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# EFFECT OF INITIAL STRESS AND GRAVITY FIELD ON MICROPOLAR THERMOELASTIC SOLID WITH MICROTEMPERATURES

Mohamed I.A. Othman
*Zagazig University, Department of Mathematics, Faculty of Science, Zagazig, Egypt*
*Taif University, Department of Mathematics, Faculty of Science, Taif, Saudi Arabia*
*e-mail: m_i_a_othman@yahoo.com*

Ramadan S. Tantawi
*Zagazig University, Department of Mathematics, Faculty of Science, Zagazig, Egypt*
*e-mail: ramadan_tantawi1@yahoo.com*

Mohamed I.M. Hilal
*Zagazig University, Department of Mathematics, Faculty of Science, Zagazig, Egypt*
*Sinai University, Department of Basic Sciences, Faculty of Engineering Sciences, El-Arish, Egypt*
*e-mail: mimhilal@yahoo.com*

The purpose of the present article is the study of the effect of the gravity field on an initially stressed micropolar thermoelastic medium with microtemperatures. The analytical method used to obtain the formula of the physical quantities is the normal mode analysis. The comparisons are established graphically in the presence and the absence of gravity, initial stress and micropolar thermoelasticity. The main conclusions state that the gravity, initial stress and the micropolar thermoelasticity are effective physical operators on the variation of the physical quantities. The microtemperatures are very useful theory in the field of geophysics and earthquake engineering.

*Keywords:* gravity, initial stress, micropolar thermoelasticity, microtemperatures

## 1. Introduction

The theory of elastic micropolar materials was introduced by Eringen (1966). The theory of continuum micropolar mechanics takes into consideration the microstructure of materials. Description of the micropolar materials is useful for fibrous, lattice or, in general, materials having microstructural construction having in each point extra rotational degrees of freedom independent of translation. The material, however, can transmit couple stress. Smith (1967) studied wave propagation in micropolar elastic solids. Parfitt and Eringen (1971) investigated reflection of plane waves from a flat boundary of a micropolar elastic half-space. Ariman (1972) also studied wave propagation in a micropolar elastic half-space solid. Eringen (1999) presented the microcontinuum field theory. Kumar and Ailawalia (2005) studied the response of a micropolar cubic crystal due to various sources. Kumar and Gupta (2010) studied propagation of waves in a transversely isotropic micropolar generalized thermoelastic half-space. Abbas and Kumar (2013) studied deformation due to a thermal source in micropolar thermoelastic media with the two-temperature effect. Recently, Othman *et al.* (2014) established the effect of rotation on a micropolar thermo-elastic solid with two temperatures. Abouelregal and Zenkour (2015) studied a thermoelastic problem of an axially moving micro beam subjected to an external transverse excitation. The concept of microtemperatures means that microelements of a thermoelastic body have different temperatures and depend homogeneously on microcoordinates of the microelements, which are based on the microstructure of the continuum. Grot (1969) established the thermodynamic theory of elastic materials with inner structures, in which microdeformations and particles possess microtemperatures. Eringen and Kafadar (1976) presented the basis for the microelements with microtemperatures. Riha (1979) presented a study of heat conduction in materials with inner structures. Iesan and Quintanilla (2000) constructed the linear theory of thermoelasticity for materials with inner structure whose particles, in addition to the classical displacement and temperature fields, possess microtemperatures. Iesan (2001, 2004) presented the mathematical model of theory of micromorphic elastic solids with microtemperatures, in which microelements possess microtemperatures and can stretch and contract independently of their translations. Casas and Quintanilla (2005) studied exponential stability in thermoelasticity with microtemperatures. Scalia and Svanadze (2006) discussed solutions of the theory of thermoelasticity with microtemperatures. Iesan (2006, 2007) presented a study of thermoelastic bodies with a microstructure and microtemperatures.

The effect of gravity on wave propagation in an elastic medium was first considered by Bromwich (1898) who treated the force of gravity as a type of a body force. Love (1965) extended the work of Bromwich investigating the influence of gravity on superficial waves and showed that the Rayleigh wave velocity is affected by the gravitational field. Sezawa (1927) studied dispersion of elastic waves propagating on curved surfaces. Othman *et al.* (2013a,b) investigated two models on the effect of the gravitational field on thermoelastic solids. The presence of initial stresses in solid materials has a substantial effect on their subsequent response to applied loads that is very different from the corresponding response in the absence of initial stresses. In geophysics, as an example, high stress developed below the Earth's surface due to gravity has a strong influence on the propagation speed of elastic waves. While in soft biological tissues initial (or residual), stresses in artery walls ensure that the circumferential stress distribution through thickness of the artery wall is close to uniform at typical physiological blood pressures. Initial stresses may arise, for example, from applying loads, as in the case of gravity, processes of growth and development in living tissue or, in the case of engineering components, from the manufacturing process. Ames and Straughan (1999) derived continuous dependence results for initially pre-stressed thermoelastic bodies. Montanaro (1999) investigated isotropic linear thermoelasticity with hydrostatic initial stress. Wang and Slattery (2002) formulated thermoelastic equations without energy dissipation for initially stressed bodies. Iesan (2008) presented a theory of Cosserat thermoelastic solids with initial stresses. Recently, Othman *et al.* (2015) discussed the effect of initial stress on a thermoelastic rotating medium with laser pulse heating.

This investigation studies the 2D problem of linear, isotropic, homogeneous initially stressed micropolar thermoelastic solid influenced by the gravity field. The application of the present model cannot be ignored in geophysics and earthquake engineering due to the importance of the microtemperature properties. The normal mode analysis is the analytical method used to obtain the solutions of the considered physical quantities which are graphically represented in the absence and presence of the studied physical effects.

