain components obtained by grids arranged in different ways is shown for model V with five holes at the loading level  $s = 0.778$ , for which, the strain state has been already determined using a traditionally affixed grid (Kozłowska, 2008).

The images of Moiré fringes at the grids affixed in different ways are shown in Fig. 13, where one can see a larger number of Moiré fringes in selected areas at the rotated grid than at the grid affixed in the direction of tensile stress.

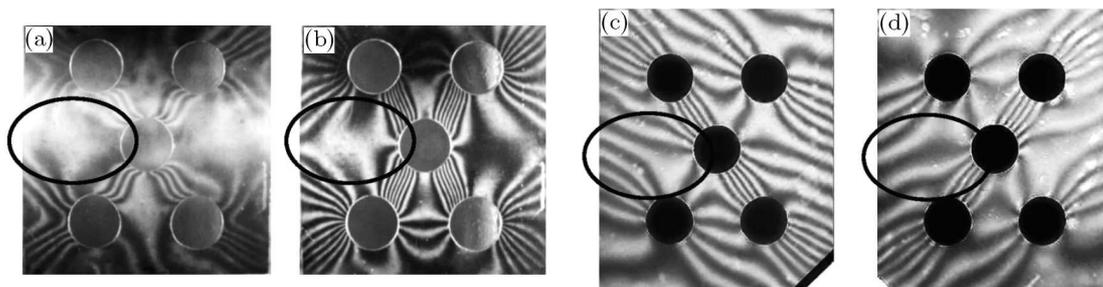


Fig. 13. Moiré pattern for model V ( $s = 0.778$ ) – grid affixed according to the direction of tension: (a)  $u(x, y)$ , (b)  $v(x, y)$ ; grid rotated by an angle of  $45^\circ$ : (c) clockwise, (d) counter-clockwise

The analysis for the rotated grid has been carried out as in previous cases (because of the double symmetry of the model and loading) for one-quarter of the tested area (Kozłowska, 2007) – Fig. 14.

For strain analysis, the coordinate system  $x-y$  associated with an element under tension and a traditionally affixed grid has been rotated by an angle of  $45^\circ$  to form a coordinate system  $x'-y'$  associated with a rotated grid (Fig. 12).

On the basis of the displacement obtained from Moiré fringes at the rotated grid, the strain components have been determined in the new coordinate system  $(x', y')$  by means of analytical differentiation.

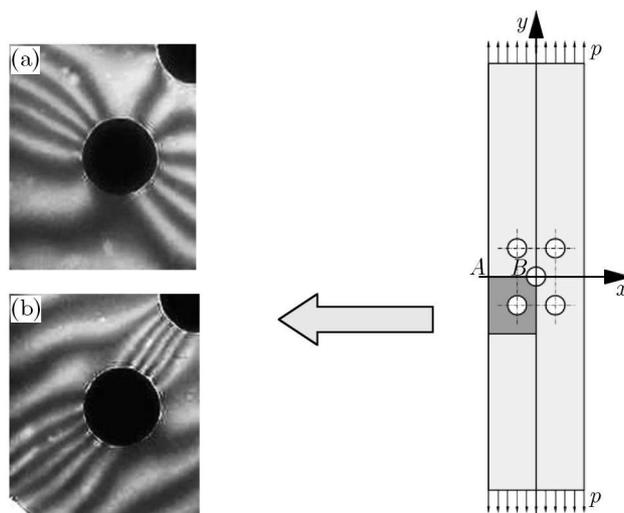


Fig. 14. Moiré pattern for the analyzed area of model V ( $s = 0.778$ ) – grid rotated by an angle of  $45^\circ$ :  
(a) clockwise, (b) counter-clockwise

Then the values of strain  $\varepsilon_x$  and  $\varepsilon_y$  in the basic coordinate system  $(x, y)$  have been calculated by making use of formulas enabling converting the strain state described in one coordinate system to another (rotated) one (1), where  $\alpha = 45^\circ$ , Fig. 15

$$\begin{aligned}
 \varepsilon_x &= \varepsilon_{x'} \cos^2 \alpha + \varepsilon_{y'} \sin^2 \alpha + \gamma_{x'y'} \sin \alpha \cos \alpha \\
 \varepsilon_y &= \varepsilon_{x'} \sin^2 \alpha + \varepsilon_{y'} \cos^2 \alpha - \gamma_{x'y'} \sin \alpha \cos \alpha \\
 \frac{1}{2} \gamma_{xy} &= (\varepsilon_{x'} - \varepsilon_{y'}) \sin \alpha \cos \alpha + \frac{1}{2} \gamma_{x'y'} (\sin^2 \alpha - \cos^2 \alpha)
 \end{aligned} \tag{5.1}$$

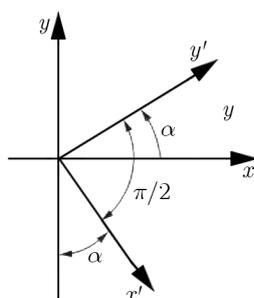


Fig. 15. Strain state transformation

To compare the obtained results with those from the analysis of Moiré images at the grid affixed according to the direction of the tensile stresses, the strain components distribution in the horizontal axis of symmetry  $x$  has been assumed (Fig. 16). In the diagram, the correction of strain components calculation resulting from the larger number of Moiré fringes (selected areas) is shown.