## 2. Basic equations

Consider the linear theory of thermodynamics for isotropic elastic materials with inner structure. According to Eringen (1999), Isean (2007) and Montanaro (1999), the field equations and the constitutive relations for a linear, homogeneous, isotropic initially stressed micropolar thermoelastic solid with microtemperatures without body forces, body couples, heat sources and first heat source moment, can be considered as

$$
\begin{aligned}
&\frac{\partial\sigma_{ij}}{\partial x_i}=\rho\frac{\partial^2 u_j}{\partial t^2} \qquad \frac{\partial m_{ij}}{\partial x_i}+\varepsilon_{ijr}\sigma_{ir}-\mu_1\frac{\partial w_j}{\partial x_i}=J\rho\frac{\partial^2\phi_j}{\partial t^2}\\
&k_6\frac{\partial^2 w_i}{\partial x_j^2}+(k_4+k_5)\frac{\partial^2 w_j}{\partial x_i\partial x_j}+\mu_1\frac{\partial}{\partial t}\frac{\partial\phi_j}{\partial x_i}-k_2w_i-b\frac{\partial w_i}{\partial t}-k_3\frac{\partial T}{\partial x_i}=0\\
&k\frac{\partial^2 T}{\partial x_i^2}-\rho C_e\frac{\partial T}{\partial t}-\gamma_1T_0\frac{\partial}{\partial t}\frac{\partial u_i}{\partial x_i}+k_1\frac{\partial w_i}{\partial x_i}=0
\end{aligned}
\tag{2.1}
$$

and

$$
\begin{aligned}
&\sigma_{ij}=\lambda\frac{\partial u_r}{\partial x_r}\delta_{ij}+\mu\Big(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\Big)+k^*\Big(\frac{\partial u_i}{\partial x_j}-\varepsilon_{ijr}\phi_r\Big)-\gamma_1T\delta_{ij}-p(\delta_{ij}+\omega_{ij})\\
&m_{ij}=\alpha\frac{\partial\phi_r}{\partial x_r}\delta_{ij}+\beta\frac{\partial\phi_i}{\partial x_j}+\gamma\frac{\partial\phi_j}{\partial x_i} \qquad q_i=k\frac{\partial T}{\partial x_i}+k_1w_i\\
&q_{ij}=-k_4\frac{\partial w_k}{\partial x_k}\delta_{ij}-k_5\frac{\partial w_i}{\partial x_j}-k_6\frac{\partial w_j}{\partial x_i} \qquad Q_i=(k_1-k_2)w_i+(k-k_3)\frac{\partial T}{\partial x_i}\\
&e_{ij}=\frac{1}{2}\Big(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\Big) \qquad \omega_{ij}=\frac{1}{2}\Big(\frac{\partial u_j}{\partial x_i}-\frac{\partial u_i}{\partial x_j}\Big)
\end{aligned}
\tag{2.2}
$$

where $\lambda$ and $\mu$ are Lamé constants, $\alpha$, $\beta$, $\gamma$, and $k^*$ are micropolar constants, $\gamma_1=(3\lambda+2\mu+k^*)\alpha_t$, while $\alpha_t$ is the linear thermal expansion coefficient, $\rho$ is density, $C_e$ – specific heat, $k$ – thermal conductivity, $u_i$ – displacement vector, $T$ – absolute temperature, $T_0$ – reference temperature chosen so that $|(T-T_0)/T_0|\ll 1$, $\phi_j$ is the microrotation vector, $\sigma_{ij}$ are components of stresses, $e_{ij}$ are components of strains, $\delta_{ij}$ is the Kronecker delta, $\varepsilon_{ijr}$ is the permutation symbol, $p$ – pressure, $m_{ij}$ are couple stresses, $J$ is microinertia, $w_i$ – microtemperature vector, $\mu_1$, $b$, $k_i$ $(i=1,2,\dots,6)$ are constitutive coefficients, $q_i$ is heat flux moment, $q_{ij}$ – first heat flux moment and $Q_i$ is the mean heat flux vector.

## 3. Formulation and solution of the problem

Consider an isotropic, linear, homogeneous, initially stressed micropolar thermoelastic solid with microtemperatures. Consider also a half-space $(y\geqslant 0)$ and the rectangular Cartesian coordinate system $(x,y,z)$ originated in the surface $z=0$. For a two-dimensional problem, assume the dynamic displacement vector as $u_i=(u,v,0)$. The microrotation vector $\phi_j$ will be $\phi_j=(0,0,\phi_3)$, consequently the microtemperature vector $w_i$ willbe $w_i=(w_1,w_2,0)$. All quantities will be a function of the time variable $t$ and coordinates $x$ and $y$. In the equations, comma denotes derivatives with respect to coordinates system.

Equations (2.1) under the effect of the gravitational field can be stated as

$$
\begin{aligned}
&\Big(\mu+k^*-\frac{p}{2}\Big)\nabla^2u+\Big(\lambda+\mu+\frac{p}{2}\Big)\frac{\partial e}{\partial x}+k^*\frac{\partial\phi_3}{\partial y}-\gamma_1\frac{\partial T}{\partial x}+\rho g\frac{\partial v}{\partial x}=\rho\frac{\partial^2 u}{\partial t^2}\\
&\Big(\mu+k^*-\frac{p}{2}\Big)\nabla^2v+\Big(\lambda+\mu+\frac{p}{2}\Big)\frac{\partial e}{\partial y}-k^*\frac{\partial\phi_3}{\partial x}-\gamma_1\frac{\partial T}{\partial y}-\rho g\frac{\partial u}{\partial x}=\rho\frac{\partial^2 v}{\partial t^2}\\
&\gamma\nabla^2\phi_3-2k^*\phi_3+(k^*-p)\Big(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\Big)-\mu_1\Big(\frac{\partial w_2}{\partial x}-\frac{\partial w_1}{\partial y}\Big)=J\rho\frac{\partial^2\phi_3}{\partial t^2}\\
&k_6\nabla^2w_1+(k_4+k_5)\frac{\partial}{\partial x}\Big(\frac{\partial w_1}{\partial x}+\frac{\partial w_2}{\partial y}\Big)+\mu_1\frac{\partial}{\partial t}\frac{\partial\phi_3}{\partial y}-k_2w_1-b\frac{\partial w_1}{\partial t}-k_3\frac{\partial T}{\partial x}=0\\
&k_6\nabla^2w_2+(k_4+k_5)\frac{\partial}{\partial y}\Big(\frac{\partial w_1}{\partial x}+\frac{\partial w_2}{\partial y}\Big)-\mu_1\frac{\partial}{\partial t}\frac{\partial\phi_3}{\partial x}-k_2w_2-b\frac{\partial w_2}{\partial t}-k_3\frac{\partial T}{\partial y}=0\\
&k\nabla^2T-\rho C_e\frac{\partial T}{\partial t}-\gamma_1T_0\frac{\partial e}{\partial t}+k_1\Big(\frac{\partial w_1}{\partial x}+\frac{\partial w_2}{\partial y}\Big)=0
\end{aligned}
\tag{3.1}
$$

where $g$ is the acceleration of gravity and $e$ is dilatation.

Define non-dimensional variables by expressions

$$
\begin{aligned}
&x'_i=\frac{\omega_1^*}{c_0}x_i && u'_i=\frac{\rho c_0\omega_1^*}{\gamma_1 T_0}u_i && \phi'_3=\frac{\rho c_0^2}{\gamma_1 T_0}\phi_3\\
&w'_i=\frac{c_0}{\omega_1^*}w_i && m'_{ij}=\frac{\omega_1^*}{\gamma_1 c_0 T_0}m_{ij} && q'_{ij}=\frac{\mu c_0^2}{\omega_1^*}q_{ij}\\
&(T',p'_2)=\frac{1}{T_0}(T,p_2) && (\sigma'_{ij},p'_1)=\frac{1}{\gamma_1 T_0}(\sigma_{ij},p_1) && t'=\omega_1^* t\\
&g'=\frac{g}{c_0\omega_1^*} && \omega_1^*=\frac{\rho C_e c_0^2}{k} && c_0^2=\frac{\lambda+2\mu+k^*}{\rho}
\end{aligned}
\tag{3.2}
$$

Assuming the potential functions $\psi_1(x,y,t)$, $\psi_2(x,y,t)$, $q_1(x,y,t)$ and $q_2(x,y,t)$ in dimensionless form, we have

$$
u=\frac{\partial\psi_1}{\partial x}+\frac{\partial\psi_2}{\partial y}\qquad v=\frac{\partial\psi_1}{\partial y}-\frac{\partial\psi_2}{\partial x}\qquad w_1=\frac{\partial q_1}{\partial x}+\frac{\partial q_2}{\partial y}\qquad w_2=\frac{\partial q_1}{\partial y}-\frac{\partial q_2}{\partial x}
\tag{3.3}
$$

To get the solution for the physical quantities, consider it in form of the normal mode as

$$
[\psi_1,\psi_2,\phi_3,q_1,q_2,T](x,y,t)=[\psi_1^*,\psi_2^*,\phi_3^*,q_1^*,q_2^*,T^*](y)\mathrm{e}^{\mathrm{i}(ax-\xi t)}
\tag{3.4}
$$

where $[\psi_1^*,\psi_2^*,\phi_3^*,q_1^*,q_2^*,T^*](y)$ are amplitudes of the physical quantities, $\xi$ is the angular frequency, $\mathrm{i}=\sqrt{-1}$ and $a$ is the wave number in the $x$ direction.

Apply equations (3.2)-(3.4) into equations (3.1) and drop the prime to obtain

$$
\begin{aligned}
&[\mathrm{D}^2-N_3]\psi_1^*-N_4\psi_2^*-N_5T^*=0 \qquad N_6\psi_1^*+[\mathrm{D}^2-N_7]\psi_2^*+c_2\phi_3^*=0\\
&-c_6[\mathrm{D}^2-a^2]\psi_2^*+[\mathrm{D}^2-N_8]\phi_3^*+c_7[\mathrm{D}^2-a^2]q_2^*=0\\
&[\mathrm{D}^2-N_9]q_1^*-N_{10}T^*=0 \qquad N_{11}\phi_3^*+[\mathrm{D}^2-N_{12}]q_2^*=0\\
&N_{14}[\mathrm{D}^2-a^2]\psi_1^*+c_{16}[\mathrm{D}^2-a^2]q_1^*+[\mathrm{D}^2-N_{13}]T^*=0
\end{aligned}
\tag{3.5}
$$

where $\mathrm{D}=d/dy$. All the constants are given in Appendix B.

Eliminating $\psi_1^*$, $\psi_2^*$, $\phi_3^*$, $q_1^*$, $q_2^*$ and $T^*$ from equations (3.5), enables one to obtain the following differential equations

$$
[\mathrm{D}^{12}-\lambda_1\mathrm{D}^{10}+\lambda_2\mathrm{D}^8-\lambda_3\mathrm{D}^6+\lambda_4\mathrm{D}^4-\lambda_5\mathrm{D}^2+\lambda_6]\{\psi_1^*(y),\psi_2^*(y),\phi_3^*(y),q_1^*(y),q_2^*(y),T^*(y)\}=0
\tag{3.6}
$$

where $\lambda_n$ $(n=1,2,\ldots,6)$ are constants.

Equation (3.6) can be factored as

$$
\begin{aligned}
&[(\mathrm{D}^2-S_1^2)(\mathrm{D}^2-S_2^2)(\mathrm{D}^2-S_3^2)(\mathrm{D}^2-S_4^2)(\mathrm{D}^2-S_5^2)(\mathrm{D}^2-S_6^2)]\\
&\quad\cdot\{\psi_1^*(y),\psi_2^*(y),\phi_3^*(y),q_1^*(y),q_2^*(y),T^*(y)\}=0
\end{aligned}
\tag{3.7}
$$

where $S_n^2$ $(n=1,2,\ldots,6)$ are the roots of the characteristic equation of (3.7).