The analysis of strain components for model V (model with five holes) shows that in its horizontal axis of symmetry  $x$ , where the data obtained at the grid affixed traditionally are relatively inaccurate (low number of Moiré fringes), the information found from the rotated grid allows one to increase the accuracy of strain determination and to correct errors resulting from differentiation of surfaces of deformation.

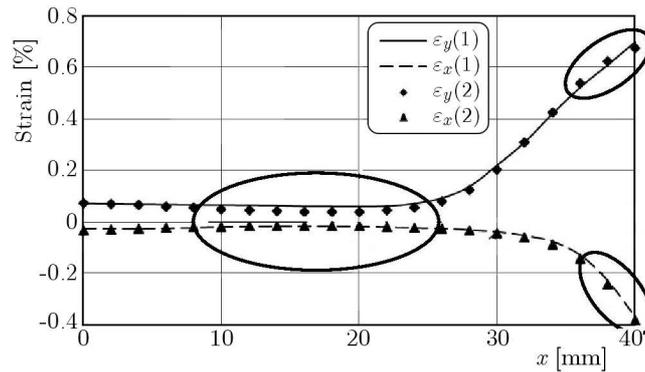


Fig. 16. Strain distribution in the axis of symmetry  $x$  (model V -  $s = 0.778$ ): (1) grid affixed according to the direction of tension, (2) grid rotated by an angle of  $45^\circ$

## 6. Conclusions

The review of experimental methods used in mechanics of solids carried out in terms of elastic-plastic analysis resulted in the selection of three of them for further investigations: the Moiré method, the method of photoelastic coating and the method of thermography. The choice has been dictated not only by their measuring capabilities in the over-elastic range of the material characteristics, but also by the simplicity of their use in practice and the availability of the equipment.

The main advantage of all three methods is the ability to conduct an experiment on real structures in working conditions, also under loading causing partial plastifying of the material and to obtain information about the strain state of the whole tested object. An additional advantage of the chosen methods (especially the method of photoelastic coating and the thermography method) is excellent visualization of the process of progressive material plastifying. The advantage of the Moiré method is also the simplicity of measurements and the work out of the experimental results.

The Moiré method and the method of photoelastic coating enable a relatively easy and quick quantitative analysis of the strain state around stress concentrators on the basis of experimental data. Thermographic tests have shown that this method allows rather getting a general view of the distribution of plastic strain components than their precise quantitative determination.

The conducted tests and detailed analysis of experimental data enabled definition of the range of applicability of each of the selected methods and determination of their capabilities in terms of the accuracy of calculation of strain components at various stages of material plastifying.

The range of application of the method of photoelastic coating (for an 2 mm thick optically active layer used in the testing of duralumin elements) is up to the maximum plastic strain  $\sim 0.9\%$ .

The Moiré method allows testing of the elements in a wider range of the material plastifying. For the used grids of 20 lines per millimeter, plastic strain can be measured up to  $\sim 1.5\%$ , while determination of the strain less than  $\sim 0.2\%$  causes difficulties due to the low number of Moiré fringes. The measurement sensitivity of the Moiré method can be locally increased by affixing the grids at different angles. Such a possibility has been verified by additional tests performed at the grid rotated by an angle of  $45^\circ$  to the axis of symmetry of the model. That gave an effect similar to applying the rosette of strain gauges and showed that the accuracy of the elastic-plastic strain distribution around stress concentrators could be increased in the areas where the number of Moiré fringes is low.

The comparison of the strain components distribution in the horizontal axis of symmetry  $x$  (perpendicular to the direction of tension) obtained from the Moiré method and the method of

photoelastic coating for the range of material plastifying, for which both of them are applicable, shows that the differences between results are about a few percent. The comparison with numerical calculations (FEM) also shows good agreement of the results.

The quantitative analysis of strain and stress components in the whole area around stress concentrators proves that the Moiré method is a little more useful. The method of photoelastic coating is more labor-consuming due to the necessity of analytical strain separation (solving of the system of partial differential equations, Kozłowska, 2013) and converting the obtained results from irregular grid nodes to the rectangular grid.

The thermography method, although not enough accurate for quantitative strain analysis, gives an opportunity of observing plastic zones developing in elements in the full range of loading until their complete failure, so it seems to be useful for the study of elastic-plastic states in cooperation with other experimental methods (e.g., Moiré method and the method of photoelastic coating).

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