The general solution to equation (3.7) bounded at $y\to\infty$ is given by

$$
\begin{aligned}
&u(x,y,t)=\sum_{n=1}^{6}G_{1n}R_n\mathrm{e}^{-S_ny+\mathrm{i}(ax-\xi t)} && v(x,y,t)=\sum_{n=1}^{6}G_{2n}R_n\mathrm{e}^{-S_ny+\mathrm{i}(ax-\xi t)}\\
&\phi_3(x,y,t)=\sum_{n=1}^{6}A_{2n}R_n\mathrm{e}^{-S_ny+\mathrm{i}(ax-\xi t)} && T(x,y,t)=\sum_{n=1}^{6}A_{5n}R_n\mathrm{e}^{-S_ny+\mathrm{i}(ax-\xi t)}\\
&m_{xz}(x,y,t)=\sum_{n=1}^{6}G_{9n}R_n\mathrm{e}^{-S_ny+\mathrm{i}(ax-\xi t)} && w_2(x,y,t)=\sum_{n=1}^{6}G_{4n}R_n\mathrm{e}^{-S_ny+\mathrm{i}(ax-\xi t)}\\
&\sigma_{xy}(x,y,t)=\sum_{n=1}^{6}G_{7n}R_n\mathrm{e}^{-S_ny+\mathrm{i}(ax-\xi t)} && q_{xy}(x,y,t)=\sum_{n=1}^{6}G_{12n}R_n\mathrm{e}^{-S_ny+\mathrm{i}(ax-\xi t)}
\end{aligned}
\tag{3.8}
$$

Here $R_n$ $(n=1,2,\ldots,6)$ are some coefficients. The other field quantities are given in Appendix A.

## 4. Applications

Consider the following non-dimensional boundary conditions to determine the coefficients $R_n$ $(n = 1, 2, \ldots, 6)$ and neglect the positive exponentials to avoid unbounded solutions at infinity. The surface of the medium satisfies the following conditions $y = 0$:

- The mechanical boundary conditions are
  - normal stress condition (mechanically stressed by the constant force $p_1$), so that

$$\sigma_{yy} = -p_1 \mathrm{e}^{\mathrm{i}(ax-\xi t)} - p \tag{4.1}$$

  - tangential stress condition (stress free)

$$\sigma_{xy} = 0 \tag{4.2}$$

- Condition of couple stress (couple stress is constant in the $y$-direction) implying that

$$m_{xz} = 0 \tag{4.3}$$

- Thermal condition (half-space subjected to thermal shock with constant temperature $p_2$ applied to the boundary) leading to

$$T = p_2 \mathrm{e}^{\mathrm{i}(ax-\xi t)} \tag{4.4}$$

- Normal and tangential heat flux moments are free, so that

$$q_{yy} = q_{xy} = 0 \tag{4.5}$$

Substituting the expressions of the considered quantities into boundary conditions (4.1)-(4.5), one obtains equations satisfied by the coefficients $R_n$ $(n = 1, 2, \ldots, 6)$. Applying the inverse of matrix method to the raised system of equations, one finds values of the coefficients $R_n$ $(n = 1, 2, \ldots, 6)$ as

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \end{bmatrix} = \begin{bmatrix} G_{61} & G_{62} & G_{63} & G_{64} & G_{65} & G_{66} \\ G_{71} & G_{72} & G_{73} & G_{74} & G_{75} & G_{76} \\ G_{91} & G_{92} & G_{93} & G_{94} & G_{95} & G_{96} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} \\ G_{111} & G_{112} & G_{113} & G_{114} & G_{115} & G_{116} \\ G_{121} & G_{122} & G_{123} & G_{124} & G_{125} & G_{126} \end{bmatrix}^{-1} \begin{bmatrix} -p_1 \\ 0 \\ 0 \\ p_2 \\ 0 \\ 0 \end{bmatrix} \tag{4.6}$$

Thus, we obtain expressions for the physical quantities of the plate surface.

## 5. Particular cases

In the present study, we consider the following particular cases:

(i) Absence of gravity by taking $g = 0$ in equations (4.1) and (4.2).

(ii) Non-initial stress effect by taking $p = 0$ in equation (4.5).

(iii) Absence of micropolar by taking $\alpha$, $\beta$, $\gamma$, $k^*$ and $j = 0$ in equations (4.1)-(4.5).

## 6. Numerical results and discussion

In order to illustrate the obtained theoretical results in the preceding Section, according to Eringen (1984), the magnesium crystal-like thermoelastic micropolar material has been chosen for the purpose of calculations. The used parameters are given in SI units. The constants are taken as $\lambda = 9.4 \cdot 10^{10}\,\mathrm{N/m^2}$, $\mu = 4 \cdot 10^{10}\,\mathrm{N/m^2}$, $k = 1.7 \cdot 10^2\,\mathrm{N/(s\,K)}$, $\rho = 1.74 \cdot 10^3\,\mathrm{kg/m^3}$, $\alpha_t = 7.4033 \cdot 10^{-7}\,\mathrm{/K}$, $C_e = 1.04 \cdot 10^3\,\mathrm{J/(kg\,K)}$, $k* = 1 \cdot 10^{10}\,\mathrm{N/m^2}$, $\gamma = 7.779 \cdot 10^{-8}\,\mathrm{N}$, $J = 2 \cdot 10^{-20}\,\mathrm{m^2}$, $T_0 = 298\,\mathrm{K}$, $k_1 = 0.0035\,\mathrm{N/s}$, $k_2 = 0.0045\,\mathrm{N/s}$, $k_3 = 0.0055\,\mathrm{N/(s\,K)}$, $k_4 = 0.065\,\mathrm{N/(s\,m^2)}$, $k_5 = 0.076\,\mathrm{N/(s\,m^2)}$, $k_6 = 0.096\,\mathrm{N/(s\,m^2)}$, $\mu_1 = 0.0085\,\mathrm{N}$, $b = 0.15 \cdot 10^{-9}\,\mathrm{N}$, $p_1 = 1\,\mathrm{N/m^2}$, $p_2 = 2\,\mathrm{K}$, $a = 1.5\,\mathrm{m}$, $t = 0.5\,\mathrm{s}$, $\xi = \eta + i\eta_1$, $\eta = 0.9\,\mathrm{rad/s}$, $\eta_1 = 2.9\,\mathrm{rad/s}$, $x = 0.5\,\mathrm{m}$, $0 \leqslant y \leqslant 6\,\mathrm{m}$.

The variation of real parts of each displacement $v$, microtemperature vector $w_2$, temperature $T$, stress $\sigma_{xy}$, couple stress $m_{xz}$, microrotation $\phi_3$ and the first heat flux moment $q_{xy}$ are obtained and represented by the distance $y$.

Figures 1-3 represent the behavior of these physical quantities against the distance $y$ in 2D when $p = 5\,\mathrm{N/m}$ and $g = 9.8\,\mathrm{m/s^2}$. Figures 4a and 4b show the behavior of these physical quantities against the distance $y$ in 2D for $g = 9.8\,\mathrm{m/s^2}$ in the case of $p = 5\,\mathrm{N/m}$. Figures 5a and 5b depict the variation of these physical quantities against the distance $y$ in 2D in the case of presence and absence of micropolar thermoelasticity when the gravity and the initial stress are present.

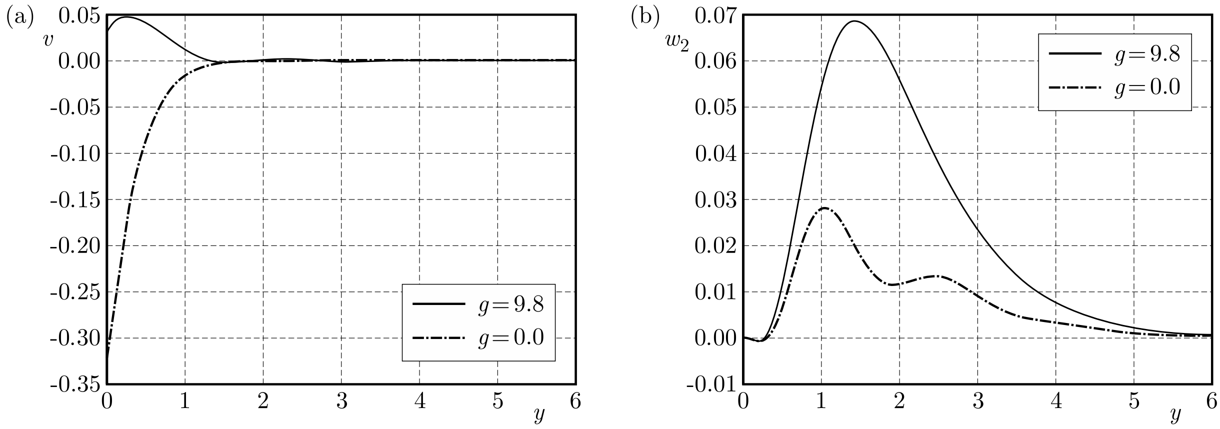


Fig. 1. Variation of displacement $v$ (a) and of microtemperature vector $w_2$ (b) against $y$

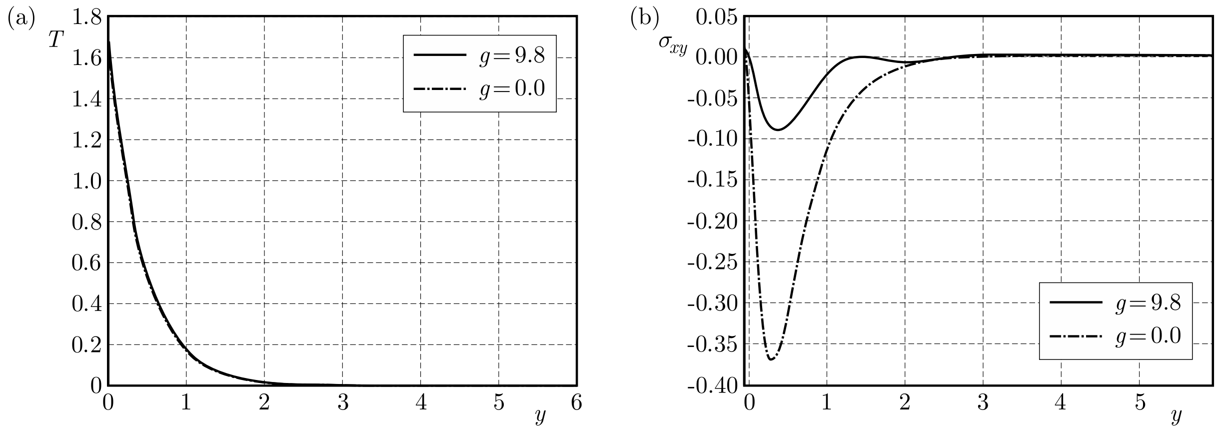


Fig. 2. Variation of temperature $T$ (a) and of stress $\sigma_{xy}$ (b) against $y$

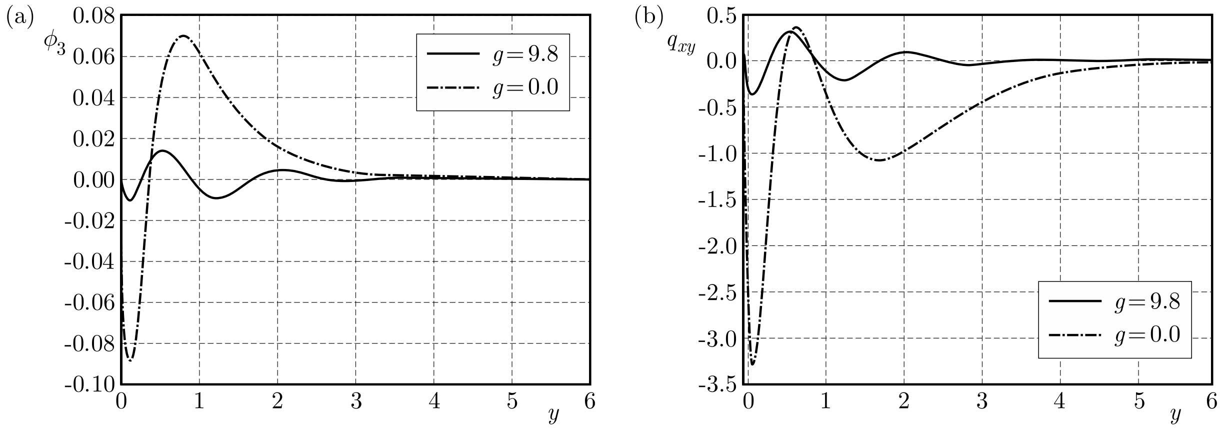


Fig. 3. Variation of microrotation vector $\phi_3$ (a) and of the first heat flux moment $q_{xy}$ (b) against $y$

Figure 1a shows that the variation of the displacement component $v$ increases with an increase in gravity for $y \geqslant 0$. Figure 1b clarifies the variation of the microtemperature vector $w_2$ which decreases with an increase in gravity for $y \geqslant 0$. It is clear from Fig. 2a that the variation of temperature $T$ decreases with an increase in gravity for $y \geqslant 0$, but for very small values it seems to be identical. This means that the effect of gravity has a small influence on the variation of temperature. Figure 2b depicts the variation of the shearing stress $\sigma_{xy}$ which increases with an increase in gravity for $y \geqslant 0$. Figure 3a explains that the variation of the microrotation vector $\phi_3$ increases in the interval $0 \leqslant y \leqslant 0.5$, while it decreases in the interval $0.5 \leqslant y \leqslant 6$, with an increase in gravity. Figure 3b determines the variation of the heat flux moment $q_{xy}$ which increases at the intervals $0 \leqslant y \leqslant 0.6$ and $1 \leqslant y \leqslant 6$, but decreases at the interval $0.6 \leqslant y \leqslant 1$ with an increase in gravity. The gravity has an effective role in the variation of all physical quantities of the problem. One can notice a change in the variation of the physical quantities while gravity is present or absent.

Figure 4a shows that the variation of the displacement component $v$ decreases in the intervals $0 \leqslant y \leqslant 0.4$, $1 \leqslant y \leqslant 1.8$ and $2.8 \leqslant y \leqslant 6$, while it increases in the intervals $0.4 \leqslant y \leqslant 1$ and $1.8 \leqslant y \leqslant 2.8$ with an increase in the initial stress. Figure 4b clarifies the variation of the microtemperature vector $w_2$ which increases with an increase in the initial stress for $y \geqslant 0$. It is clear that all functions are continuous and all the curves converge to zero. The initial stress has a significant role in the variation of all physical quantities in the problem. This can be deduced from changing of the manner of variation of the physical quantities while the effect of the initial stress is present or absent.

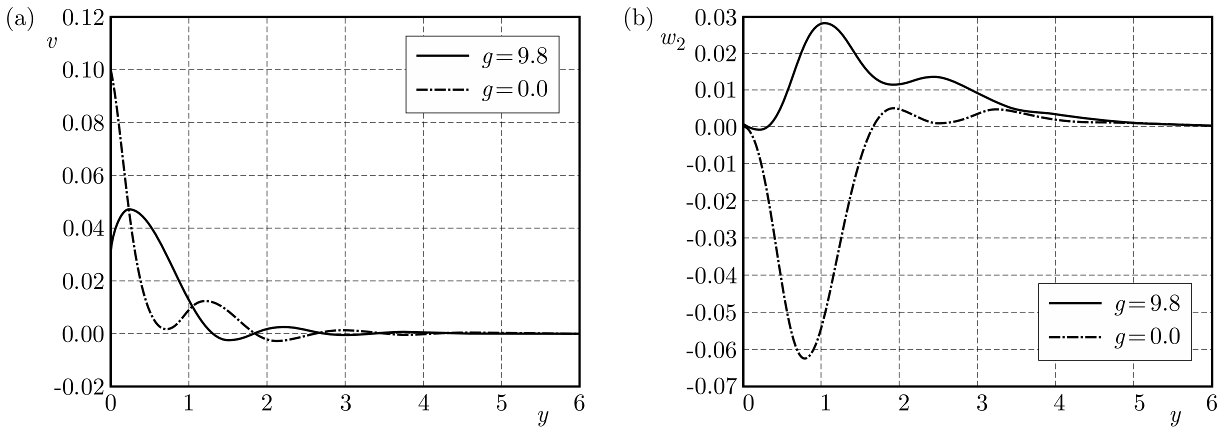


Fig. 4. Variation of displacement $v$ (a) and of microtemperature vector $w_2$ (b) against $y$

Figure 5a shows that the variation of the displacement component $v$ increases in the intervals $0 \leqslant y \leqslant 2$ and $4.4 \leqslant y \leqslant 6$, while it decreases in the interval $2 \leqslant y \leqslant 4.4$ with an increase in the micropolar thermoelasticity. It is clear from Fig. 5b that the variation of temperature $T$ decreases with an increase of the micropolar thermoelasticity for $y \geqslant 0$ in observable behavior. It is clear that all functions are continuous and all the curves converge to zero. The micropolar thermoelasticity plays an important role in the variation of all physical quantities in the problem. The micropolar thermoelasticity is a very important property in thermoelastic materials with a microstructure.

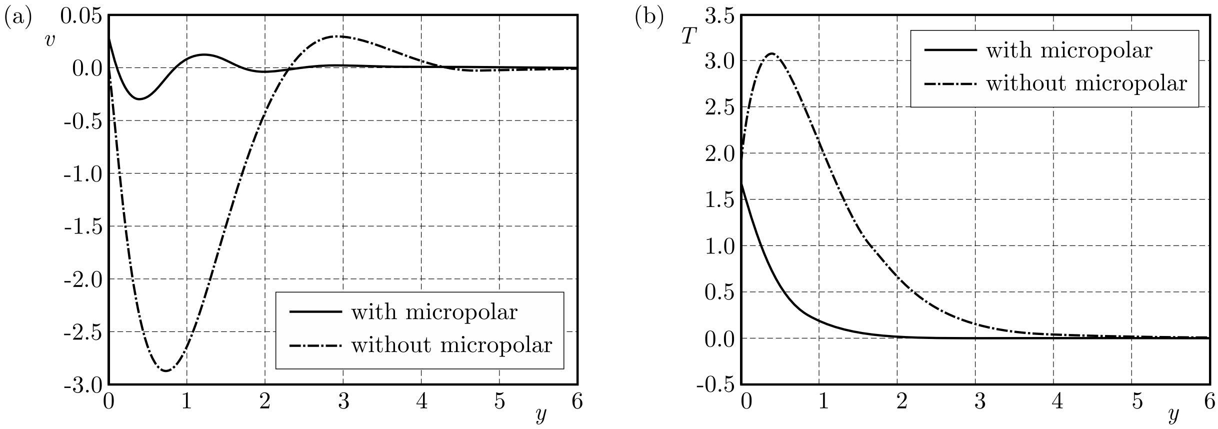


Fig. 5. Variation of displacement $v$ (a) and of temperature $T$ (b) against $y$ with and without micropolar

The 3D curves of the quantities $v$ and $w_1$ are shown in Figs. 6a and 6b for $g = 9.8\,\mathrm{m/s^2}$ and $p = 5\,\mathrm{N/m}$ with the presence of the micropolar thermoelasticity at $t = 0.5\,\mathrm{s}$. These figures depict the dependence of these quantities on the distances $x$ and $y$ while they are moving during wave propagation.

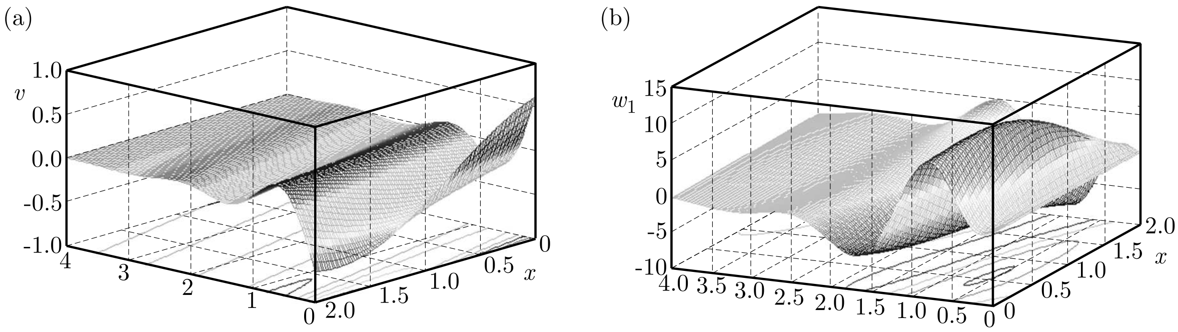


Fig. 6. Variation of displacement $v$ (a) and of microtemperature vector $w_1$ (b) versus distances $x$ and $y$

## 7. Conclusion

From the above analytical solutions, we conclude that:

1. Gravity and initial stress are effective physical factors having an important role in the variation of the physical quantities.
2. The micropolar thermoelasticity is an important property. The presence or the absence of this property is an observable effect in the variation of the considered physical quantities, for example in the variation of temperature.
3. The microtemperature is a very useful theory in the field of geophysics and earthquake engineering and for seismologists working in the field of mining tremors and drilling into the earth's crust.

4. Values of all physical quantities converge to zero with an increase in the distance $y$, and all functions are continuous.

## Appendix A

$$\psi_1(x,y,t)=\sum_{n=1}^{6} R_n \mathrm{e}^{-S_n y+\mathrm{i}(ax-\xi t)} \qquad \psi_2(x,y,t)=\sum_{n=1}^{6} A_{1n}R_n \mathrm{e}^{-S_n y+\mathrm{i}(ax-\xi t)}$$

$$q_1(x,y,t)=\sum_{n=1}^{6} A_{3n}R_n \mathrm{e}^{-S_n y+\mathrm{i}(ax-\xi t)} \qquad q_2(x,y,t)=\sum_{n=1}^{6} A_{4n}R_n \mathrm{e}^{-S_n y+\mathrm{i}(ax-\xi t)}$$

$$w_1(x,y,t)=\sum_{n=1}^{6} G_{3n}R_n \mathrm{e}^{-S_n y+\mathrm{i}(ax-\xi t)} \qquad \sigma_{xx}(x,y,t)=\sum_{n=1}^{6} G_5 n R_n \mathrm{e}^{-S_n y+\mathrm{i}(ax-\xi t)}$$

$$\sigma_{yy}(x,y,t)=\sum_{n=1}^{6} G_{6n}R_n \mathrm{e}^{-S_n y+\mathrm{i}(ax-\xi t)} \qquad \sigma_{xz}(x,y,t)=\sigma_{yz}(x,y,t)=0$$

$$m_{yz}(x,y,t)=\sum_{n=1}^{6} G_{8n}R_n \mathrm{e}^{-S_n y+\mathrm{i}(ax-\xi t)} \qquad m_{xz}(x,y,t)=\sum_{n=1}^{6} G_{9n}R_n \mathrm{e}^{-S_n y+\mathrm{i}(ax-\xi t)}$$

$$q_{xz}(x,y,t)=0 \qquad q_{xx}(x,y,t)=\sum_{n=1}^{6} G_{10n}R_n \mathrm{e}^{-S_n y+\mathrm{i}(ax-\xi t)}$$

$$q_{yy}(x,y,t)=\sum_{n=1}^{6} G_{11n}R_n \mathrm{e}^{-S_n y+\mathrm{i}(ax-\xi t)} \qquad q_{yz}(x,y,t)=0$$

## Appendix B

$$N_1=c_1+1 \qquad N_2=c_9+1 \qquad N_3=a^2-\frac{c_3\xi^2}{N_1} \qquad N_4=\frac{\mathrm{i}ac_4}{N_1}$$

$$N_5=\frac{c_3}{N_1} \qquad N_6=\mathrm{i}ac_4 \qquad N_7=a^2-c_3\xi^2 \qquad N_8=a^2+2c_5-c_7\xi^2$$

$$N_9=a^2+\frac{c_{11}-\mathrm{i}\xi c_{12}}{N_2} \qquad N_{10}=\frac{c_{13}}{N_2} \qquad N_{11}=-\mathrm{i}\xi c_{10} \qquad N_{12}=a^2+c_{11}-\mathrm{i}\xi c_{12}$$

$$N_{13}=a^2-\mathrm{i}\xi c_{14} \qquad N_{14}=\mathrm{i}\xi c_{15} \qquad A_{1n}=\frac{S_n^6-l_1S_n^4+l_2S_n^2-l_3}{N_4S_n^4-l_4S_n^2+l_5}$$

$$A_{2n}=\frac{-N_6-A_{1n}(S_n^2-N_7)}{c_2} \qquad A_{3n}=\frac{N_{10}A_{5n}}{S_n^2-N_9} \qquad A_{4n}=\frac{-N_{11}A_{2n}}{S_n^2-N_{12}}$$

$$A_{5n}=\frac{-N_{14}S_n^4+(N_9N_{14}+N_{14}a^2)S_n^2+N_9N_{14}a^2}{S_n^4-(N_9+N_{13}+c_{16}N_{10})S_n^2+N_9N_{13}-c_{16}N_{10}a^2}$$

$$A_{6n}=c_{14}(\mathrm{i}aH_{1n}-S_nH_{2n})+\mathrm{i}ac_{15}H_{1n}-A_{5n} \qquad A_{9n}=-c_{18}S_nA_{2n}$$

$$A_{7n}=c_{14}(\mathrm{i}aH_{1n}-S_nH_{2n})-S_nc_{15}H_{2n}-A_{5n} \qquad A_{10n}=\mathrm{i}ac_{18}A_{2n}$$

$$A_{8n}=c_{16}(-S_nH_{1n}+\mathrm{i}aH_{2n})+c_{17}(\mathrm{i}aH_{2n}-A_{2n}) \qquad c_1=\frac{2(\lambda+\mu)+p}{2(\mu+k^*)-p}$$

$$c_2=\frac{2k^*}{2(\mu+k^*)-p} \qquad c_3=\frac{2\rho c_0^2}{2(\mu+k^*)-p} \qquad c_4=\frac{2\rho g c_0^2}{2(\mu+k^*)-p}$$

$$c_5=\frac{2k^*c_0^2}{\gamma\omega_1^{*2}} \qquad c_6=\frac{c_0^2(k^*-p)}{\gamma\omega_1^{*2}} \qquad c_7=\frac{\rho\mu_1c_0^4}{\gamma\gamma_1T_0\omega_1^{*2}} \qquad c_8=\frac{j\rho c_0^2}{\gamma}$$

$$c_9=\frac{k_4+k_5}{k_6}\qquad c_{10}=\frac{\mu_1\gamma_1T_0}{\rho\omega_1^*k_6}\qquad c_{11}=\frac{k_2c_0^2}{k_6\omega_1^{*2}}\qquad c_{12}=\frac{bc_0}{k_6\omega_1^*}$$
$$c_{13}=\frac{k_3T_0c_0^2}{k_6\omega_1^{*2}}\qquad c_{14}=\frac{\rho C_ec_0^2}{k\omega_1^*}\qquad c_{15}=\frac{\gamma_1^2T_0}{\rho k\omega_1^*}\qquad c_{16}=\frac{k_1}{kT_0}\qquad c_{17}=\frac{\lambda}{\rho c_0^2}$$
$$c_{18}=\frac{2\mu+k^*}{\rho c_0^2}\qquad c_{19}=\frac{\mu+p}{2\rho c_0^2}\qquad c_{20}=\frac{2(k^*+\mu)-p}{2\rho c_0^2}\qquad c_{21}=\frac{k^*}{\rho c_0^2}$$
$$c_{22}=\frac{\gamma\omega_1^{*2}}{\rho c_0^4}\qquad c_{23}=-k_4\mu\omega_1^*\qquad c_{24}=-(k_5+k_6)\mu\omega_1^*\qquad c_{25}=-k_5\mu\omega_1^*$$
$$c_{26}=-k_6\mu\omega_1^*\qquad l_1=N_3+N_9+N_{13}-c_{16}N_{10}-N_5N_{14}$$
$$l_2=N_3(N_9+N_{13}-c_{16}N_{10})+N_9N_{13}-c_{16}N_{10}a^2-N_5N_{14}(N_9+a^2)$$
$$l_3=N_3(N_9N_{13}-c_{16}N_{10}a^2)-N_5N_9N_{14}a^2\qquad l_4=N_4(N_9+N_{13}-c_{16}N_{10})$$
$$l_5=N_4(N_9N_{13}-c_{16}N_{10}a^2)\qquad G_{1n}=(\mathrm{i}a-S_nA_{1n})\qquad G_{2n}=-(S_n+\mathrm{i}aA_{1n})$$
$$G_{3n}=\mathrm{i}aA_{3n}-S_nA_{4n}\qquad G_{5n}=c_{17}(\mathrm{i}aG_{1n}-S_nG_{2n})+\mathrm{i}ac_{18}G_{1n}-A_{5n}-p$$
$$G_{4n}=-(S_nA_{3n}+\mathrm{i}aA_{4n})\qquad G_{6n}=c_{17}(\mathrm{i}aG_{1n}-S_nG_{2n})-S_nc_{18}G_{2n}-A_{5n}-p$$
$$G_{7n}=-c_{19}S_nG_{1n}+\mathrm{i}ac_{20}G_{2n}-c_{21}A_{2n}\qquad G_{8n}=-c_{22}S_nA_{2n}$$
$$G_{9n}=\mathrm{i}ac_{22}A_{2n}\qquad G_{10n}=c_{23}(\mathrm{i}aG_{3n}-S_nG_{4n})+\mathrm{i}ac_{24}G_{3n}$$
$$G_{11n}=c_{23}(\mathrm{i}aG_{3n}-S_nG_{4n})-S_nc_{24}G_{4n}\qquad G_{7n}=-c_{15}S_nG_{3n}+\mathrm{i}ac_{26}G_{4n}$$
$$n=1,2,\ldots,6$$

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